

Report on "Feedback capacity of stationary Gaussian channels" by Young-Han Kim

This paper (referred to as [Kim] below) deals with the feedback capacity for stationary Gaussian channels.

The followings are main outcomes of this paper.

(a) Prove that stationary sources achieve the feedback capacity and derive the related capacity formula (Theorem 3.2).

(b) Prove that the Butman-Schalkwijk-Kailath coding scheme achieves the capacity (Theorem 6.1).

About (a), some useful backgrounds are available. In particular, the two references listed below address the problem for the ARMA case. Theorem 6 in [YKT] together with Proposition 5 in [LE] gives the feedback capacity formula for an ARMA process. Theorem 6 in [YKT] (refined in Proposition 5 in [LE]) can be seen as a time domain equivalence of the frequency domain result of Theorem 3.2 in [Kim]. The main difference is that Kim's results are more general, applicable to not only ARMA processes but also general stationary ones. I believe that the proof of the "stationarity" part is the main contribution of Kim. I regard this as a very clever technique that circumvents the difficulty related to the Cuff-Permuter-Cover approach.

In summary, I basically agree with Gerhard with his following statement.

"Anyone who worked on the feedback problem (there were/are not as many as for the broadcast channel) likely expected that stationary distributions were optimal, and expected that Butman's coding scheme was optimal. The "stationarity" part didn't turn out to be too difficult to prove if one starts with the Szego-Kolmogorov-Krein Theorem (see the proof of Thm. 3.2). But after that it took some work to get to Theorem 5.3 which states that Butman's code is optimal."

About (b), [LE, Proposition 9] showed that the Schalkwijk-Kailath coding method is optimal. This is for ARMA processes, so is [Kim]. Therefore, they appear equivalent, although they may approach the problem from different angles: LE stayed with ARMA from the beginning to end, whereas Kim from stationary to ARMA.

I noticed that [LE] has not been reference in the [Kim]. In my view, [LE] should be referenced provided if Kim is aware of this paper, since their scopes are similar. I noticed that there is an acknowledgement in [LE] (footnotes in page 1, published in 2005) on useful discussions with Kim. This implies that Kim may be aware of [LE].

As a final remark, I have tried my best to understand these papers but I have never worked on these topics before. I would put the confidence level of this

report to about 70%.

References:

[LE] J. Liu and N. Elia, "Achieving the stationary feedback capacity for Gaussian channels." Proc. 43rd Annual Allerton Conf. on Communication, Control, and Computing, Sept. 2005. Available at

http://cslgreenhouse.csl.illinois.edu/allertonarchives/allerton05/PDFs/Papers/I_H_7.pdf.

[YKT] S. Yang, A. Kavčič, and S. Tatikonda, "Feedback capacity of finite state machine channels," IEEE Trans. Inf. Theory, vol. IT-51, no. 3, pp. 799–810, Mar. 2005.