Summary of: "Optimal, systematic, q-ary codes correcting all asymmetric and symmetric errors of limited magnitude"

I. Background

In multi-level flash memories, each cell can be in one of q levels, hence storing $\log_2 q$ bits of information. If, due to some physical effect, cell levels are changed, errors are introduced to the stored bits. It has been observed in [3],[4], that the common error mechanisms in flash devices are much more likely to introduce errors with small magnitudes compared to the full range of levels. In addition, due to the inherent asymmetry between programming (adding charge) and erasing (removing charge) of flash cells, errors in one direction may be significantly more frequent than errors in the other direction. These two facts motivate the study of q-ary codes for asymmetric limited-magnitude errors.

Asymmetric limited-magnitude errors were first addressed in [1]. In both [1] and the subsequent work [2], the number of asymmetric limited-magnitude errors T is always as large as the number of symbols in the block N. This special case of $T = N$ is called "all asymmetric limited-magnitude errors", and is also the case of study in the current paper [5].

For practical flash devices, it is the general case of $T \leqslant N$ that seems more applicable. Errors are not expected to happen in every cell, and so implementing a code for $T = N$ is wasteful in terms of redundancy. The general case $T \leqslant N$ was studied in [4]. Among other contributions, constructions for *systematic* codes were first proposed. Systematic codes are codes in which K of the q-ary symbols are pure information, and the remaining $N - K$ symbols are parity. The main motivation to implement a systematic code is to lower the number of cells to be updated after an information update.

II. Contributions

The current paper [5] proposes a systematic code for the case $T = N$, and shows that the resulting number of parity symbols is optimal. In the original non-systematic code [2], each cell in the block independently protects itself from the errors by storing one of only $\lceil q/(\ell+1) \rceil$ levels, spaced at $\ell + 1$ levels between adjacent usable levels (ℓ is the magnitude-limit of the asymmetric errors). The objective of the current paper is to have K information cells that can be at any of the q levels, plus $N - K$ parity cells whose contents allow recovering the information in the presence of the errors occurring in both the information and parity cells.

The proposed code, which has a nicely proved matching lower bound, turns out to be quite simple. Rather than constraining the modulo $\ell + 1$ of each cell (as in the original $T = N$ code), we group these values and store them in the dedicated parity symbols. To ensure access to these correct values in the presence of parity-symbol errors, we use only $\lceil q/(\ell+1) \rceil$ of the levels at the parity symbols. Therefore, the number of q-ary parity symbols can easily be found to be:

$$N - K = \frac{K \log(\ell + 1)}{\log \left\lceil \frac{q}{\ell+1} \right\rceil}$$

III. Discussion

While the scope of the paper is exclusively the $T = N$ case, this simple technique can be generalized to transform more practical $T < N$ codes into systematic form. However, this approach is inferior (in terms of rate K/N) to the systematic construction of [4] for all but very low-rate codes. Given a non-systematic code with k pure-information symbols and $n - k$ mixed information-parity symbols, we denote $\rho = k/n$. Given the parameters q and ℓ, it is possible to find the cutoff ρ_0, such that the construction of [4] is superior to the generalization of [5] for every $\rho > \rho_0$. For example, for $q = 32$ and $\ell = 3$, it can be found that $\rho_0 = 0.5$. The cutoff happens at lower ρ values as the error-magnitude level increases. For example if $q = 32$ and $\ell = 6$, then $\rho_0 = 0.356$. Hence the method that generalizes the construction of [5] applies only to cases where a significant fraction of the block cells are expected to have asymmetric limited-magnitude errors.

References

[1] R. Ahlswede, H. Aydinian, and L. Khachatrian, "Unidirectional error control codes and related combinatorial problems," in *proc. of the Eighth International Workshop on Algebraic and Combinatorial Coding Theory (ACCT-8), St. Petersburg, Russia (Extended version available at: http://arxiv.org/abs/cs/0607132)*, 2002, pp. 6–9.

[2] R. Ahlswede, H. Aydinian, L. Khachatrian, and L. Tolhuizen, "On q-ary codes correcting all unidirectional errors of a limited magnitude," Arxiv.org, http://arxiv.org/abs/cs/0607132, Tech. Rep. cs.IT/0607132, 2006.

[3] Y. Cassuto, M. Schwartz, V. Bohossian, and J. Bruck, "Codes for multi-level Flash memories: correcting asymmetric limited-magnitude errors," in *Proc. of the IEEE International Symposium on Info. Theory*, Nice, France, June 2007, pp. 1176–1180.

[4] ——, "Codes for asymmetric limited-magnitude errors with application to multilevel flash memories," *IEEE Transactions on Information Theory*, vol. 56, no. 4, pp. 1582–1595, 2010.

[5] N. Elarief and B. Bose, "Optimal, systematic, q-ary codes correcting all asymmetric and symmetric errors of limited magnitude," *IEEE Transactions on Information Theory*, vol. 56, no. 3, pp. 979–983, 2010.