Appraisal of paper: Channel coding rate in the finite block length regime by Y. Polyanskiy, H.V. Poor and S. Verdu

Brief description of paper:

With 53 transactions pages and 67 Theorems, Lemmas and Corollaries, it is difficult to summarize the paper and do justice to it. It looks at the maximum channel coding rate achievable at a given finite block length and error probability, in contrast to the usual asymptotic result. Upper and lower bounds are given on the size of any code for a given block length and error probability (referred to as converse and achievability bounds). These bounds are tighter than existing bounds for wide ranges of parameter sets and block lengths as short as 100. One key result shows analytically that the maximum rate achievable with error probability ϵ with block length n is closely approximated by

$$C - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon)$$

where C is the channel capacity V is a quantity measuring channel dispersion and Q is the complementary Gaussian cumulative distribution function. The bounds will prove useful in determining the highest rate achievable for a given block length and error probability.

Section II reviews previous work on the problem. Section III gives new results on achievability of rates (lower bounds) using random coding, random coding union bound, dependence testing bound, and for converses (upper bounds), maximal probability of error, average probability of error and maximal probability of error. While the bounds are for quite general channels, the specific cases of BSC, BEC and AWGN are given as examples of the theory. Curves of rate vs block length for a given channel parameter and error probability demonstrate the closeness of the bounds obtained even for block lengths down below 200. Section IV considers an asymptotic analysis of the achievable rate for given block length. The quantity *channel dispersion* is defined (measured in squared information units per channel use) and it is shown for both DMC's and Gaussian channels that

$$\log M^*(n, \epsilon) = nC - \sqrt{nV} Q^{-1}(\epsilon) + O(\log n).$$

The paper makes numerous contributions to existing knowledge and draws many conclusions on the results. Of particular interest (to me) is the fact that the tightness of the bounds (upper and lower) on rates for random like codes (such as for many LDPC type codes) 'not only achieve capacity but also do not sacrifice much performance for all but very short block lengths. Numerical results with state-of-the-art codes show that about one half of the gap to capacity is due to the fundamental back off due to finite block length. The other half is bridgeable with future advances in coding theory'.

COMMENTS:

The work seems particularly appropriate given the recent advances with codes that approach capacity, albeit at very large block lengths. So much of work in the area is for the asymptotic case and the approach of this paper is long overdue.

It is a truly impressive and masterful piece of work, comprehensive in its scope, meticulous in its detail and very focused in its approach. It is also a very relevant and important piece of work, given the recent advances on capacity achieving codes at large block lengths. A problem that will receive much attention in the future will be to achieve similar performance as for longer block lengths, at shorter block lengths, and this work will serve as a guide as to how much back off is sacrificed by the shorter code lengths.

The breadth and depth of the results of this paper are unsurpassed in my experience. The importance of the problem addressed in this work and the depth of the results obtained might place it slightly above the paper of Borade et al which I also ranked very highly.

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