MESON-MESON SCATTERING IN MESON FIELD THEORY

V. V. Sudakov

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Unlike the Green's function and the vertex part, the asymptotic expressions for which are functions of a single argument [1, 2], the same kind of asymptotic expression for meson-meson scattering depends in general on two arguments. I. T. Diatlov and K. A. Ter-Martirosian [3] have set up a system of equations which determine the asymptotic behavior of the amplitude and have found its limiting values. In the general case these equations can be solved only by numerical integration.

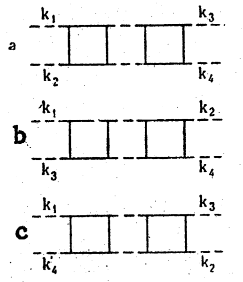

Fig. 1.

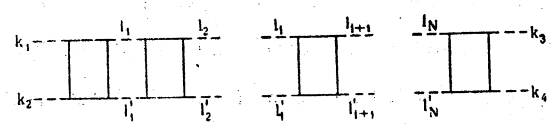

Fig. 2.

It would be possible to confine ourselves to the most interesting case where the two arguments of P coincide so that P is a function of a single argument (in this case the momenta of all four mesons k_1, k_2, k_3 and k_4 as well as their paired sums must be of the same order of magnitude). However in the set of equations given in [3] the amplitude P which depends on a single argument is not independent,and must be determined simultaneously with the P which is dependent on two arguments.

We shall show that it is nevertheless possible to obtain a simple closed expression for a P which is dependent on only a single argument (our derivation will use neutral pseudoscalar meson theory). For this purpose we consider the quantity $F(k_1, k_2, k_3, k_4)$ of [3], which is the sum of all diagrams reduced to the diagram in Fig. 1,a (we recall that the reduction of a diagram is the replacement of two squares connected by two meson lines by a single square). Each diagram of this sum is reducible with respect to the separation of momenta k_1, k_2 from k_3, k_4, i. e., it can be divided into two parts which are connected by only two meson lines, with k_1, k_2 connected to one part and k_3, k_4 to the other. Such a diagram consists of two or more parts connected by a chain of meson lines l_i, $l'_i = -l_i + k_1 + k_2$ (see Fig. 2) and not further reducible with respect to the separation of approaching pairs of meson lines l_i, l'_i and l_{i+1}, l'_{i+2} (<u>i</u> assumes all possible values).

We shall denote the contribution of the <u>i</u>th part of type n_i by f_{n_i} (assuming that we have numbered all possible irreducible parts):

$$f_{n_i}(z_i, z_{i+1}) = f_{n_i}[\max(z_i, z_{i+1})], \quad z_i = \ln(-l_i^2/m^2).$$

The contribution of this type of diagram is

$$f_{n_0, n_1, \ldots, n_N}(\xi) =$$
$$= \left(\frac{-g_0^2}{4\pi}\right)^N \int_\xi^L dz_1 \int_\xi^L dz_2 \ldots \int_\xi^L dz_N f_{n_0}(\xi, z_1)\, d^2(z_1)\, f_{n_1}(z_1, z_2)\, d^2(z_2) \ldots d^2(z_N)\, f_N(z_N, \xi). \tag{1}$$

We divide the region of integration over $z_1, \ldots\ldots, n_N$ into N regions; the <u>j</u>th region is defined by the inequality

$$z_j \leqslant z_1, z_2, \ldots, z_{j-1}, z_{j+1}, \ldots, z_N; \quad j = 1, \ldots, N.$$

The right-hand side of (1) goes over into the sum of N terms. In the <u>j</u>th term we integrate over all z_k, $k \neq j$. Then using directly the formulas obtained from (1)

$$\left(\frac{-g_0^2}{4\pi}\right)^{j-1} \int_{z_j}^L dz_1 \ldots \int_{z_j}^L dz_{j-1} f_{n_0}(\xi, z_1)\, d^2(z_1) \ldots d^2(z_{j-1})\, f_{n\,j-1}(z_{j-1}, z_j) =$$
$$= f_{n_0, \ldots, n_{j-1}}(z_j),$$

$$\left(\frac{-g_0^2}{4\pi}\right)^{N-j} \int_{z_j}^L dz_{j+1} \ldots \int_{z_j}^L dz_N f_{n_j}(z_j, z_{j+1})\, d^2(z_{j+1}) \ldots d^2(z_N)\, f_N(z_N, \xi) =$$
$$= f_{n_j, \ldots, n_N}(z_j),$$

we have

$$f_{n_0, n_1, \ldots, n_N}(\xi) = \sum_{j=1}^N \left(\frac{-g_0^2}{4\pi}\right) \int_\xi^L f_{n_0, \ldots, n_{j-1}}(z_j)\, d^2(z_j)\, f_{n_j, \ldots, n_N}(z_j)\, dz_j. \tag{2}$$

