Decentralized code construction and network coding for multicast with a cost criterion

Overview

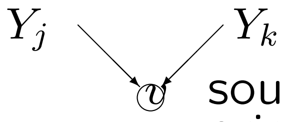
- Randomized construction and its error behavior
- Performance of distributed randomized construction - case studies
- Traditional methods based on flows - a review
- Trees for multicasting - a review
- Network coding with a cost criterion - flow-based methods for multicasting through linear programming

- Distributed operation - one approach
- A special case - wireless networks
- Sample ISPs

Linear network coding

$$\begin{tikzcd} Y_j \arrow[dr] & & Y_k \arrow[dl] \\ & v & \end{tikzcd}$$

Linear network coding for multicast



source X_i
originating at v

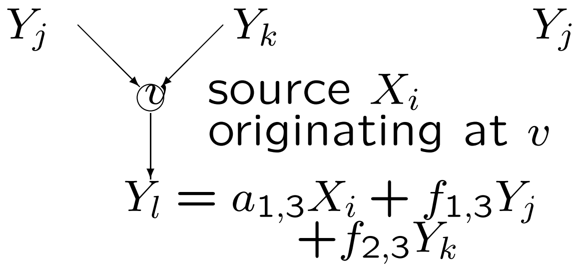
Linear network coding for multicast

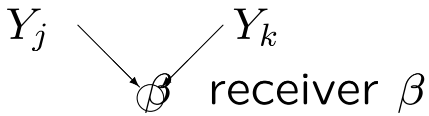
$$\begin{tikzcd} Y_j \arrow[dr] & & Y_k \arrow[dl] \\ & v \arrow[d] & \\ & Y_l & \end{tikzcd}$$

source X_i
originating at v

$$Y_l = a_{1,3}X_i + f_{1,3}Y_j + f_{2,3}Y_k$$

Linear network coding for multicast





Linear network coding for multicast

Y_j Y_k

v source X_i originating at v

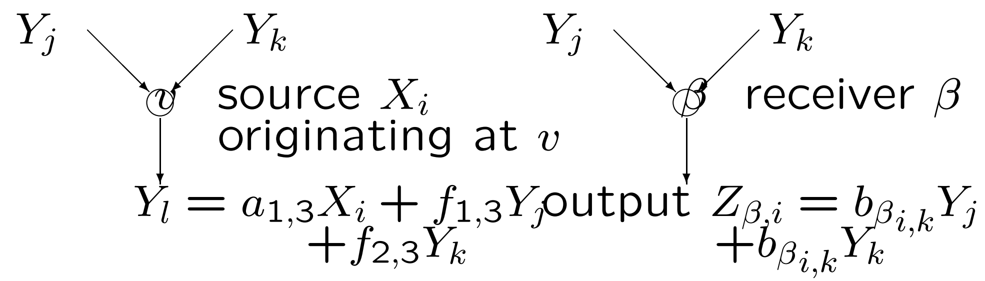
$Y_l = a_{1,3}X_i + f_{1,3}Y_j + f_{2,3}Y_k$

Y_j Y_k

β receiver β

output $Z_{\beta,i} = b_{\beta_{i,k}}Y_j + b_{\beta_{i,k}}Y_k$

Linear network coding for multicast

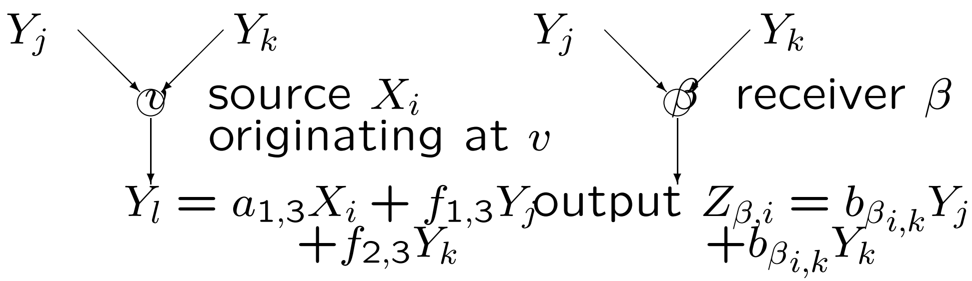


- Coefficients $\{a_{i,j}, f_{l,j}, b_{\beta_{i,l}}\}$ give network-constrained transfer matrices $(A, F, \{B_\beta\})$, a network code

