Frequency-shift keying for ultrawideband - achieving rates of the order of capacity

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Abstract

In wideband systems that decorrelate in time and frequency, capacity can be reached in the limit of infinite bandwidth by using impulsive frequency-shift keying (FSK) with vanishingly small duty cycle. The richness of the codebook is then created through the large bandwidth, thus requiring very large bandwidths in order to approach capacity. We show that FSK with small duty cycle can achieve rates of the order of capacity in ultrawideband systems with limits on bandwidth and peak power. The richness of the codebook is then built in both time and frequency. We consider single tone and two-tone FSK schemes. Our results indicate that, for typical wireless conditions, we can achieve rates within a few dB of the energy-limited, infinite-bandwidth capacity with these simple FSK schemes. At high signal-to-noise ratios (SNRs), the FSK schemes are bandwidth-limited rather than energy-limited. In the high SNR regime, multi-tone FSK is significantly less bandwidth-limited than single-tone FSK.

1 Introduction

The capacity of the infinite-bandwidth general fading multipath-fading channel is nonzero and is equal to the capacity of the infinite-bandwidth additive white Gaussian noise (AWGN) channel with the same average received power constraint. This result has been shown by Kennedy[6], Gallager[4, §8.6] and Telatar and Tse[12]. The proofs are constructive and use frequency-shift keying (FSK) with non-coherent detection in a system that transmits at a vanishingly low duty cycle. This capacity-achieving transmission scheme, which we term impulsive FSK, is thus "peaky" both in frequency and time, a special case of flash signaling ([13]). Such a scheme is quite different from the wideband spread-spectrum schemes that create signals that mimic white Gaussian noise (WGN), such as would be optimal for infinite bandwidth non-fading channel. Indeed, the use of signals akin to WGN, which we term bandwidth-scaled, with moments that scale inversely with bandwidth, has been shown to yield a vanishing capacity in the limit of infinite bandwidth ([12], [9], [11]).

While the results of [6] and [12] only provide a capacity-achieving method for vanishingly small duty cycle and infinite bandwidth, the results do not show how and how fast a system can approach this limit. Recent results show that this limit is approached slowly. In [13], by considering the relation among SNR, capacity and spectral efficiency, Verdú shows that approaching capacity may require extremely large bandwidths and peak-to-average signal ratios. A similar conclusion is reached by Lun, Médard and Abou-Faycal in [2], [1] by considering the per codeword probability of error as a function of peak energy, duty cycle and total bandwidth for Rayleigh and general fading channels using impulsive FSK.

From the above discussion emerge two main themes. First, bandwidth-scaled signals, of the type used in commercial direct-sequence code-division multiple access (DS-CDMA) systems, perform poorly in the wideband regime for fading channels. Second, the optimal schemes in the infinite bandwidth regime require extremely high bandwidth and peak power to approach capacity. Both of these results may be viewed as negative results, indicating what certain types of signaling cannot do. What, then, is the appropriate way of transmitting for ultrawideband communications, over bandwidths of the order of several GHz? For such bandwidths and moderate signal energy, bandwidth-scaled signals perform poorly. On the other hand, the limitations on bandwidth and peak power in practical systems are well below those required to approach the infinite-bandwidth capacity.

Our goal in this paper is to present a signaling scheme which achieves a capacity of the order of the infinite-bandwidth capacity for bandwidths and peak power constraints that are large but well below those required to achieve the infinite bandwidth capacity. We study the performance of single-tone and multi-tone FSK schemes in Rayleigh fading channels. These schemes maintain some of the characteristics of impulsive FSK, but build the codebook in time as well as frequency, thus obviating the need for the vanishingly small duty cycles of flash signaling. Our FSK scheme assumes no channel side information at the sender or at the receiver. The multi-tone FSK schemes make greater use of the diversity in frequency and can be applied at lower bandwidths than traditional single-tone FSK schemes.

We consider a Rayleigh fading channel that decorrelates in time and frequency. We study the interplay among the capacity, bandwidth, power, and "peakiness" of the scheme. We show that single and multi-tone FSK schemes with limited bandwidth and "peakiness" can achieve performance that is of the same order of that of the scheme with infinitely large bandwidth and vanishing duty cycle.

