Anyon Excitons

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We propose a model of an anyon exciton consisting of a hole and several anyons, and apply it to the spectroscopy of an incompressible quantum liquid. Fractionalization of the electron charge makes properties of such entities quite different from those of usual magnetoexcitons. The model describes a number of properties established by few-particle simulations, including an abrupt change in emission vs electron-hole asymmetry of the system. The attractive field of a hole may eliminate the hard core constraint for anyons. The effect of exciton-magnetoroton coupling is discussed.

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Phenomena in two-dimensional (2D) electron systems related to the fractional quantum Hall effect (FQHE) [1, 2] and Wigner crystallization were originally discovered by means of magnetotransport. Later on the formation of an incompressible quantum liquid (IQL) [3], underlying the FQHE, and related phenomena have been investigated by optical experiments, which have become a powerful tool in the field [4]. These findings stimulated theoretical activity on the optical spectroscopy of the FQHE [5–8]. It has been shown that a hidden symmetry, which is inherent in 2D systems with charge symmetric electron-hole interaction ($V_{ee} = V_{hh} = -V_{eh}$), results in exact cancellation of the effect of the electron background on optical spectra. Optical spectra of symmetric systems are trivial, since they coincide with the spectra of free magnetoexcitons, and are insensitive to electron phase transitions. Therefore, the spectroscopy of charge-asymmetric systems acquires a special importance. Gap widths for IQLs determined from cusp strengths [5(a)] in extrinsic optical emission spectra were reported [4(a)] for such systems.

It is one of the most remarkable properties of IQLs that their elementary excitations carry fractional charge [3], and are anyons [9, 10], i.e., obey fractional statistics [11, 12]. The theory [5(c),8] predicts that fractional charges should manifest themselves by dramatic changes in the position and the intensity of the emission band vs the asymmetry parameter. The ratio h/l, where h is a distance between electron and hole confinement planes, and $l = (c\hbar/eH)^{1/2}$ is the magnetic length, may be chosen as such a parameter. Numerical simulations for few-electron systems are accessible only for small values of $h/l \lesssim 1$. For the opposite limit case of strongly asymmetric systems, $h/l \gtrsim 1$, the approach based on the anyon concept seems to be most promising. An exciton, appearing against a background of an electron IQL, is a neutral entity consisting of a valence hole and several anyons, e.g., if the filling factor $\nu = 1/3$, the charge of anyons $e^* = -e/3$, their statistical charge $\alpha = -1/3$ (for comparison, $\alpha = 0$ for bosons, and $\alpha = 1$ for fermions) [9], and the number of anyons $N = 3$. If $h \gg l$, the mean separation between anyons in an exciton is about h, which is larger than the anyon size, l. Therefore, anyons are well defined particles, anyon-anyon and anyon-hole interactions follow a Coulomb law in the leading approximation, and the Coulomb energy is small as compared to the IQL gap width, Δ. When $h \lesssim l$, the exciton radius is about $l^* = l\nu^{-1/2}$, which is larger than l but comparable to it, and the anyon-hole binding energy is even larger than Δ. Only qualitative results may be expected from the anyon exciton (AE) model in this limit. Nevertheless, we show that they are quite encouraging. Recently there has been a significant activity in the hierarchy theory of the FQHE [13–15], anyon superconductivity [16], and mechanics of anyons [17], and some experimental data have been discussed in these terms [18]. While the hierarchies provide a level classification for free anyons, AEs may also give insight into the effect of an external (Coulomb) field and the treatment of optical data.

We consider a model of an AE consisting of a hole and two semions, anyons with $e^* = -e/2$, $\alpha = -1/2$. If the hard-core constraint [12, 14] is imposed, one should choose $\alpha = 3/2$. For spin polarized IQLs $e^* = -e/q$, where q is odd. Our two anyon model is the simplest one giving insight into the properties of the more realistic models with $q \geq 3$. We assume that the magnetic field is strong enough, i.e., the Coulomb energy $\varepsilon_C = e^2/\epsilon l \ll \hbar\omega_c$, ω_c is the cyclotron frequency, and use dimensionalless variables scaled in units of ε_C, l, and e. Such an AE, consisting of three particles, is described by three quantum numbers. Since it is a neutral entity, a two component momentum $\mathbf{K}$ may be ascribed to it [19]. Therefore, the internal motion in it is characterized by a single quantum number, and the charge fractionalization (CF) should result in the multiplicity of energy bands, instead of the single band of a usual magnetoexciton.

