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Inversion domains and densities of states in gapless semiconductors in a quantising magnetic field

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Abstract. Gapless semiconductors with the linear electron spectrum are considered taking into account a quantising magnetic field. Deformation potential interactions with phonons are shown to induce the spontaneous destruction of the zero-gap state due to the lowering of the effective space dimension in the magnetic field. The interaction with impurities is shown to shift the band inversion point. The Bogoliubov–de Gennes equations are derived; their inhomogeneous solutions are kinks linking the normal and inverted band domains. The electronic densities of states for two- and three-dimensional zero-gap states in a magnetic field are exactly calculated taking into account the Gaussian random potential and their singularities in the vicinity of band crossing points are studied. Possible consequences of these results for experiments with narrow-gap semiconductors near the inversion point are discussed.

1. Introduction

Ever since the theoretical work of Abrikosov and Beneslavsky (1970, 1971) on the problem of the Fermi point existence the interest in gapless semiconductors has persisted up to the present time (Gel'mont *et al* 1976, Tsidilkovski *et al* 1985). In contrast to the type II gapless semiconductors (α-Sn, HgTe etc), which have been well studied, the problem of type I gapless semiconductors is yet to be solved. The first kind of gapless state (GS I) can for example, be realised in solid solutions $Pb_{1-x}Sn_xTe$,. The zero-gap state in this case is not a result of symmetry but as a result of the play of parameters (x, pressure and temperature).

The aim of this work is to study GS I in an ultra-quantum magnetic field in the presence of random impurities and electron–phonon interactions. The present paper is organised as follows. In §2 we define the two-band model of GS I and construct eigen-states of Dirac's electron in a strong magnetic field. We use the relativistic holomorphic representation for these states in the first Landau band. In §3 we show the instability of GS I due to the electron–phonon interaction. We use an effective one-dimensionalisation of the system (in the general case the dimension is lowered to two). In §4 we suggest the possibility for domains of the normal and inverted bands to exist. Sections 5 and 6 are devoted to the evaluation of the electronic density of states in the presence of random impurity fields in two and three dimensions, respectively. Our results are summarised and discussed in the concluding §7.

2. Excitations in gapless semiconductors

The Hamiltonian for electronic GS I in a one-electron approximation is

$$\mathscr{H}_0 = s\rho_1\sigma\pi \tag{2.1}$$

where s is a limiting velocity of electrons in Cohen–Blount model (Cohen and Blount 1960), $\pi = P - (e/c)A$, ρ_1 and σ are the Pauli matrices in band and Kramers spaces respectively. The corresponding electron spectrum has the form

$$E_{\sigma n}(P_z) = \pm\left[(sP_z)^2 + (sP_\perp)^2\left(n + \frac{1-\sigma}{2}\right)\right]^{1/2} \tag{2.2}$$

where $n = 0, 1, 2\ldots$ are Landau level numbers, $\sigma = \pm 1$, $sP_\perp = 2^{1/2}\hbar s/l_H; = (\hbar c/eH)^{1/2}$ is a magnetic length; the electron spectrum (2.2) gives the following density of states (DOS)

$$\rho_H(E) = \rho_{0H}\sum_{n=0}^{n_{max}}\sum_{\sigma} E\left[E^2 - (sP_\perp)^2\left(n + \frac{1-\sigma}{2}\right)\right]^{-1/2}. \tag{2.3}$$

We have to sum over all n preserving the positiveness of the expression under the radical.

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Let us consider the ultra-quantum limit ($\mu(H) < sP_\perp$; $\mu(H)$ is a chemical potential). For $n=0$, $\sigma=1$ we have for spectrum and DOS (Petrov 1979):

$$E_{0+} = \pm sP_z \tag{2.4}$$

$$\rho_{0H} = (2\pi^2\hbar s l_H^2)^{-1} = \frac{1}{\pi\hbar s}\frac{1}{2\pi l_H^2}. \tag{2.5}$$

The linear size L of the system is equal to 1.

So one can see that a strong magnetic field preserves GS I and in contrast to the one-band case the DOS has a flat domain between the zero-point and the next Landau level (see figure 1). DOS (2.5) has one-dimensional form $(\pi\hbar s)^{-1}$ and $1/2\pi l_H^2$ is a degeneracy factor ($L^2=1$).