2 System model

The transmission scheme we examine is an impulsive FSK with small duty cycle and large bandwidth. The system is studied in Rayleigh fading channel conditions, which are common in wireless communication scenarios. Related work has considered the capacity of FSK schemes. In [10], Stark determined the capacity of FSK schemes under non-selective Rician fading with receiver side information. A similar system is considered in [8] for channels with erasures. A simulator for obtaining capacity for binary FSK for a general FIR filter was considered in [7]. However, we consider channels with decorrelation in time and bandwidth and thus none of the above results apply to our model.

We send single-frequency signals which are selected from a large set of frequencies and transmitted using a low duty cycle. Because we use a large set of frequencies in a wide bandwidth, the frequency difference between two successive symbols we assume to be greater than the coherence frequency. Moreover, the low duty cycle means successive symbols are generally separated in time by more than a coherence time. Hence, the probability of sending two successive signals within a coherence band in the same coherence time is negligible. We can thus assume different symbols experience independent fading.

Assume the FSK system has M frequencies. In each symbol time, a signal may be or be not transmitted according to a probability θ $(0 < \theta \leq 1)$. θ is the duty cycle. In the interval in which some signal is transmitted, one of the M frequencies is sent. The transmitted signal can expressed as

$$x(t) = \begin{cases} \exp(2\pi i f_m t), & 0 \leq t \leq T_s; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

where $f_m (1 \leq m \leq M)$ is the frequency of FSK signal and T_s is the length of symbol interval. The received signal is

$$y(t) = \alpha(t) \sqrt{P} \exp\{2\pi i f_m t\} + n(t) \tag{2}$$

where $\alpha(t)$ is a circularly symmetric complex Gaussian process, K is the number of paths, P is the power of the signals, and $n(t)$ is circularly symmetric complex Gaussian process with double-sided power density $\frac{N_0}{2}$ per dimension.

The coherence time T_c is the duration of time over which the channel remains essentially time-invariant. The delay spread T_d represents the uncertainty in the delay of the paths. In this paper, we focus on the case where the symbol time, T_s, is much less than the coherence time of the channel and the delay spread is less than the symbol time. During the interval $[T_d, T_s]$, we can assume $\alpha(t) = A$ is constant, then the expression of received signal is

$$y(t) = A\sqrt{P} \exp\{2\pi i f_m t\} + n(t) \tag{3}$$

where A is circularly symmetric Gaussian variable. Without loss of generality, A can be assumed as a complex Gaussian variable with variance $\frac{1}{2}$ per dimension.

At the receiver, we use a bank of matched bandpass filters with central frequencies $\{f_n\}$ to detect signals. $n\,(1 \leq n \leq M)$ is the index of frequencies. In a certain symbol slot i, the output of the nth matched filter is

$$\widetilde{y}_n(t) = \int_{(i-1)T_s}^{t} y(\tau) e^{2\pi j f_n (t-\tau)} d\tau \quad (i-1)\,T_s \leq t \leq iT_s. \tag{4}$$

Let the frequency difference between two adjacent f_n's be $F_s = \frac{1}{T_s - T_d}$ and f_n be an integer multiple of F_s. The whole bandwidth of the FSK system is $F = \frac{M-1}{T_s - T_d}$. When the mth symbol is sent, the outputs of the nth filter at time $(T_s - T_d) + iT_s$ are

$$\widehat{y}_n = \delta_{n,m} A\sqrt{P}(T_s - T_d) + \widehat{v}_n \tag{5}$$

where $\widehat{v}_n$ is a complex Gaussian variable. The $\widehat{v}_n$s are mutually independent and $Cov[\widehat{v}_n] = (T_s - T_d)\,N_0$. The normalized output of the nth matched filter is

$$R_n = \frac{1}{\sqrt{N_0\,(T_s - T_d)}} \widehat{y}_n = S_n + Z_n \tag{6}$$

where Z_n is a complex Gaussian variable with variance $\frac{1}{2}$ per dimension. Let $\zeta = \frac{P(T_s - T_d)}{N_0}$. When $n = m$, S_n is a complex Gaussian variable with variance $\frac{\zeta}{2}$ per dimension, otherwise S_n is 0.