In the strong field limit the preexponential factor in the wave function of a positively (negatively) charged particle is a polynomial in complex coordinate z ($\bar{z}$), $z = x + iy$. We introduce for the two anyons the Jacobi coordinates $z = z_1 - z_2$ and $z_0 = (z_1 + z_2)/2$, and relate the anyon center-of-mass coordinate, z_0, to a hole coordinate, z_3, by the usual exciton transformation. The new complex coordinates are z, $\zeta = z_0 - z_3$, and $Z = (z_0 + z_3)/2$, and the wave functions of the three noninteracting particles with a momentum $\mathbf{K}$, built from Halperin pseudo wave functions [9], are

$$\Psi_{\mathbf{K}n}(z_1, z_2, z_3|\alpha) = B_n(\alpha)\exp\{i\mathbf{K}\cdot\mathbf{R} + i(\rho_x Y - \rho_y X)/2\}\exp\{-(1/4)(\boldsymbol{\rho} - \boldsymbol{\kappa})^2)\}|z|^\alpha \bar{z}^n \exp(-|z|^2/16).$$

Vectors $\mathbf{R}$, $\boldsymbol{\rho}$, and $\mathbf{r}$ correspond to complex coordinates Z, ζ, and z; $\boldsymbol{\kappa} = \hat{\mathbf{z}} \times \mathbf{K}$, $\hat{\mathbf{z}}$ is a unit vector in the direction perpendicular to the confinement plane, $-\boldsymbol{\kappa}$ is the dipole moment of the exciton, and $B_n(\alpha)$ is a normalization factor. The quantum number $n \geq 0$ is the relative angular momentum of anyons. The Coulomb interaction may be written as

$$V = V_{aa} + V_{ah}, \quad V_{aa} = 1/4r,$$
$$V_{ah} = -(1/2)\{|\boldsymbol{\rho} + \mathbf{r}/2 + h\hat{\mathbf{z}}|^{-1} + |\boldsymbol{\rho} - \mathbf{r}/2 + h\hat{\mathbf{z}}|^{-1}\}.$$

The first term is diagonal in n:

$$\langle n_1|V_{aa}|n_2\rangle = \delta_{n_1n_2}\frac{\Gamma(n_1 + \alpha + 1/2)}{8\sqrt{2}\Gamma(n_1 + \alpha + 1)}. \quad (1)$$

When $m = |n_1 - n_2| =$ odd, all $\langle n_1|V_{ah}|n_2\rangle = 0$, which ensures the correct interchange symmetry; only even n have a physical meaning. For even m,

$$\langle n_1|V_{ah}|n_2\rangle = -\left\{\Gamma\left(\frac{n_1 + n_2 + m}{2} + \alpha + 1\right)/2^{m/2}m![\Gamma(n_1 + \alpha + 1)\Gamma(n_2 + \alpha + 1)]^{1/2}\right\}$$
$$\times \int_0^\infty q^m \exp(-q^2/2 - qh)J_m(Kq)\Phi\left(\frac{n_1 + n_2 + m}{2} + \alpha + 1, m + 1; -q^2/2\right)dq, \quad (2)$$

Φ is the confluent hypergeometric function. For $\alpha = -1/2$ the integral $\langle n|V_{aa}|n\rangle$ diverges logarithmically for $n = 0$. This is the price for using the oversimplified model, $N = 2$. We use a cutoff $V_{aa} = \frac{1}{4}(r^2 + a^2)^{1/2}$. For the hard-core model, $\alpha = 3/2$, and for an exotic exciton built of two quasiholes and one electron, $\alpha = 1/2$; hence, all integrals are regular.

For $K = 0$ only diagonal matrix elements survive, and Eqs. (1) and (2) give the energy spectrum by quadrature. When $h = 0$, Eq. (2) takes a simple form:

$$\langle n_1|V_{ah}|n_2\rangle = -\delta_{n_1n_2}\sqrt{\pi/2}(2/3)^{n_1+\alpha+1} \times F\left(\frac{n_1 + \alpha + 1}{2}, \frac{n_1 + \alpha}{2} + 1; 1; 1/9\right), \quad (3)$$

where F is the hypergeometric function. Equation (3) and its generalization for $h \neq 0$ [diagonal matrix elements in Eq. (2)] give exact solutions of the three-particle problem.

