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ELEMENTARY EXCITATIONS OF CHARGE-CONJUGATE INCOMPRESSIBLE QUANTUM LIQUIDS

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Exciton spectra of a $\nu = 2/3$ Incompressible Quantum Liquid (IQL) are investigated and compared with those of a $\nu = 1/3$ IQL. Difference in exciton spectra of these IQL's is related to the asymmetry in the statistical and dynamical properties of quasielectrons and quasiholes of the same IQL which follows from the composite fermion theory. Energy spectra and electron form factors of excitons of a $\nu = 2/3$ IQL are in a satisfactory agreement with the anyon exciton model. They are also compatible with the assignment of magnetorotons as quasiparticles with the intrinsic angular momentum 2.

Keywords: D. fractional quantum Hall effect, D. optical properties.

Spectra of elementary excitations of IQLs[1] comprise charged and neutral excitations. Charged quasiparticles (QPs) are quasielectrons (QEs) and quasiholes (QHs) carrying fractional charges[1] e^* and obeying fractional statistics,[2] i.e., they are anyons. For an IQL with a filling factor $\nu = p/q$ the charge e^* equals $e^* = \pm e/q$. A collective mode known as a magnetoroton (MR) branch[3] forms a lower part of the spectrum of neutral excitations. Optical experiments on intrinsic photoluminescence and resonant Raman scattering[4] involve excitons, a different type of neutral excitations. Presence of an incompressible condensate strongly influences the internal structure of excitons. When the separation, h, between electron and hole confinement planes is large as compared to the magnetic length, l, exciton spectrum shows a multiple-branch structure.[5] Multiplicity of branches arises because of charge fractionalization, and excitons of some of these branches are anyon excitons consisting of a valence hole and q QEs. Multiple-branch exciton spectra have been also found in recent finite-size calculations for an $\nu = 1/3$ IQL.[6, 7, 8] Charged complexes which include anyons (anyon ions[9a]) can also arise.[9] Fine structure of the optical spectra resulting from all these entities should not disappear even in the $h \to \infty$ limit.[10]

Anyon exciton branches are related [6, 8] to the low energy sector of the electron subsystem[11] which is believed to be the anyon sector. This relation connects excitons to the basic physics of the fractional QHE and opens the possibility to investigate it by means of the optical experiments. Because of the charge symmetry, QEs of a $\nu = (1 - p/q)$ IQL have the same properties as QHs of a $\nu = p/q$ IQL. Therefore, investigation of excitons of charge conjugate IQLs, $\nu = p/q$ and $\nu = 1 - p/q$, permits one to probe the properties of QEs ans QHs of the same IQL. The very fact that the filling factors of incompressible states are non-equal to $1/2$ implies difference both in statistical and dynamic properties of QEs and QHs. The difference in dynamic properties, e.g., in form factors, is a well established fact.[12] Difference in the dimensions of the Hilbert spaces of QEs and QHs has been established numerically.[13] We argue that this difference in the statistic properties of QEs and QHs is an intrinsic property of IQLs which follows from composite fermion (CF) theory by Jain[14] and does not require any involvement of dynamic correlations between QPs. We show that the energy spectra of excitons of $\nu = 2/3$ and $\nu = 1/3$ IQLs differ drastically, and that exciton spectra of a $\nu = 2/3$ IQL may be related to the anyon sector of the electronic subsystem in conformity with the approach developed previously.[6, 15] Exciton spectra of a $\nu = 2/3$ IQL support the assignment[6, 15] of MRs as QPs possessing an intrinsic angular momentum $L = 2$.

Let us consider Laughlin liquids, $\nu = 1/m$, in a spherical geometry[12] and apply to them the CF theory. The flux $2S_{CF}$ in the CF representation equals

$$2S_{CF} = N + N_{QH} - N_{QE} - 1 \ , \tag{1}$$

where N, N_{QH}, and N_{QE} are the numbers of electrons, QHs, and QEs, respectively. Since QHs inhabit the lowest Landau level, the dimension of the Fock space is