When the mth signal in the M frequencies is transmitted, the received $|R_m|^2$ has the probability density given by (7), otherwise, $|R_n|^2$ $(n \neq m)$ has the density given by (8).

$$P_{|R_n|^2}(r) = \frac{1}{1+\zeta} \exp\left[\frac{-r}{1+\zeta}\right] \qquad (r > 0) \tag{7}$$

$$p_{|R_n|^2}(r) = \exp[-r] \qquad (r > 0)\,. \tag{8}$$

Keeping the system's average power constant, a change in the duty cycle parameter θ will affect the signal power P.

To decide which signal was transmitted, we use the maximum-a-posteriori (MAP) rule based on the observation of $|R_n|^2$ at the receiver. The probability system transmits nothing in a symbol slot is $1-\theta$. Assume the M signals have equal probabilities to be transmitted, then the probability of transmitting the mth $(m = 1, 2, ..., M)$ signal is $\frac{\theta}{M}$.

When there is no signal being transmitted in a slot, the joint probability density of $\left(|R_1|^2, |R_2|^2, .., |R_M|^2\right)$ in the slot is

$$p_{|R_1|^2|R_2|^2,..,|R_M|^2}(r_1, r_2, ..., r_M) = \prod_{i=1}^{M} \exp(-r_i). \tag{9}$$

Otherwise, if one signal m is sent, the joint probability density in the slot should be

$$p(r_1, r_2, ..., r_M) = \frac{1}{1+\zeta} \exp\left[\frac{-r_m}{1+\zeta}\right] \prod_{i=1, i\neq m}^{M} \exp(-r_i). \tag{10}$$

The threshold used for MAP rule is

$$Z = \frac{\zeta+1}{\zeta} \ln\left((1+\zeta)\frac{(1-\theta)M}{\theta}\right). \tag{11}$$

If no $|R_n|^2$ is greater than Z, the receiver will decide that no signal was transmitted. Otherwise, the receiver decides that the n corresponding to the largest $\{|R_n|^2\}$ was transmitted.

We can determine the probability of error in this detection scheme. Let P_1 denote the probability of missing a signal, i.e. the receiver decides that nothing has been transmitted when a signal was transmitted. P_1 is

$$P_1 = \left(1 - \exp\left[\frac{-Z}{1+\zeta}\right]\right)(1 - \exp(-Z))^{M-1}. \tag{12}$$

Let P_2 denote the error probability of detecting a transmission in error, i.e. the receiver decides one signal has been transmitted when actually another signal was transmitted. P_2 is given by

$$\begin{aligned} P_2 &= \int_Z^\infty \left(1 - (1-\exp(-x))^{M-1}\right)\left(\frac{\exp\left[\frac{-x}{1+\zeta}\right]}{1+\zeta}\right) dx \\ &+ \left(1 - \exp\left[\frac{-Z}{1+\zeta}\right]\right)\left[1 - (1-\exp[-Z])^{M-1}\right]. \end{aligned} \tag{13}$$

Another kind of error occurs when the receiver decides that a signal has been transmitted when nothing was transmitted. The probability of this type of error is denoted by P_3:

$$P_3 = 1 - (1 - \exp(-Z))^M. \tag{14}$$

The additive noise is stationary and white. The complex gain of the channel, A, decorrelates in any two symbol slots because the symbols experience independent fading. We compute the capacity of this system using a discrete memoryless channel(DMC) model.

In each symbol slot, we choose to transmit nothing or one of M signals. Hence, the discrete model has an input alphabet with size $M+1$, denoted as a_0, a_1, a_2, ..., a_M. a_0 means no signal is transmitted, and a_j $(1 \leq j \leq M)$ means the jth FSK signal is transmitted. The output is decided by the vector $\{|R_1|^2, |R_2|^2, |R_3|^2, ..., |R_M|^2\}$.