Figure 1 shows the effect of statistics on the distribution of electron density, $d(\rho)$, around a hole in an AE for $K = n = 0$. The functions $d(\rho)$ for a free exciton, $d_{\rm ex}(\rho) = \exp(-r^2/2)/2\pi$, and an exciton in the presence of an IQL ($\nu = 1/3, h = 0$), are also shown. In the last case the excess density, $d_{\rm L}(\rho)$, is plotted [8]. The most striking property of $d_{\rm L}$ is a considerable increase in the spread of the density as compared to $d_{\rm ex}$, which is caused by the Pauli exclusion principle. This property is reproduced by the anyon model, primary because of the increase in the magnetic length, $l^* > l$. The curve $d_{-1/2}(\rho)$ is even in reasonable quantitative agreement with $d_{\rm L}$; a realistic comparison may be done only for odd q. The first excited state of an $\alpha = -1/2$ exciton coincides with the ground state of an $\alpha = 3/2$ exciton. The latter curve, having a flat minimum at $\rho = 0$, highly resembles the distribution of the electronic density for an exciton against the background of an IQL, $\nu = 1/3$ (Fig. 2, curve 3 in Ref. [8]). This similarity in $d(\rho)$ for the two lowest states found by both simulations and for AEs strongly suggests that when h is small the hard-core constraint for quasiparticles [12,14] is violated in an exciton by the attractive field of a hole.

There are several distinctive features of AEs caused by CF. If $K = 0$, and h increases, $\Psi_{\mathbf{K}n}$ remain exact eigenfunctions, but the level arrangement is changed; the larger h is, the higher is the value of n for the ground level, and the wider is the density distribution, $d(\rho)$, for it. The $n = 0$ and $n = 2$ levels interchange at $h_{\rm cr} \approx 1.66$ (for $a = 1$); at this point $d(\rho)$ changes abruptly from $\alpha = -1/2$ for $\alpha = 3/2$ curve (Fig. 1). Since the K dependence of diagonal matrix elements, $\langle n|V|n\rangle$, is stronger the less n is, energy levels draw together at different values of K. At $K = 0$, nondiagonal elements of V vanish, and the level rearrangement shows the pat-

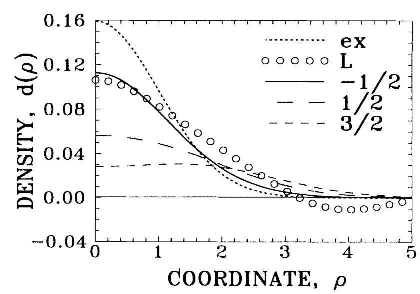

FIG. 1. Electron density, $d(\rho)$, for the ground state of a free magnetoexciton (ex), an exciton in the presence of the IQL with $\nu = 1/3$ (L), and for anyon excitons with statistical charges of anyons $\alpha = -1/2$, $1/2$, and $3/2$; $K = 0, h = 0$. The last curve also describes the first excited state for $\alpha = -1/2$.

terns of the level crossing, while at $K \neq 0$ of the level anticrossing, since V_{ah} mixes states, $\Psi_{\mathbf{K}n}$, with different n. This behavior is seen in Fig. 2 where dispersion laws, $\varepsilon(K)$, are shown for $h = 0$ and $h > h_{\rm cr}$. Dispersion is strongly suppressed as compared to a usual exciton (Fig. 2), since in the $K \to \infty$ limit only one anyon moves away from the hole, while another remains in a bound state and makes with the hole an ion (in a conventional exciton all the electron charge is moved away from the hole in the $K \to \infty$ limit). If $h = 0$, $\varepsilon(K) \approx -\sqrt{\pi}/4 - 1/8K$ for the lowest spectrum branch in the $K \to \infty$ limit [Fig. 2(a)]. The K dependence is the same as for magnetorotons [20, 21]. If $h > h_{\rm cr}$, dispersion in the ground state is even more suppressed [Fig. 2(b)]. Both the abrupt change in the ground state with increasing h and the suppression of the dispersion are in conformity with the patterns found by simulations [5, 8].

