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MULTIPLE-BRANCH EXCITON ENERGY SPECTRA IN THE FQHE REGIME

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Spectrum of excitons in a FQHE regime depends on the ratio h/l, h is a distance between electron and hole confinement planes, l is the magnetic length. It comprises a single exciton branch and continuum at $h \leq l/2$ and a multiplicity of branches at $h > l/2$. New branches arise due to charge fractionalization. Their quantum numbers are angular momenta of the internal motion in an exciton. Connection of excitons to the low-energy sector of the quasielectron Hilbert space is established. Projecting rules from a spherical into plane geometry are generalized for composite particle spectra.

Keywords: D. fractional quantum Hall effect, D. optical properties.

It is the spectrum of elementary excitations which reflects the most fundamental properties of incompressible quantum liquids[1] underlaying the Fractional Quantum Hall Effect (FQHE). Charged excitations are anyons, quasiparticles which carry a fractional charge[1] and obey a fractional statistics.[2,3] For an incompressible quantum liquid (IQL) with a filling factor $\nu = p/q$ these excitations are quasielectrons (QE) and quasiholes (QH) having electrical charges $(-e/q)$ and (e/q), respectively. The existence of a gap Δ is a specific property of the dispersion law $\varepsilon_{\mathrm{MR}}(k)$ of low energy neutral elementary excitations, magnetorotons (MR). Magnetorotons may be described either in terms of charge density excitations,[4] or in terms of quasiexcitons consisting of a QE's and QH's.[5,6] The basic achievements of the theory of IQLs, including the hierarchical classification[7,2] and composite fermion description of them,[8] were based originally on magnetotransport data. More recently there was a great interest in the interband magnetospectroscopy of the FQHE. Different signatures of the formation of IQLs were found in the spectra of both the extrinsic[9] and intrinsic emission,[10] and Raman spectra.[11] *In this paper a relation between the energy and optical spectra of excitons and the low-energy physics of the FQHE is established.* An exciton is a neutral quasiparticle consisting of a valence hole screened by the polarization of an IQL; it possesses a 2D quasimomentum $\mathbf{k}$. There is a governing parameter in the physics of such excitons: the separation h between electron and hole confinement planes. The screening patterns strongly depend on h. When h is small, $h < l$, where l is the magnetic length, the perturbation exerted by a hole is strong, and there exists no simple picture for the internal structure of an exciton. It is only known that i) because of the existence of a gap an exciton remains a stable entity, and ii) for $h = 0$ a hidden symmetry exists, owing to which the frequences of allowed optical transitions are not affected by the electron-electron interaction. The states to (from) which the transitions are allowed are called multiplicative states. A single exciton branch and a continuum above it exist for small h values.[12-14] In the opposite limit, $h \gg l$, the perturbation produced by a hole is weak and an exciton looks like a flat quasiatom consisting of q QE's (anyons) having "small" charges $(-e/q)$ and a hole carrying a "large" charge e. An anyon exciton has a size about h and possesses $(q-1)$ internal degrees of freedom. As a result, a single-branch spectrum of a conventional magnetoexciton ($q = 1$) splits into a multiple-branch spectrum of an anyon exciton.[15] Multiplicity of branches is a fingerprint of the participation of fractional charges. It should be most accessible for experimental observation if it develops at the values of h which are not too large as compared to l.

Our computations performed in a spherical geometry[7] revealed for the first time that the multiple-branch spectrum appears at moderate values of h/l, $h \approx l$. This fact has important consequences also for the theory. We were able i) to find the general structure of the exciton energy spectrum and to follow up the evolution of it *vs* h/l, ii) to give symmetry classification of the exciton branches in terms of the exciton internal angular momentum L_{ex}, iii) to relate the L_{ex} values to the specific quantum states in the low-energy sector[16,17] of the QE Hilbert space,[18] iv) to establish the existence of two types of exciton branches (related to the low-energy sector and originating from high-energy electronic states) and of the critical angular momentum separating these two manifolds, and v) to show that the order in which the former branches emerge is determined by the momentum decomposition of the QE Hilbert space. The above results relate the physics of excitons to the low-energy physics of the FQHE: for $h > l$ excitons display the few-particle physics of QE's for which the hierarchies are the many-particle physics. Since the dynamic space of an exciton is a plane, and not a sphere, and the quantum states at these manifolds possess different quantum numbers, we have also established vi) a generalized prescription for the sphere-onto-plane projecting for composite quasiparticles which includes the internal angular momentum of a composite, and vii) selection rules for optical transitions.