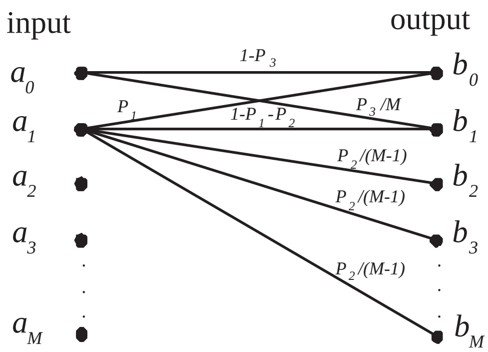

Figure 1: The discrete mode

The capacity of this DMC channel is

$$C = H\left(\bar{b}\right) - H\left(\bar{b}|\bar{a}\right). \tag{15}$$

The capacity of the FSK system is $\frac{C}{T_s}$. In the following section, we use numerical methods to calculate this capacity using the error probabilities given above.

3 Bounds on capacity

Before we discuss the capacity of our system, we first consider some bounds on this capacity. From [12], we know that the capacity of the Rayleigh fading channel is the same as that of the AWGN channel in the limit of infinite bandwidth. So a tight bound on capacity is

$$C = \frac{T_s - T_d}{T_s} F \ln(1 + \frac{P}{2N_0 F}) \tag{16}$$

where F is the bandwidth of the system, P is the average signal power, N_0 is single-sided power density per dimension for additive noise. The factor $\frac{T_s - T_d}{T_s}$ is introduced because the effective time of transmission is $[T_d, T_s]$. When F goes to infinity, the bound approaches $(1 - \frac{T_d}{T_s})\frac{P}{2N_0}$. We term this bound the *limited energy bound*.

Another bound on capacity is deduced from the discrete model of the system. Owing to the limited number of input symbols, an upper bound on capacity is given by

$$C \leq \frac{\ln(M+1)}{T_s}. \tag{17}$$

This bound is tight when power is very large, as we shall see in following discussion. We denote this bound as the *limited bandwidth bound*.

4 Numerical results for FSK

A high peak signal power makes transmission reliable. When the bandwidth is very large, which means there is a large number of transmission symbols, we need to improve the peak power of the signal to get reliable transmission. However, when the average received signal power is constant, we should lower the signal duty cycle in order to improve the peak signal power, which will put a limitation on the data rate. Hence, we need to adjust the duty cycle parameter θ to optimize the system capacity. In our simulation, all results are optimized with respect to θ.

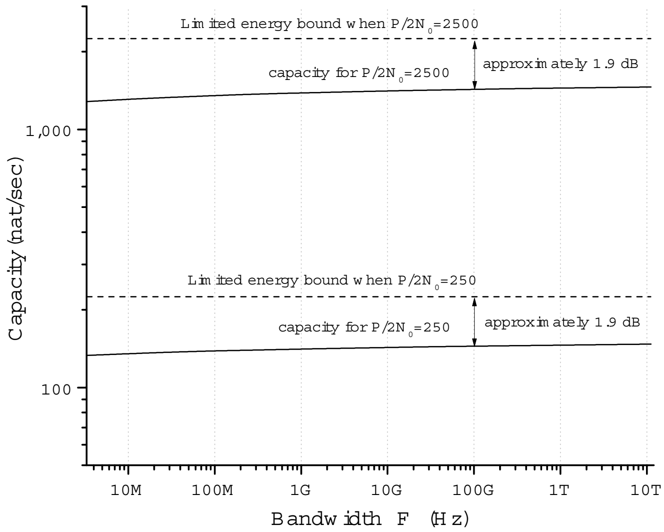

Figure 2: Capacity vs. System Bandwidth. $T_s = 10\mu s$, $T_d = 1\mu s$.

In Figure 2, we fix $\frac{P}{2N_0}$ and let the symbol time T_s be $10\mu s$, and find, as expected, that the capacity increases with the bandwidth of system. However, it grows very slowly, and has roughly a gap of 2dB with the infinite bandwidth bound when the system bandwidth is between $1MHz$ and $10THz$. Note that our scheme achieves at moderate bandwidths capacities very close to those achievable under very large bandwidths.

