An asymptotic analysis of Gibbs-type priors

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Abstract

The Dirichlet process, introduced by T. Ferguson in 1973, is the most famous and successful example of discrete nonparametric prior in Bayesian statistics. Among many desirable features, such process is posterior consistent, that is the posterior distribution accumulates, as the sample size grows to infinity, to a mass point at the "true" distribution, say P_0, that has generated the data, whatever such distribution is. Here the asymptotic problem is set in a frequentist sense, namely the data are assumed to be identically and independently distributed (i.i.d.) from the "true" probability distribution P_0.

It is of interest to investigate whether the property of posterior consistency can be extended to a more general class of random probability measures, to be used as priors in a Bayesian framework. A natural extension of the Dirichlet process is given by the class of Gibbs- type priors; they are random probability measures which are almost surely discrete and with a convenient predictive structure. In this thesis, after a preliminary introduction to the Bayesian nonparametric framework and some standard results, we analyze the main features of Gibbs-type priors and focus on posterior consistency for some notable examples in this class. We provide an algorithmic framework to test the consistency of random measures of Gibbs type with respect to arbitrary underlying base distributions P_0 by means of Monte Carlo simulations.