Comments on "Information Spectrum Approach to Second-Order Coding Rate in Channel Coding" by M. Hayashi

This paper presents some original ideas concerning the behavior of the probability of error of channel codes for rates in the vicinity of channel capacity. The results are presented in terms of the second-order coding rate. The author extends his own results of similar nature on source coding [6], using the method of information spectrum proposed by Han and Verdu' [4] and developed by Han [5]. It is apparent that the author was not aware of the work of Strassen in 1962 [3], and was alerted about it by a colleague.

In this extension to channel codes, the author reports on the difficulties of applying the method to channels with cost constraints, and indicates the way around the problem by considering the log likelihood ratio between the conditional output distribution and a defined distribution.

The results seem to be very innovative and shed light on some nebulous areas such as the behavior of the Gallager bound in the neighborhood of capacity, giving examples in which that behavior is not optimal, contrarily to common belief (cf. Fig. 2).

The use of the language is somewhat nonconventional and the paper is not easy to read. A second reading helps, and so does a third. Some definitions are given in unusual order. It is also a more mathematical paper in nature, rather than an engineering oriented paper, like that of Polyanskiy et al.

Using the information spectrum method, the author clarifies the evolution of probability of error for various classes of channels and derives the optimum second-order transmission rate for DMCs, DMCs with cost constraints, a class of Markov channels and the additive Gaussian channel with energy constraint . The author also comments about the interest on the behavior of third-order coding rates, and on the associated difficulties. It is still not clear whether these rates evolve as O(1) or as O(log n), according to the author.

The paper shows only five citations on Google Scholar. This number is probably going to rise significantly as the paper gets more exposure. The results are genuinely novel and interesting, like those of Polyanskiy et al., that also shed light on this difficult area. It is somewhat interesting that the earlier results occurred on the source channel version of the problem [6], and then on channel coding. In many other advances, the order seems to have been reversed.

This paper was originally submitted in August 2008, and is therefore essentially concurrent with the paper by Polyanskiy et al, submitted in November 2008. I think the results of this paper are similar in nature to those of Polyanskiy et al., and complementary to that work in some aspects. The combined papers form a strong entry to the IT Paper Award.

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