The Bose branch of the spectrum can be obtained from the following vertex function

$$\Gamma(\omega,q) = \frac{4\pi e^2}{\varepsilon_0 q^2 - 4\pi e^2 \Pi(\omega, q)} \tag{2.6}$$

which is a polarisation operator, and which must be calculated with the Green function:

$$G(r_1, r_2, \omega_1) = \frac{1}{\pi^{1/2} l_H \hbar s}\int \frac{dk_y dk_z}{(2\pi)^2} \exp\left[ik_y(y_2-y_1) + ik_z(z_2-z_1)\right]\hat{M}(\omega_1, k_z)$$

$$\times \exp\left(-\frac{1}{2l_H^2}[(x_1 - l_H^2 k_y)^2 + (x_2 - l_H^2 k_y)^2]\right) \tag{2.7}$$

where

$$\hat{M}(\omega_1, k_z) = \begin{bmatrix} \dfrac{1}{\omega_1 - sk_z} & 0 \\ 0 & \dfrac{1}{\omega_1 + sk_z} \end{bmatrix}$$

ε_0 is a dielectric constant taking into account only lattice polarisation; the Landau gauge was taken. After some calculations we have

$$\Pi(\omega, q) = \frac{\exp[-l_H^2 q_\perp^2/2]}{\hbar s l_H^2}\left[\Pi_+(q_z, \omega) + \Pi_-(q_z, \omega)\right] \tag{2.8}$$

where $q_\perp^2 = q_x^2 + q_y^2$

$$\Pi_+(q_z, \omega) = \frac{sq_z}{\omega - sq_z} \qquad \Pi_-(q_z, \omega) = -\frac{sq_z}{\omega + sq_z}. \tag{2.9}$$

Equation (2.8) gives us the generalisation to symmetry 0_h of the results for GS I in semiconductors with symmetry D_n (Kulikov 1974). In particular (2.8) takes into account left (Π_-) and right (Π_+) particles.

The quasi-one-dimensional character of the excitation spectrum is obvious from (2.8) and (2.9). It is easy to see that the polarisation operator coincides with the one obtained for the Luttinger model without a magnetic field. From the vertex poles we have for collective excitations (cf Kulikov 1974)

$$\omega = \pm sq_z\left(1 + \frac{e^2}{\varepsilon_0\hbar s}\frac{8\pi}{l_H^2(q_z^2+q_\perp^2)}\right)^{1/2}. \tag{2.10}$$

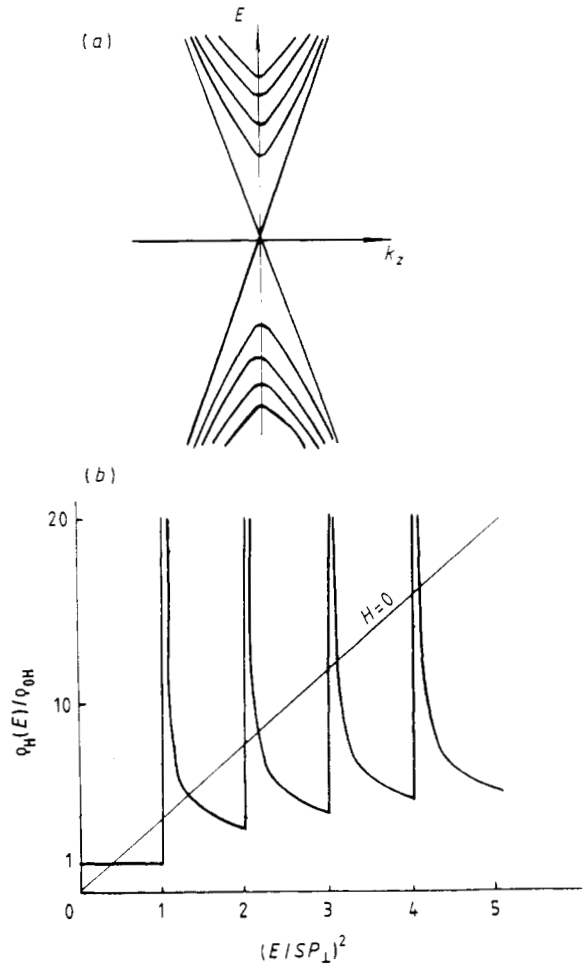

Figure 1. (*a*) Energy spectrum and (*b*) density of states of unperturbed states in a quantising magnetic field.