The meson-meson scattering amplitude is expressed as follows in terms of the quantities $f_{n_0, \ldots, n_N}$:

$$P(\xi) = \sum_{N=0}^\infty 2^{-N} \sum_{n_0, \ldots, n_N} f_{n_0, \ldots, n_N}(\xi).$$

The factor 2^{-N} represents the fact that in summing over all possible irreducible parts n_j we obtain 2^N identical resulting diagrams. Inserting in

$$F(\xi) = \sum_{N=1}^\infty 2^{-N} \sum_{n_0, \ldots, n_N} f_{n_0, n_1, \ldots, n_N}(\xi)$$

the Expression (2) for $f(\xi)$ we obtain

$$F(\xi) = \left(\frac{-g_0^2}{8\pi}\right) \int_\xi^L P^2(z)\, d^2(z)\, dz. \tag{3}$$

The total amplitude P is expressed [3] as the sum of six squares shown in Fig. 3 and denoted by us as $R_0(\xi)$, and the sum of three $F(\xi)$ corresponding to the sets of diagrams reduced to Figs. 1a, b, c:

$$P(\xi) = R_0(\xi) + 3F(\xi). \tag{4}$$

Substituting (3) in (4) we finally obtain

$$P(\xi) = R_0(\xi) - \frac{3g_0^2}{8\pi}\int_\xi^L P^2(z)\,d^2(z)\,dz,$$
$$R_0 = 24(1-x), \quad x = [1 + (5g_0^2/4\pi)(L-\xi)]^{-1/5}. \tag{5}$$

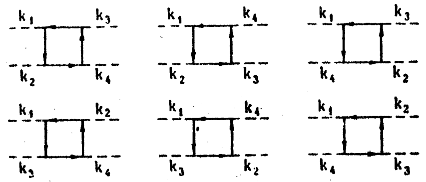

Fig. 3.

Our derivation can be extended with very insignificant complications associated with the appearance of isotopic indices to the symmetric pseudoscalar theory and gives the following equations:

$$F_{\tau_1\tau_2,\,\tau_3\tau_4}(\xi) = -\frac{g_0^2}{8\pi}\int_\xi^L P^2(z)\,d^2(z)\,dz\,[2\delta_s + 5\delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4}], \tag{3'}$$

$$P_{\tau_1\tau_2\tau_3\tau_4}(\xi) = P(\xi)\delta_s, \quad \delta_s = \delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4} + \delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} + \delta_{\tau_1\tau_4}\delta_{\tau_2\tau_3},$$
$$P(\xi)\,\delta_s = \rho_0\delta_s + F_{\tau_1\tau_2,\,\tau_3\tau_4}(\xi) + F_{\tau_1\tau_3,\,\tau_2\tau_4}(\xi) + F_{\tau_1\tau_4,\,\tau_2\tau_3}(\xi), \tag{4'}$$

$$P(\xi) = \rho_0(\xi) - \frac{11g_0^2}{8\pi}\int_\xi^L P^2(z)\,d^2(z)\,dz, \tag{5'}$$

where $\rho_0(\xi)\,\delta_s$ is the sum of six squares (Fig. 3);

$$\rho_0(\xi) = {}^{16}/_3\,(y-1), \quad y = [1 + (5g_0^2/4\pi)(L-\xi)]^{3/5}.$$

The integral Equations (5) and (5') reduce to the differential equations

$$\frac{dP}{dx} = -24 + {}^3/_2\left(\frac{P}{x}\right)^2, \quad P(1) = 0, \tag{6}$$
$$\frac{dP}{dy} = {}^{16}/_3 - {}^{11}/_6\left(\frac{P}{y}\right)^2, \quad P(1) = 0. \tag{6'}$$

The exact solutions of (6) and (6') are

$$P(x) = \frac{\sqrt{145}+1}{3}\,x\,\frac{1 - x^{\sqrt{145}}}{1 + [(\sqrt{145}+1)/(\sqrt{145}-1)]\,x^{\sqrt{145}}}, \tag{7}$$

$$P(y) = \frac{16}{11}\,y\,\frac{1 - y^{-19/3}}{1 + {}^8/_{11}x^{-19/3}}. \tag{7'}$$

It is seen from (7) and (7') that P in both cases is of the same order of magnitude as the contributions R_0 and ρ_0 from the very simple diagrams of Fig. 3. This supports the conclusion in [4] that the meson has zero charge.

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