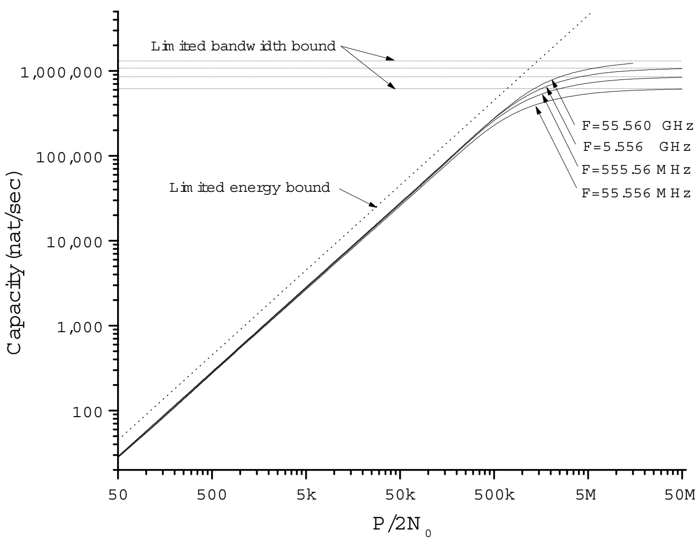

Figure 3: Capacity vs. $\frac{P}{2N_0}$ performance. $T_s = 10\mu s$, $T_d = 1\mu s$.

Figure 3 shows the capacity vs. power performance under different bandwidth constraints. We see that it is only for very large power that the limited bandwidth bound is below the limited energy bound on the capacity. Thus, for moderate received power, the limitation in bandwidth does not hamper our scheme, which achieves capacities of the order of the limited energy bound.

The symbol time T_s is also an important parameter for the FSK system. On the one hand, the greater T_s, the greater ζ, which will reduce the error probability, and thus improve capacity. On the other hand, increasing T_s will lower the symbol rate. If the average power and bandwidth are fixed, we can adjust T_s to maximize the capacity. With two different bandwidths, we show how symbol time T_s affects capacity in Figure 4, where $\frac{P}{2N_0}$ is 2500.

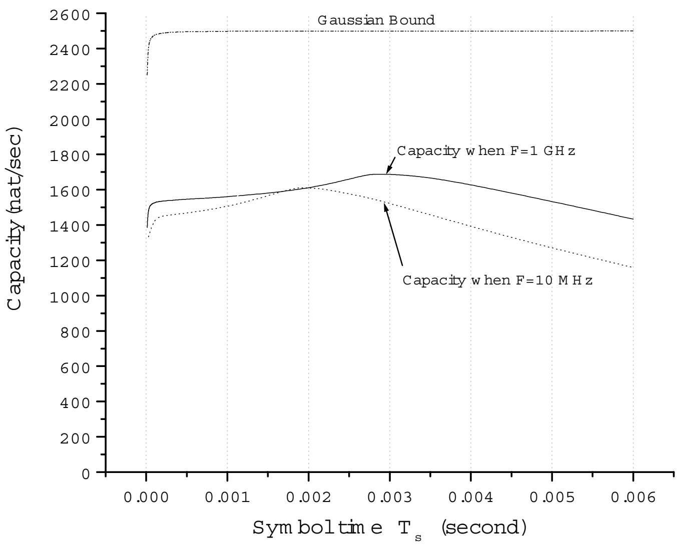

Figure 4: Capacity vs. Symbol Time T_s. $\frac{P}{2N_0} = 2500$, $T_d = 1\mu s$.

Assume the noise density parameter N_0 is fixed. Our numerical results indicate that, when power is reduced, the peaks of the curves move to the right, which means that a greater symbol time improves system capacity when average signal power is lower. Another observation is that the duty cycle for optimizing capacity is a non-decreasing function of power.

For high power, the limited bandwidth bound begins to affect our performance. For lower power, θ decreases and T_s increases. When the power is very small, we need vanishingly a low duty cycle and very long T_s to reach the maximum.

5 Multi-tone FSK

Now we introduce a modulation scheme which we term *multi-tone Frequency Shift Keying*. We use several frequencies at a time in transmitting a symbol instead of one frequency per symbol scheme. When several frequencies are used in the representation of a symbol, the size of the symbol alphabet is increased. For example, if there are M frequency points we can use in a certain bandwidth, and we use two frequencies at a time to represent a symbol, then the size of the alphabet is $M(M-1)/2$ which is greater than M when $M \geq 3$. On the other hand, the power of the transmission at each frequency is reduced in order to maintain the overall transmission power constant.