The level intersections at $K = 0$ have important implications for optical transitions. The matrix elements for them are

$$M_n(\alpha) \propto \int \Psi_{0n}(z_1, z_2, z_3|\alpha)\delta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) d\mathbf{r}_1\, d\mathbf{r}_2.$$

Here $\delta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ is a δ-shape function of r_{13} and r_{23} having a width about 1, in units of l. After the angular integration over $\mathbf{r}$, only the $M_n(\alpha)$ with $n = 0$ survive. Therefore, exciton transitions are allowed in the emission at $T = 0$, i.e., from the ground state, only if $h < h_{\rm cr}$. This result is in agreement with numerical data [5(c),8] which show that only weak transitions assisted by magnetorotons (MR) are allowed at $h > h_{\rm cr} \sim 1$.

The effect of the CF becomes even more spectacular when the ground state density, $d(\boldsymbol{\rho})$, is plotted for $K \neq 0$. In Fig. 3 $d(\boldsymbol{\rho})$ is shown along the symmetry line, $\boldsymbol{\rho} \parallel \boldsymbol{\kappa}$, for $K = 0$ and $K = 2$; it is symmetric with respect to $\rho = -K$ for all K. At $K \approx 1.59$ ($a = 1, h = 0$) a single humped distribution changes into a camelback type. The right hand part, centered near $\rho = 0$, corresponds to the ground state of the ion, and the left hand part to a free anyon. When K increases, the separation between maxima approaches $K/|e^*| = 2K$, and the electron density distribution in both wings approaches $d(\rho) = \exp(-\rho^2/4)/8\pi$, which describes the shape of both a free anyon and ion.

We have concentrated on the small h region, the least favorable for the AE model, since the comparison with numerical data is available only for it. All the more, the success of the model is impressive. However, there are two peculiarities of the exciton ground state found by simulations [8] which the simple AE model does not describe: (i) At $h = 0$ the function $d(\boldsymbol{\rho})$ is of a single-hump type and only feebly depends on K and (ii) at $h = h_{\rm cr}$ the minimum of $\varepsilon(K)$ shifts from $K = 0$ to $K_{\rm min} \neq 0$, where $K_{\rm min}$ is close to the roton minimum [20]. We argue here that these facts unambiguously signal the AE-MR coupling should be invoked. The importance of it is implied by Fig. 3. The separation between an anyon and ion increases with K, and an AE produces a strong Coulomb field acting on the IQL, which is known to become unstable when an external charge about e^* approaches it [22]. A simple idea that the AE creates a virtual MR and makes a bound state with it describes the behavior at $h \approx 0$ very well. When h is small, the dispersion of a bare AE is strong [Fig. 2(a)]. Therefore, the momentum of an AE-MR complex is carried by the MR, while the AE momentum $K \approx 0$, which explains a narrow distribution of $d(\boldsymbol{\rho})$ [8]. When h increases, the exciton dispersion curve flattens [Fig. 2(b)]. As a result, the AE acquires a larger share in the total momentum, K, and a new ground state with a broken symmetry and the momentum $K_{\rm min}$ appears. In this state the shape of $d(\boldsymbol{\rho})$ found by simulations, Ref. [5(c)], is reminiscent of the curve $K = 2$ (Fig. 3). AE-MR coupling manifests itself also in the oscillatory behavior of $d_{\rm L}(\rho)$ (Fig. 1) (similarly to the screening of charged impurities [20, 22]).

In conclusion, we have proposed a model of anyon excitons for the description of optical properties of IQLs. We show that the model reflects different distinctive features

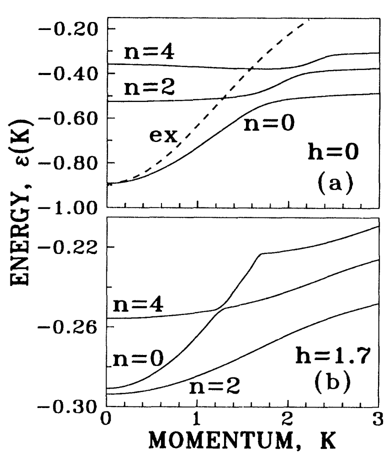

FIG. 2. Dispersion law, $\varepsilon(K)$, for the three lowest spectrum branches; $\alpha = -1/2$. When h increases, anticrossings become narrower, and $\varepsilon(K)$ for the ground state flatter. The curve for a free exciton (ex) is shifted to facilitate a comparison with the $n = 0$ curve.