We consider an ideal model one in which electrons and a hole move on the same sphere, and separation between their confinement planes, h, is taken into account by a modified Coulomb interaction $(h^2 + |\mathbf{r}_e - \mathbf{r}_h|^2)^{-1/2}$. For $\nu = 1/3$ the energy spectrum is quite different for $h/l < 0.5$ and $h/l > 0.5$. For $h/l \leq 0.5$ it is shown in Fig. 1 for a system consisting of a $\nu = 1/3$ IQL of 6 electrons and an exciton ($N_e = 7, N_h = 1$; N_e and N_h are the numbers of electrons and holes, respectively). For $h = 0$ the exciton branch L_0 with the dispersion law $\varepsilon(k)$ and a continuum above it are shown. For $k > 1.5/l$ (or $L \geq 4$) the exciton branch and continuum draw together. When h increases, $\varepsilon(k)$ becomes flatter, and a gap opens between the exciton branch and continuum. We attribute the spectrum flattening in the $kl > 1$ region to the formation of a composite consisting of a QE and an anyon ion separated by the distance qkl^2.[15] This picture strongly resembles a quasiexciton.[5,6] The spectrum changes dramatically at $h > 0.5l$, Fig. 2. At first the branch L_0 shows no considerable change, but new branches split off of the lower part of the continuum, Fig. 2a. Every branch starts at some value of L which remains invariable when h changes. We enumerate the branches by their minimal momenta, L_m. The first to appear is the branch L_3. When h increases, the branches L_0 and L_3 get closer and finally, at $h \approx l$, L_3 finds itself below L_0. The L_5 branch, which first arises in Fig. 2b, passes below L_0 in Fig. 2c. For systems with $5 \leq N_e \leq 8$ the values of L_m do not depend on N_e, hence, on the radius of the sphere R. Therefore, *L_m are quantum numbers which survive in the macroscopic limit.*

We attribute the quantum numbers L_m to the angular momenta connected with the internal degrees of freedom of an exciton, and check this assignment by calculation of the relevant momenta. In the Jain's projection[8] any Laughlin liquid ($\nu = 1/q$) fills completely the first sphere, while QE's live on the second sphere and are a subject of the Pauli exclusion principle. The maximum angular momentum of n QE's equals

$$(L_{QE})_{\max} = nN_e/2 - n(n-1). \quad (1)$$

Hence, $(L_{QE})_{\max} = 3N_e/2 - 6$ for $n = 3$. Momentum of

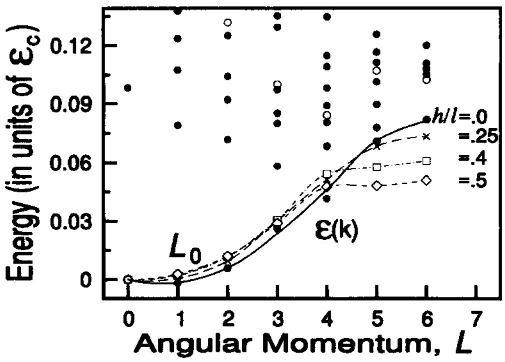

FIG. 1. Energy spectra of a system with $N_e = 7, N_h = 1$, and the flux $2S = 15$ for $h \leq 0.5l$. In absence of an exciton ($N_e = 6, N_h = 0$) an IQL with $\nu = 1/3$ forms. Maximum MR momentum equals $L = N_e = 6$. Since $\varepsilon(k = 0)$ is chosen as an origin, the multiplicative states (open dots) display the MR dispersion law. For $h = 0$ the exciton dispersion, $\varepsilon(k)$, and the continuum are shown. For different values of h only $\varepsilon(k)$ is shown.