We first establish the fact that multi-tone FSK can be used to achieve capacity in the infinite bandwidth limit. The discussion below proves the following theorem. The proof is similar to the method used in [12], which proves the analogous result for FSK.

Theorem 1 *Multi-tone FSK can achieve the capacity of the multipath fading channel without bandwidth constraint which is*

$$(1-\frac{T_d}{T_s})\frac{P_a}{N_0}. \tag{18}$$

The average received power of the system is P_a.

Proof: We analyze the scheme with two frequencies first, which can be called 2-tone FSK. We will prove this theorem in 2-tone FSK case, and then extend to the general multi-tone FSK case.

Suppose we wish to transmit one of $M(M-1)/2$ messages in a 2-tone FSK system. We pick two frequencies from the M frequency points we can use and send them in the same power level. Assume the total received signal power is P, which is averaged upon two frequencies, so each frequency has power $P/2$. We have already discussed the probability density of the signal power after a sinusoid waveform passed through a multipath fading channel. Using previous results, we know $|R_i|^2$ and $|R_j|^2$ which are proportional to the expected received power of two frequencies have probability density

$$P_{|R_n|^2}(r) = \frac{1}{1+\zeta}\exp\left[\frac{-r}{1+\zeta}\right] \qquad (r>0, n = i\, or\, j) \tag{19}$$

where $\zeta = P\left(T_s - T_d\right)/(2N_0)$. When corresponding frequency is not sent, $|R_n|^2$ has density

$$p_{|R_n|^2}(r) = \exp\left[-r\right] \qquad (r>0)\,. \tag{20}$$

To transmit a message, we will repeat the transmission of two corresponding frequencies on N disjoint time intervals to average over the fading of the channel. The receiver will get $|R_{n,k}|^2$ for each possible frequency $1 \le n \le M$ and each disjoint interval $1 \le k \le N$. The decoder will form the decision variables

$$S_n = \frac{1}{N}\sum_{k=1}^{N}|R_{n,k}|^2 \tag{21}$$

A threshold rule will be used to decide on a message: if exactly two S_n's exceed $A = 1 + (1-\varepsilon)P(T_s - T_d)/(2N_0)$, then the corresponding message will be declared to be transmitted. Otherwise, a decoding error will be declared. $\varepsilon \in (0,1)$ is a fixed constant which can be arbitrarily small. By ergodicity of the fading process, the two time averages corresponding to the two frequencies sent will exceed the threshold with probability arbitrarily close to 1 for any $\varepsilon > 1$ as N gets large.

If there is another S_n which is greater than A, then an error occurs. We will bound the probability $\Pr[S_n \ge A]$ using a Chernoff bound

$$\Pr[S_n \ge A] \le [\exp(-sA)E[\exp(s\,|R_n|^2)]]^N = \exp(-N\Phi(A)) \tag{22}$$

where

$$\begin{aligned}\Phi(A) &= \sup_s[sA - \ln(E[\exp(s\,|R_n|^2)])] \\ &= \sup_s[sA + \ln(1-s)])] \\ &= A - 1 - \ln(A)\end{aligned} \tag{23}$$

Using the union bound, the probability that one of the averages $S_n(n \neq i, j)$ exceeds A is bounded by

$$(M-1)\exp(-N\Phi(A)) = \exp(-N[\Phi(A) - \frac{1}{N}\log(M-1)]). \tag{24}$$

The probability decays to zero exponentially in N as long as

$$\frac{1}{N}\ln(M-1) < A - 1 - \ln(A) \tag{25}$$

We know $\ln(M-1) < \ln M$, so when $\frac{1}{N}\ln(M) < A - 1 - \ln A$, (25) will also be satisfied. Then the union bound will decay to zero in N as long as

$$\frac{1}{N}\ln(M-1)M < 2(A - 1 - \ln A) \tag{26}$$

Substituting the value for A, (26) can be rewritten as

$$\begin{aligned} \frac{1}{NT_s}\ln\frac{M(M-1)}{2} &\leq (1-\varepsilon)(1-\frac{T_d}{T_s})\frac{P}{N_0} - \frac{1}{NT_s}\ln 2 \\ &- \frac{2}{T_s}\ln[1 + (1-\varepsilon)\frac{P(T_s - T_d)}{2N_0}] \end{aligned} \tag{27}$$