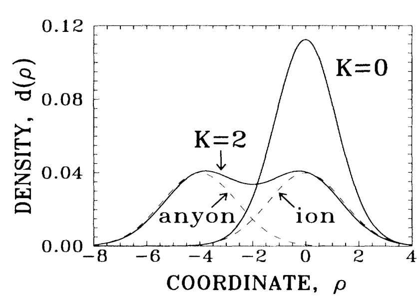

FIG. 3. Distribution of the electron density in the ground state of an anyon exciton along the direction $\boldsymbol{\rho} \perp \mathbf{K}$.

of excitons in the presence of an IQL as compared to free magnetoexcitons, including broadening of the electron density, suppression of the exciton dispersion, and an abrupt change of the emission spectrum with increasing separation between electron and hole confinement planes. The results of simulations and the predictions of the AE model are compared, and the properties of excitons, for which their coupling to magnetorotons is of crucial importance, are established.

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[1] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[2] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer, New York, 1990); T. Chakraborty and P. Pietiläinen, *The Fractional Quantum Hall Effect* (Springer-Verlag, Heidelberg, 1988).
[3] R. B. Laughlin, Phys. Rev. Lett. **50**, 13 (1983).
[4] (a) I. V. Kukushkin *et al.*, Surf. Sci. **263**, 30 (1992); (b) B. B. Goldberg *et al.*, *ibid.* **263**, 9 (1992); (c) A. J. Turberfield *et al.*, *ibid.* **263**, 1 (1992); (d) E. M. Goldys *et al.*, Phys. Rev. B **46**, 7957 (1992), and references in (a)–(d).
[5] V. M. Apal'kov and E. I. Rashba, (a) Pis'ma Zh. Eksp. Teor. Fiz. **53**, 420 (1991) [JETP Lett. **53**, 442 (1991)]; (b) *ibid.* **54**, 160 (1991) [**54**, 155 (1991)]; (c) *ibid.* **55**, 38 (1992) [**55**, 37 (1992)].
[6] B.-S. Wang, J. L. Birman, and Z.-B. Su, Phys. Rev. Lett. **68**, 1605 (1992).
[7] A. H. MacDonald, E. H. Rezayi, and D. Keller, Phys. Rev. Lett. **68**, 1939 (1992).
[8] V. M. Apalkov and E. I. Rashba, Phys. Rev. B **46**, 1628 (1992).
[9] B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).
[10] D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984).
[11] F. Wilczek, Phys. Rev. Lett. **49**, 957 (1982).
[12] Y.-S. Wu, Phys. Rev. Lett. **52**, 2103 (1984); **53**, 111 (1984).
[13] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).
[14] S. He, X.-C. Xie, and F.-C. Zhang, Phys. Rev. Lett. **68**, 3460 (1992).
[15] G. Dav and J. K. Jain, Phys. Rev. Lett. **69**, 2843 (1992).
[16] R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988).
[17] M. Sporre *et al.*, Phys. Rev. Lett. **67**, 1813 (1991); M. V. N. Murthy *et al.* , *ibid.* **67**, 1817 (1991); S. Levit and N. Sivan, *ibid.* **69**, 363 (1992); G. Dunne *et al.*, Nucl. Phys. **B370**, 601 (1992).
[18] R. G. Clark *et al.*, Phys. Rev. Lett. **60**, 1747 (1988); S. Kivelson, *ibid.* **65**, 3369 (1990); S. W. Hwang *et al.*, Phys. Rev. B **44**, 13 497 (1991); F. G. Pikus and A. L. Efros (to be published).
[19] L. P. Gor'kov and I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. **53**, 717 (1967) [Sov. Phys. JETP **26**, 449 (1968)].
[20] S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. **33**, 2481 (1986).
[21] C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).
[22] C. Zhang *et al.*, Phys. Rev. **32**, 6920 (1985); E. H. Rezayi and F. D. M. Haldane, Phys. Rev. **32**, 6924 (1985).