$$G_{QH} = 2S_{CF} + 1 = N + N_{QH} \ . \tag{2}$$

QEs inhabit the next Landau level, hence, the dimension of the QE Fock space is

$$G_{QE} = 2(S_{CF}+1)+1 = N - N_{QE} + 2 \,. \qquad (3)$$

For $N_{QE} = N_{QH} = 1$ both dimensions coincide and (2) and (3) give a well known result $G_{QE} = G_{QH} = N + 1 \equiv G_1$.[12] However, for $N_{QE} = N_{QH} \neq 1$ the dimensions differ:

$$G_{QH} - G_{QE} = 2(N_{QP} - 1) \,. \qquad (4)$$

Eq. (4) was proposed in Ref. 13 to explain the numerical finite-size data and treated in terms of a hard core constraint for QEs. The above arguments show that it has a statistical rather the dynamical origin.[16] Bosonic Haldane dimension[17] of QHs, d_B, equals $d_B = G_1$. Bosonic dimension of QEs equals $d_B = G_1 - 2(N_{QE} - 1)$, while fermionic dimension of them equals $d_F = G_1 - (N_{QE} - 1)$. Therefore, statistical suppression of the space dimension is stronger for QEs than for usual fermions.

Maximum total angular momentum of several QEs or QHs in a $\nu = 1/q$ IQL may be found in the CF representation by analogy with Refs. 6 and 15:

$$(L_{QE})_{\max} = N_{QE}N/2 - N_{QE}(N_{QE}-1) \,, \qquad (5)$$
$$(L_{QH})_{\max} = N_{QH}N/2 \,. \qquad (6)$$

These states correspond to minimum relative momenta of QPs, hence, to most compact configurations of them. This statement is supported by numerical data on the electron density distribution. For a $\nu = 1/3$ IQL the energy spectrum of a 3-QH system is shown in Fig. 1, and the QH density distribution in Fig. 2; for the data for 3-QE systems see Ref. 15. Compact states are insensitive to the system size. Indeed, energies of large L states nearly coincide for the systems of different size when plotted *vs* $L - (L_{QP})_{\max}$. With increasing N the value of $(L_{QP})_{\max}$ increases as it follows from (5) and (6), and the low L part of the spectrum (which corresponds to large QP distances) develops. The low energy sector has a macroscopic limit when plotted *vs* $L - (L_{QP})_{\max}$. It is a remarkable property of the data of Fig. 1 and Ref. 15 (which are in agreement with the data of Ref. 13) that the energies of $L = (L_{QP})_{\max}$ and $L = (L_{QP})_{\max} - 2$ states are high for 3-QH systems, while low for 3-QE systems. Therefore, a short range repulsion of anyons is strong for QHs rather than for QEs, which is in conformity with the difference in their form factors.[12]

The properties of 3-QH and 3-QE quantum states for a $\nu = 1/3$ IQL provide, because of the charge symmetry arguments, the properties of 3-QE complexes in a $n = 2/3$ and $\nu = 1/3$ IQLs, respectively. Binding of these complexes to a hole results in forming anyon exciton branches. The data on 3-QE complexes permit one to find angular momenta of these branches and to foresee which branches appear in the low energy part of the exciton spectrum for moderate values of $h/l \approx 1-2$.

In Fig. 3 the evolution of the lower part of the exciton spectrum of a $\nu = 2/3$ IQL is shown for $0 \leq h/l \leq 1.6$. Only a single exciton branch, L_0, exists for $h = 0$. The gap between the L_0 branch and the quasi-continuum above it is wider than for a $\nu = 1/3$ IQL. With increasing h/l new branches develop just as for a $\nu = 1/3$ IQL.[6,7,8] First the L_2 branch appears, then L_4, and for $h/l = 2$

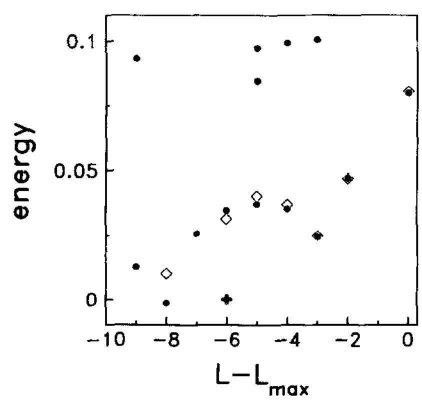

FIG. 1. Energy of a $\nu = 2/3$ IQL with one extra electron plotted *vs* $L - L_{\max}$. Dots – $N = 15$, diamonds – $N = 13$ (only the low energy sector is shown). The same spectrum possesses a $\nu = 1/3$ IQL with 3 QHs. Energy in units of $e^2/\epsilon l$.