We transmit information in a fraction $\theta (0 < \theta < 1)$ of time. During this time, the signal power P equals to P_a/θ, and the rest of the time the transmitter transmits nothing. This scheme will maintain the average power to be P_a. The data rate that the scheme achieves is

$$\begin{aligned} \frac{\theta}{NT_s}\ln\frac{M(M-1)}{2} &< (1-\varepsilon)(1-\frac{T_d}{T_s})\frac{P_a}{N_0} - \frac{\theta}{NT_s}\ln 2 \\ &- \frac{2\theta}{T_s}\ln[1 + (1-\varepsilon)\frac{P_a(T_s - T_d)}{2\theta N_0}] \end{aligned} \tag{28}$$

Let θ approach 0 and ε be chosen arbitrarily small, this rate will achieve

$$(1 - \frac{T_d}{T_s})\frac{P_a}{N_0} \tag{29}$$

The proof for 2-tone FSK case is finished. For Multi-tone FSK case, we can prove that

$$\begin{aligned} \frac{\theta}{NT_s}\ln\begin{pmatrix} M \\ K \end{pmatrix} &< (1-\varepsilon)(1-\frac{T_d}{T_s})\frac{P_a}{N_0} - \frac{\theta}{NT_s}\ln K! \\ &- \frac{K\theta}{T_s}\ln[1 + (1-\varepsilon)\frac{P_a(T_s - T_d)}{K\theta N_0}] \end{aligned} \tag{30}$$

where K is the number of frequencies we used to transmit a message. In general, $K << M$. Let θ approach 0 and ε be chosen arbitrarily small, this rate will also achieve (29).

□

As the discussion before, we can call this capacity bound as *Limited energy bound* for multi-tone FSK.

We now present numerical results to study the performance of multi-tone FSK. We restrict ourselves to the 2-tone case. First, we set up a discrete system model. The transmitter sends one of $M(M-1)/2$ messages which contains two equal power sinusoid waveforms with different frequencies or sends nothing. At the receiver, a bank of matched filters with central frequencies $f_n(1 \leq n \leq M)$ are used to detect signals. The detector decodes the message based on $|R_n|^2$. According to the MAP rule, we use following strategy for decoding: select the two largest $|R_n|^2$'s, add them together, and get the sum. We then compare the sum to a threshold Z. If the sum is greater than Z, then the message corresponding to the two frequencies are declared transmitted. Otherwise, the decoder declares that nothing was transmitted. The threshold is determined by the MAP rule

$$Z = \left(\frac{1+\zeta}{\zeta}\right) \ln \left[(1+\zeta)^2 \frac{1-\theta}{\theta} \begin{pmatrix} M \\ 2 \end{pmatrix}\right] \tag{31}$$

The discrete model associated with this decoding scheme has $M(M-1)/2+1$ inputs and $M(M-1)/2+1$ outputs. With probability θ, the transmitter transmits one of $M(M-1)/2$ messages. Each message has an input probability $\frac{2\theta}{M(M-1)}$. With probability $1-\theta$, it transmits nothing. The distribution of $|R_n|^2$ is known and the decoding rule is determined, so the transition probabilities can be get. The mutual information of the system is optimized on the probability of input, and thus equals capacity for our input alphabet, input distribution constraint and channel transition probabilities.