Only for $q_\perp = 0$ there is a gap $\omega_0 = s(\sqrt{\gamma})/l_H$ in the spectrum

$$\omega = \pm(\omega_0^2 + s^2 q_z^2)^{1/2} \tag{2.11}$$

with $\gamma = 8\pi e^2/\varepsilon_0\hbar s \sim 1$.

When $q_\perp \neq 0$ we have the gapless spectrum which can be written in a simple form for $q_z \ll q_\perp$

$$\omega = \pm \tilde{s} q_z \tag{2.12}$$

where

$$\tilde{s} = s\left(1 + \frac{\gamma}{l_H^2 q_\perp^2}\right)^{1/2} \tag{2.13}$$

and for $q_\perp \ll l_H^{-1}$, $\tilde{s} = \omega_0/q_\perp > s$.

The excitation spectrum is given in figure 2.

2.1. Holomorphic representation for GS I

Holomorphic representation (HR) is rather fruitful for studying GS I in ultra-quantum limit (Ktitorov and Petrov 1986). We choose the symmetric gauge for the vector potential

$$A = \tfrac{1}{2}[\boldsymbol{H} \times \boldsymbol{r}] \qquad H \| e_z. \tag{2.14}$$

The natural generalisation of the HR for the Schrödinger equation with a magnetic field to the relativistic (formal)

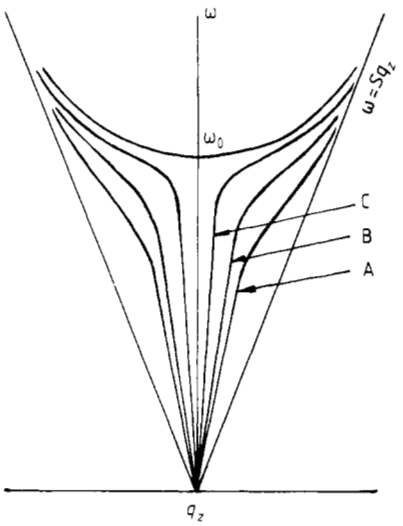

Figure 2. Excitations spectrum $\omega(q_z, q_-)$ for various $q_\perp$: $q_{\perp 3} < q_{\perp 2} < q_{\perp 1}$.

case is (Ktitorov and Petrov 1986)

$$\psi_i(x, y) = e^{-\zeta\bar{\zeta}} u_i(\zeta) \tag{2.15}$$

where

$$\zeta = \frac{x + iy}{2l_H} \qquad \bar{\zeta} = \frac{x - iy}{2l_H} \tag{2.16}$$

where u_i is the ith component of the spinor; u_i is a holomorphic function of ζ in the Fock–Bargmann space (Bargmann 1961)

$$\frac{\partial u_i(\zeta)}{\partial \bar{\zeta}} = 0 \qquad \frac{\partial \bar{u}_i(\bar{\zeta})}{\partial \zeta} = 0. \tag{2.17}$$

The normalised one-electron gapless state with energy given by (2.4), quasi-momentum k_z and $\sigma_z = 1$ read as

$$|0, m> = \frac{\exp[\pm iEt + ik_z z]}{\sqrt{2}} \frac{(b^+)^m}{\sqrt{m!}} |0, 0> \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2.18}$$

where $m = 0, 1, 2, \ldots$

$$|0, 0> = [(2\pi)^{1/2} l_H]^{-1/2} \exp[-\zeta\bar{\zeta}]. \tag{2.19}$$

Ladder Bosonic operators b and b^+ can be expressed in terms of ζ and $\bar{\zeta}$:

$$b = \frac{1}{\sqrt{2}} \left(\bar{\zeta} + \frac{\partial}{\partial \zeta} \right) \qquad b^+ = \frac{1}{\sqrt{2}} \left(\zeta - \frac{\partial}{\partial \bar{\zeta}} \right) \tag{2.20}$$