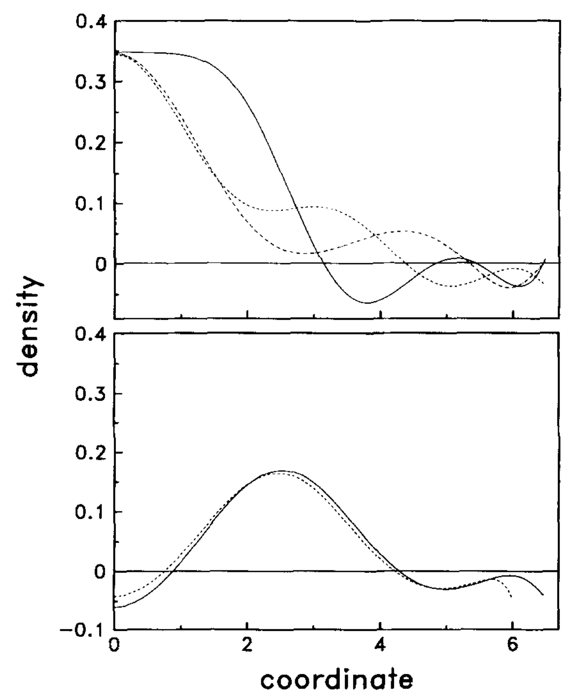

FIG. 2. Electron density distribution for a $\nu = 2/3$ IQL with one extra electron; $N = 15$. Density of the incompressible background is subtracted. Upper part: $L - L_{\max} = 0$ – solid line, 2 – dashed line, 4 – dash-dotted line. Lower part: $L - L_{\max} = 3$ – solid line; the data for a $N = 13$ system – dashed line. Density in units of $(2S+1)/4\pi S$, coordinate in units of l. The same distribution describes 3 QHs in a $\nu = 1/3$ IQL.

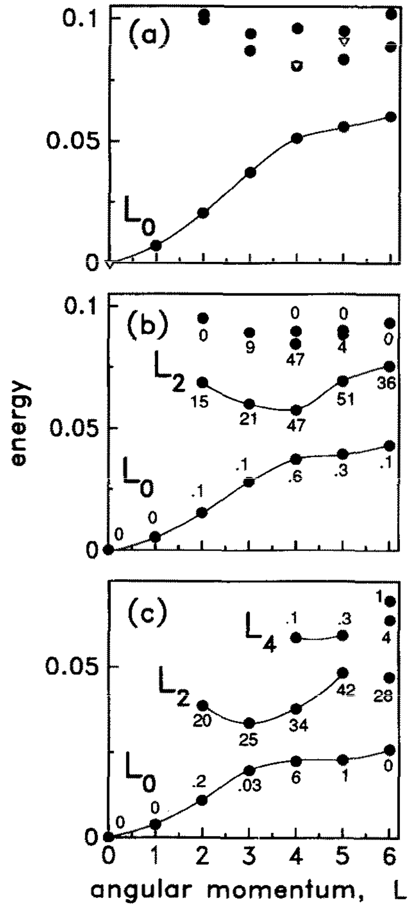

FIG. 3. Energy spectra and intensities of single-MR assisted transitions for $\nu = 2/3$; $N = 13$. Numbers - intensities in units of transition intensities from multiplicative states ($h = 0$). $h/l = 0$ - (a), 1.0 - (b), 1.6 - (c).

also L_3 branch is distinctly seen above L_4. Indices n of L_n branches indicate the values of the angular momenta the corresponding branches start from; they are quantum numbers which do not depend on the system size. It is remarkable that the sets of of these quantum numbers are different for $\nu = 1/3$ and $\nu = 2/3$ IQLs. E.g., for a $\nu = 1/3$ IQL the branch L_3 appears at first, and there is no low lying L_2 and L_4 branches.