Similarly to the FSK case, the *limited bandwidth bound* of multi-tone FSK can be deduced from discrete model:

$$C \leq \frac{\ln\left(\begin{pmatrix} M \\ K \end{pmatrix} + 1\right)}{T_s}. \tag{32}$$

Let $T_d = 1\mu s$, $T_s = 10\mu s$, and $F = 1MHz$, we compare the capacity of 2-tone FSK system and that of FSK system in different $\frac{P}{2N_0}$. Suppose we can use the optimal θ in transmission which means the capacities are optimized on θ. The simulation result is shown in Figure 5. The capacity of 2-tone FSK system is less than that of FSK system where the *limited energy bound* is a main limitation for FSK system. When $\frac{P}{2N_0}$ is very large, the *limited bandwidth bound* will limit the system performance mainly. In this region, the capacity of 2-tone FSK will exceed that of FSK, because Multi-tone FSK has higher *limited bandwidth bound.*

The *limited bandwidth bound* will increase with M while the *limited energy bound* is not changed with bandwidth. Figure 6 shows how the capacities of 2-tone FSK and FSK are changed with M. We use $T_s = 0.1s$ and $\frac{P}{2N_0} = 40$ here.

6 Conclusions

When the received signal power is very large, the limited bandwidth bound will mainly constrain the capacity of the FSK system in a Rayleigh fading channel. Otherwise, the capacity grows slowly with bandwidth, and is nearly 2 dB lower than the infinite bandwidth bound for bandwidths commensurate with general mobile communication conditions. Using a moderate bandwidth, we can approach the capacity achieved by using very large bandwidth. To achieve the best performance in such a FSK system, we need to select optimal symbol time and duty cycle. Large θ and small T_s are needed for high SNR, small θ and long T_s are needed for low SNR. Multi-tone FSK can get a higher capacity than FSK when bandwidth is limited and power can be very large.

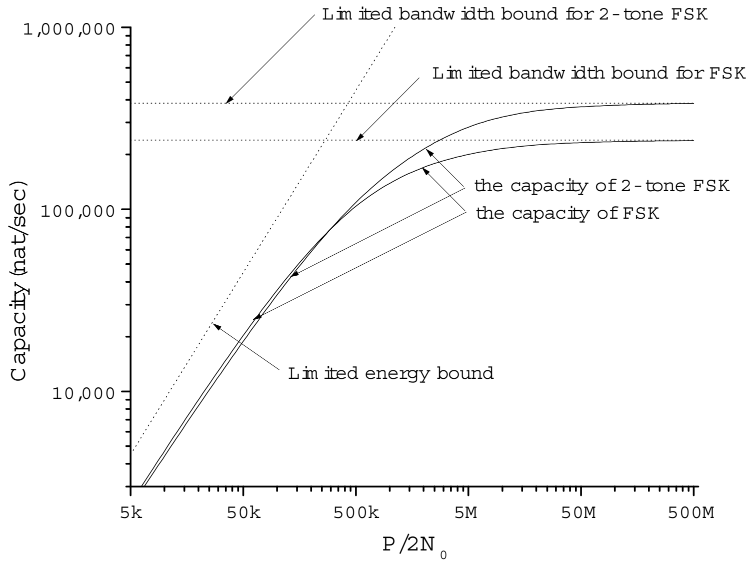

Figure 5: 2-tone FSK vs. FSK, $T_d = 1\mu s$, $T_s = 10\mu s$, and $F = 1MHz$.

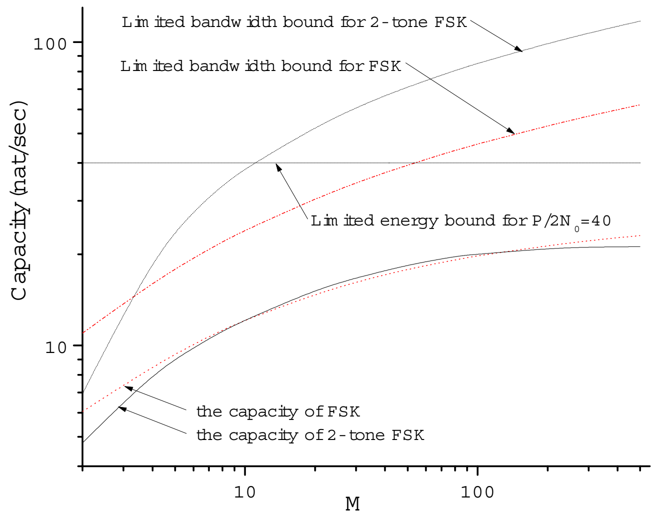

Figure 6: Capacity vs. M, $T_s = 0.1s$, $T_d = 1\mu s$, and $\frac{P}{2N_0} = 40$.

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