$$[b, b^+] = 1 \qquad [b, b] = [b^+, b^+] = 0. \tag{2.21}$$

Now we can construct the operators $\hat{\zeta}$, $\hat{\bar{\zeta}} \equiv \partial/\partial\zeta$, that only change the quantum numbers corresponding to the angular momentum component in the magnetic field direction but which don't change the Landau band numbers. Commutation rules for $\hat{\zeta}$, $\hat{\bar{\zeta}}$ are

$$[\hat{\bar{\zeta}}, \hat{\zeta}] = 1 \qquad [\hat{\zeta}, \hat{\zeta}] = [\hat{\bar{\zeta}}, \hat{\bar{\zeta}}] = 0. \tag{2.22}$$

Equation (2.18) for $|0, m\rangle$ can be written in terms of ζ, $\bar{\zeta}$:

$$|0, m> = \frac{\exp[ik_z z]}{\sqrt{2}} \frac{1}{\sqrt{2\pi}\, l_H} \frac{\zeta^m}{\sqrt{m!}} e^{-\zeta\bar{\zeta}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{2.23}$$

An arbitrary linear combination of the states (2.23) with the different m can be expressed in the form (2.15). In order to establish the relativistic HR we can use the well known connection of HR with the relativistic coherent state representation (Malkin and Man'ko 1968). The coherent state with the Landau numbers n, $\sigma_z = 1$ can be written as

$$\psi_{n, \sigma_z = 1}(x, y) = \frac{1}{\sqrt{2}} \begin{bmatrix} |n, \beta> \\ \dfrac{(\sqrt{n})\hbar s}{l_H} |n-1, \beta> \\ |n, \beta> \\ 0 \end{bmatrix} \tag{2.24}$$

For GS I $n = 0$ the small components of the spinor are equal to zero. So we have

$$\psi_{0, \sigma_z = 1}(x, y) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} |0, \beta> \tag{2.25}$$

where $|0, \beta>$ are the eigen-states of the annihilation operator b

$$b|0, \beta\rangle = \beta|0, \beta\rangle \tag{2.26}$$

$$|0, \beta\rangle = (\pi^{1/2} l_H)^{-1} \exp\left[-\zeta\bar{\zeta} + 2^{1/2}\beta\zeta - \frac{|\beta|^2}{2} \right]. \tag{2.27}$$

The function $|0, \beta\rangle$ is the generating function for $|0, m\rangle$ states with a given m.

We give here some useful formulae generalising the Dirac δ-function on the Bargmann space

$$\begin{aligned}
&\frac{1}{\pi} \int d\zeta\, d\bar{\zeta}\, e^{\bar{z}\zeta - |\zeta|^2} f(\bar{\zeta}) = f(\bar{z}) \\
&\frac{1}{\pi} \int d\zeta\, d\bar{\zeta}\, e^{\bar{z}\zeta - |\zeta|^2} \zeta^n f(\bar{\zeta}) = \left(\frac{\partial}{\partial \bar{z}} \right)^n f(\bar{z}) \\
&\frac{1}{\pi} \int d\zeta\, d\bar{\zeta}\, e^{z\bar{\zeta} - |\zeta|^2} f(\zeta) = f(z) \\
&\frac{1}{\pi} \int d\zeta\, d\bar{\zeta}\, e^{z\bar{\zeta} - |\zeta|^2} (\bar{\zeta})^n f(\zeta) = \left(\frac{\partial}{\partial z} \right)^n f(z)
\end{aligned} \tag{2.28}$$

3. Spontaneous mass generation

In the absence of a magnetic field electron–phonon and electron–impurity interactions don't generate electron mass (gap) spontaneously. Indeed, Dyson's equation in the case of three-dimensional crystal can be written

$$M - \tilde{m} = M\lambda \int_0^\Lambda \frac{P^2 dP}{P^2 + M^2} + Mg^2 \int_{-\infty}^{\infty} d\varepsilon \int_0^\Lambda \frac{P^2 dP}{P^2 - \varepsilon^2 + M^2} \tag{3.1}$$

where $\tilde{m}$ and M are bare and dressed masses respectively; λ and g are electron–impurity and electron–phonon interaction constants, respectively; $\Lambda \sim 1/a$ is maximum momentum; a is lattice spacing. One can see, that because

of infrared divergency absence in (3.1) while $M \to 0$, the spontaneous mass (gap) generation is not possible.