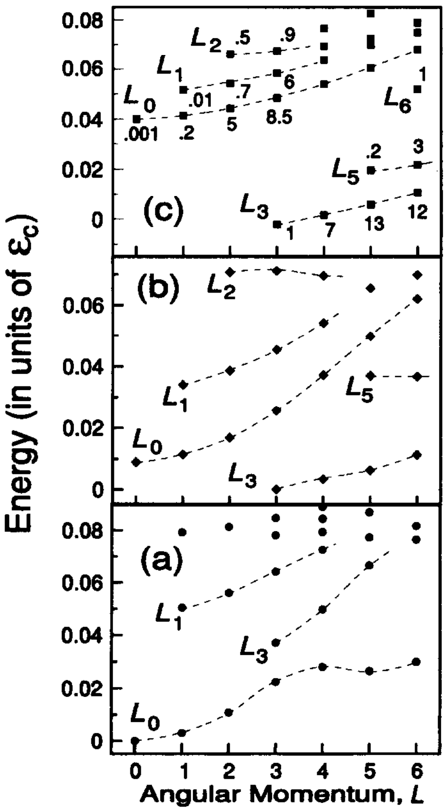

FIG. 2. Energy spectrum of a system with $N_e = 7, N_h = 1$ and $2S = 15$ for $h/l = 0.7$ (a), 1.0 (b), and $h/l = 1.5$ (c). Figure shows emerging new branches and the movement of them when h changes. Dashed lines are guides for eye. Numbers (c) show the probabilities of single-MR optical transitions in percents of the transition probability from multiplicative states (at $h = 0$). They are shown for all states having unambiguous assignments.

a hole in the presence of n QE's equals

$$L_h = S = (N_e - 1)/2\nu - n/2. \quad (2)$$

Therefore, $L_h = 3N_e/2 - 3$ for $\nu = 1/3$, $n = 3$, and

$$L_h - (L_{QE})_{\max} = 3 \quad (3)$$

holds for arbitrary N_e, including the macroscopic limit. Eq. 3, in conjunction with the standard procedure of the addition of angular momenta, suggests that *the minimum angular momentum which may possess an exciton built of a hole and three QE's equals* $(L_{ex})_{\min} = 3$. This condition selects just the L_3 branch which has been discussed in the previous paragraph. To find the momenta of different branches built of three QE's and a hole, we decompose the Hilbert space spanned by QE's on subspaces with different momenta.[19] For $N_e = 7$ the decomposition is $3/2 \oplus 5/2 \oplus 9/2$ with the two last subspaces having the lower, and nearly equal, energies. For $L_h = 15/2$ this decomposition suggests that L_3 must be followed by L_5. This is just what is seen in Figs. 2b, c. *The existence of the critical momentum, L_3, manifests itself most dramatically in Fig. 2c.* The branches L_1 and L_2 remain above L_0. The branches with $L_m \geq (L_{ex})_{\min} = 3$ drop down (relative to L_0) with increasing h. This is natural for entities originating from QE's belonging to the low-energy sector. The exciton spectrum of a $N_e = 8, N_h = 1$ system shows an analogous behavior. For $N_e = 8$ the decomposition is $0 \oplus 2 \oplus 3 \oplus 4 \oplus 6$ with the subspaces $L_{QE} = 2$, 4 and 6 having the lower energies,[16] and L_h equals $L_h = 9$. The branches L_3 and L_5 appear consequently below L_0. Since the separations between L_m branches come from *intra exciton* interactions, they should take finite values in the macroscopic limit. The above data show that the *rotational symmetry* of the excitons with $L \geq L_3$ is compatible with the picture of anyon-hole complexes. Additional work is needed to establish the region of h values where this picture works also on a *dynamical* level.

The order in which different branches appear may be specified as follows: first emerge exciton branches related to subspaces L_{QE} having lower energies, and between the excitons emerging from subspaces with comparable energies first come those which have lower values of $L_{ex} = L_h - L_{QE}$. The equation $L_{ex} = L_h - L_{QE}$ comprises an assumption that the $\{QE\}h$-coupling of the angular momenta operates, i.e., the momenta of three QE's form the momentum L_{QE} which interacts but weakly with L_h (even for h values as small as $h \approx l$). We have checked that this coupling scheme works: all low-lying exciton branches, from their beginning to the end, are formed as a decomposition of the product $L_h \otimes L_{QE} = (L_h - L_{QE}) \oplus \ldots \oplus (L_h + L_{QE})$. Implications of the $\{QE\}h$-coupling for the interaction of anyons will be discussed elsewhere.

Exciton quantum numbers in the plane and spherical geometries are k_x, k_y and L, L_z, respectively. The energy depends on k and L, and the phase of k and L_z form the spaces of degeneracy. Therefore, the projection rule should relate L and k. To establish it one should take into account that for $k = 0$ the projection $M < 0$ of the internal angular momentum enumerates spectrum branches, and all the branches start at $k = 0$.[15] These data suggests the procedure for projecting the energy spectra from the spherical into the plane geometry. For every branch the minimum angular momentum L_m *is identified with* $|M|$, $L_m = |M| = -M$, *and* $L - L_m$ *with the quasimomentum* k, $L - L_m = kR$. Therefore, the starting points of all branches are identified with $k = 0$. This procedure generalizes, for composite particles having internal angular momenta, the Haldane-Rezayi prescription $L = kR$.[20] It is self-consistent; drawing down of the branches $L_m \geq 3$ corresponds in the plane geometry to the increase in the extension of the QE cloud and the $|M|$ values for low-lying exciton states. Since all our results were obtained for $h < R/2$, using them for projecting onto the plane seems ligitimate.