- Matrix $M_\beta = A(I-F)^{-1}B_\beta^T$ gives transfer function from sources to outputs [KM01]:

$$[X_1\ X_2 \dots X_r]\ M_\beta = [Z_{\beta,1}\ Z_{\beta,2} \dots Z_{\beta,r}]$$

Linear network coding for multicast



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Feasibility and code construction

Determining feasibility

- min-cut max-flow bound satisfied for each receiver [ACLY00]
- transfer matrix $A(I-F)^{-1}B_{\beta}^{T}$ for each receiver β is non-singular [KM01]

Constructing linear solutions

- Centralized

- Direct algebraic solution using transfer matrix of [KM01]
- Algorithms using subgraph consisting of flow solutions to individual receivers [SET03, JCJ03]

- Decentralized
  - A distributed randomized network coding approach [HKMKE03]

Randomized network coding

- Interior network nodes independently choose random linear mappings from inputs to outputs

- Coefficients of aggregate effect communicated to receivers

Randomized network coding

- Interior network nodes independently choose random linear mappings from inputs to outputs
- Coefficients of aggregate effect communicated to receivers
- Receiver nodes can decode if they receive as many independent linear combinations as the number of source processes

Success probability

[HKMKE03, HMSEK03] For a feasible d-receiver multicast connection problem on a network with

- independent or linearly correlated sources
- a network code in which code coefficients $a_{i,j}$, $f_{l,j}$ for η links are chosen independently and uniformly over $\mathbb{F}_q$

the success probability is at least $(1 - d/q)^\eta$ for $q > d$. Error bound is of the order of the inverse of the field size, so error probability decreases exponentially with codeword length

Proof outline

- Recall transfer matrix $M_\beta = A(I-F)^{-1}B_\beta^T$ for each receiver β must be non-singular

- We show an equivalent condition connected with bipartite matching: the Edmonds matrices $\begin{bmatrix} A & 0 \\ I-F & B_\beta^T \end{bmatrix}$ (in the acyclic delay-free case) or $\begin{bmatrix} A & 0 \\ I-DF & B_\beta^T \end{bmatrix}$ (in the case with delays) are non-singular

- This shows that if η links have random coefficients, the determinant polynomial

- has maximum degree η in the random variables $\{a_{x,j}, f_{i,j}\}$

- is linear in each of these variables

Proof outline (cont'd)

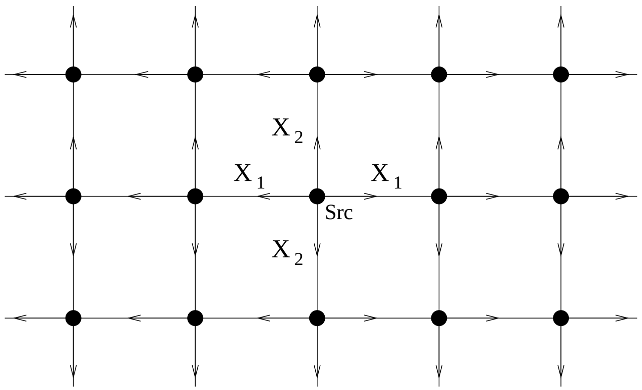
- We want the product of the d receivers' determinant polynomials to be nonzero

- We can show inductively, using the Schwartz-Zippel Theorem, that for any polynomial $P \in \mathbb{F}[\xi_1, \xi_2, \ldots]$ of degree $\leq d\eta$, in which each ξ_i has exponent at most d, if $\xi_1, \xi_2, \ldots$ are chosen independently and uniformly at random from $\mathbb{F}_q \subseteq \mathbb{F}$, then $P = 0$ with probability at most $1 - (1 - d/q)^\eta$ for $d < q$

- Particular form of the determinant polynomials gives rise to a tighter bound than the Schwartz-Zippel bound for general polynomials of the same total degree

Utility of distributed network coding

- Decentralized scenarios



Receiver position		(2,4)	(4,4)	(8,10)	(10,10)
Randomized flooding upper bound		0.563	0.672	0.667	0.667
Randomized Coding	$\mathbb{F}_{2^6}$ lower bound	0.882	0.827	0.604	0.567
	$\mathbb{F}_{2^8}$ lower bound	0.969	0.954	0.882	0.868