Angular momentum of an anyon exciton on the background of an IQL is equal to the difference between the hole angular momentum L_h and the angular momentum of a m-QE complex, L_{QE}. L_h always equals S, the half of the flux. Therefore, it is convenient to rewrite $(L_{QE})_{\max}$ in terms of S. Eq. (5) for a $\nu = 1/m$ IQL takes the form $(L_{QE})_{\max} = S - m(m-2)$. The analogous equation for a $\nu = (1 - 1/m)$ IQL is $(L_{QE})_{\max} = S$; it follows from (6). These equations determine the minimum value of the exciton angular momentum, $(L_{\rm ex})_{\min}$, for $\nu = 1/m$:[6]

$$(L_{\rm ex})_{\min} = m(m-2) \ , \qquad (7)$$

and for $\nu = (1 - 1/m)$:

$$(L_{\rm ex})_{\min} = 0 \ . \qquad (8)$$

Eq. (7) clarifies the special role of the L_3 branch for a $\nu = 1/3$ IQL, while (8) determines angular momenta of the exciton branches for $\nu = 2/3$: $L_{\rm ex} = 0,\ 2,\ 3, \ldots$. $L_{\rm ex} = 1$ is absent because of restrictions imposed on CFs by Fermi statistics. The factors favoring low values of the exciton energy are i) a low energy of the QE state and ii) a small difference $(L_{QE})_{\max} - L_{QE}$; the latter condition ensures a considerable QE-hole attraction for moderate values of $h/l \sim 1$. The energy of the $L_{QE} = 0$ state is high. As a result, there is only a small, about 25%, anyon contribution to the original L_0 branch, and there is no additional L_0 branches. L_n branches with $n \geq 2$ are related to the low energy sector of the electron subsystem and are, in this sense, anyon branches. For $h/l \approx 1$ the $L_{QE} = 2$ state has optimal properties for forming excitons; the energy of this state is sufficiently low, Fig. 1, and the QE density is high near the origin, Fig. 2. Although the energy of the $L_{QE} = 3$ state is lower than the energy of $L_{QE} = 2$, the density of $L_{QE} = 3$ is widely spread, Fig. 2, which strongly reduces a Coulomb attraction. As a result, $L_{QE} = 3$ branch approaches $L_{QE} = 2$ only for larger values of h/l about $h/l \approx 2$.

Properties of L_2 excitons provide important data for assignment of the MR branch. We argued[6] that the probabilities of single-MR assisted transitions for some branch L_n can both i) have a considerable magnitude and ii) show a slow L-dependence only if $L_n = L_{\rm MR}$, where $L_{\rm MR}$ is the angular momentum of MRs. The data for a $\nu = 1/3$ IQL favor the L_2 assignment of MRs.[6] However, for a $\nu = 1/3$ IQL the L_2 branch appears high in the energy spectrum, for large values of h, and can not be reliably separated from adjacent branches. L_2 branch in Fig. 3 gives stronger arguments in favor of the same assignment of MRs. Indeed, for it the intensities of single-MR transitions weakly depend on L and are much higher than for L_0 and for the states lying above L_2. L_2 is well isolated, and exists in the region $h/R \sim 0.5$, R is the sphere radius, where the spherical geometry is rather accurate.

It is instructive to compare the above data for QEs of a $\nu = 2/3$ IQL with the results of an analytic study of anyon excitons.[5b] The correspondence is impressive if to compare QE complexes and excitons which angular momenta are related as $L_{\rm ex} = (L_{QE})_{\max} - L_{QE}$. Indeed, there is no $L_{\rm ex} = 1$ excitons, and only one type of exciton species for each value of $L_{\rm ex} = 0,\ 2-5$. The density distributions are similar, including a very special distribution for $L_{\rm ex} = 3$. In both cases two species with the same value of the angular momentum appear for the first time for $L_{\rm ex} = (L_{QE})_{\max} - L_{QE} = 6$. Therefore, the simple anyon model of Ref. 5b works well for excitons of a $\nu = 2/3$ IQL. The success may be attributed, in part, to a small size of QEs of this IQL. The data imply that QEs of a $\nu = 2/3$ IQL possess no intrinsic angular momenta in agreement with the recent conjecture[19] on the absence of anyon spins in the plane limit. Complications inherent in a $\nu = 1/3$ liquid will be discussed elsewhere.

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