Energy spectra of excitons for $\nu = 2/5$ are similar to the spectra described above. In Fig. 3 the data are given for a system with $N_e = 9, N_h = 1$. The flux $2S = 16$ ensures formation of a $\nu = 2/5$ IQL of eight electrons. The value of L_h is $L_h = S = 8$, while $(L_{QE})_{\max} = 5$. Therefore, $(L_{ex})_{\min}$ for an exciton of five QE's equals $(L_{ex})_{\min} = 3$. The L_3 branch emerges at $h/l = 0.75$ and drops down very fast; for $h/l = 1.1$ it passes below L_0. Data for $N_e = 11, N_h = 1$ are in agreement with the above results.

The data of Figs. 2 and 3 showing the interchange in the mutual positions of the L_0 and L_3 branches at $h \approx l$ suggest the repopulation of them at low temperatures T, and the inversion in the intensity ratio of the exciton-doublet components near the branch intersection.[10,21] This inversion, if observed with increasing magnetic field under the conditions $\nu = p/q$=const, will provide an evidence of the multiplicity of exciton branches caused by fractional charges.

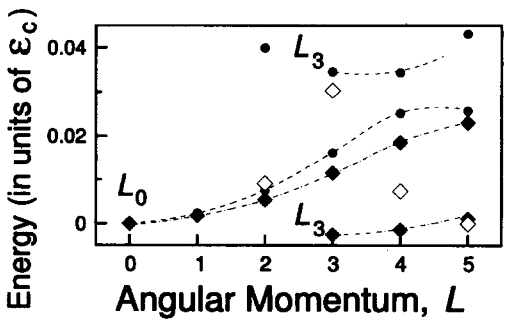

FIG. 3. Energy spectrum of a system with $N_e = 9, N_h = 1$ and $2S = 16$ for $h/l = 0.75$ (dots) and 1.1 (full diamonds). In the absence of an exciton ($N_e = 8, N_h = 0$) an $\nu = 2/5$ IQL forms. Maximum MR momentum equals $L = N_e/2 + 1 = 5$. L_3 branch vaguely seen at $h/l = 0.75$ appears below L_0 at $h/l = 1$. The low-energy spectrum in the absence of a hole is shown by empty diamonds.

The above classification of exciton spectra prompts reconsidering the MR dispersion law from the same stand point. It was proposed[4] that $L = 0$ state is a bound state of two MR's with $k \approx l^{-1}$. The idea was supported by computations.[22] It implies that the $L = 0$ state is of a different nature than MR states with $L \neq 0$. The fact that the lowest $L = 1$ state is canceled by the projection procedure in the charge density excitation model[17,23] implies that MRs should be assigned as $L_{\mathrm{MR}} = L_2$ excitations. Recent calculations[24] have shown that the quasiexciton model describes the low-energy neutral excitations up to $L = 1$; they favor the L_1 assignment of MR's, as well as the concepts of Ref. 25. The generalized prescription for the sphere-onto-plane projection results in new selection rules for optical transitions. At $k = 0$ the angular momentum should not change. Therefore, for simple energy bands direct transitions to the ground state are allowed only for L_0 excitons. Single-MR transitions from a $k = 0$ state are allowed only if $L_m = L_{\mathrm{MR}}$. The data of Fig. 2c may be used for determining L_{MR}. For forbidden transitions the probability should be small for $L = L_m$ and increase strongly with $L - L_m$. The branches L_0, L_1, L_3, and L_5 show such a behavior. Only L_2 shows a slow increase in intensity with $L - L_m$. The transition seems to be weak, but allowed. Therefore, the data of Fig. 2c favor the assignment $L_{\mathrm{MR}} = L_2$.

In conclusion, we have shown that the existence of the fractional charges in the FQHE regime manifests itself in emerging the multiplicity of exciton branches.[26]

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[26] After our work has been completed, we have received a preprint by J. Zang and J. L. Birman where a multiple branch spectrum has been also obtained.