Effective lowering of the dimensions because of the magnetic field influence gives us some hope that infrared divergence appears with resulting mass generation.

We start from the generating functional

$$Z[J, \bar{J}] = Z_0^{-1} \int \mathrm{D}\varphi \, \mathrm{D}\psi \mathrm{D}\bar{\psi} \, \mathrm{e}^{\mathrm{i}\mathscr{A}[\varphi, \psi, \bar{\psi}]} \tag{3.2}$$

with the action $\mathscr{A}[\varphi, \psi, \bar{\psi}]$.

$$\mathscr{A} = \mathscr{A}_{\mathrm{el}} + \mathscr{A}_{\mathrm{Ph}} + a_{\mathrm{int}} \tag{3.3}$$

$$\mathscr{A}_{\mathrm{el}} = \int \mathrm{d}t \, \mathrm{d}^3x \sum_{a=1}^{N} \bar{\psi}_a \left\{ \hbar s \left[\mathrm{i}\gamma \left(\nabla - \frac{\mathrm{i}eA}{c} \right) \right] + \tilde{m}s^2 \right\} \psi_a \tag{3.4}$$

$$\mathscr{A}_{\mathrm{Ph}} = \frac{\hbar^2 \omega_0^2}{2} \int \mathrm{d}t \, \mathrm{d}^3x \, \varphi^2(x, t) \tag{3.5}$$

$$\mathscr{A}_{\mathrm{int}} = g \int \mathrm{d}t \, \mathrm{d}^3x \left[\sum_{a=1}^{N} \varphi(x,t) \bar{\psi}_a(x, t) \psi_a(x, t) + \sum_{a=1}^{n} (\bar{J}_a(x, t) \psi_a(x, t) + \bar{\psi}_a(x, t) J_a(x, t)) \right] \tag{3.6}$$

where ψ and $\bar{\psi}$ are Grassman fields, $\varphi(x, t)$ is a phonon field, g is a Fröhlich electron–phonon constant, $\bar{J}/J$ are anti-fermion/fermion sources, a is a number of the electron valley running from 1 to N, where N is the number of valleys and Z_0 is a normalisation factor. Carrying out the functional integration over the phonon field φ and introducing the Grassman HR (Faddeev 1976) we have

$$Z = Z_0^{-1} \int \mathrm{D}u \, \mathrm{D}\bar{u} \exp[-\mathrm{i}\mathscr{A}_{\mathrm{hol}}^{\mathrm{eff}}] \tag{3.7}$$

$$\mathscr{A}_{\mathrm{hol}}^{\mathrm{eff}} = \int \mathrm{d}\zeta \, \mathrm{d}\bar{\zeta} \, \mathrm{d}\omega \, \mathrm{d}k \, \mathrm{e}^{-\zeta\bar{\zeta}/2} \sum_a u_{ka}(\bar{\zeta}) \times (\hbar s k \gamma_3 - \omega\gamma_0 + \tilde{m}s^2) u_{ka}(\zeta) + \Gamma \int \mathrm{d}\zeta \, \mathrm{d}\bar{\zeta} \, \mathrm{d}\kappa_i \, \mathrm{d}\omega \, \mathrm{e}^{-\zeta\bar{\zeta}} \times \sum_{a,b} \bar{u}_{k_1 a}(\bar{\zeta}) u_{k_2 a}(\zeta) \bar{u}_{k_3 b}(\bar{\zeta}) u_{k_4 b}(\zeta) \delta\left(\sum_{i=1}^{4} k_i \right) + \int \mathrm{d}\zeta \, \mathrm{d}\bar{\zeta} \, \mathrm{d}\omega \, \mathrm{d}k \sum_a (\bar{J}_a u_a + \bar{u}_a J_a) \tag{3.8}$$

where $\Gamma = g_{\mathrm{opt}}^2/\omega_0^2$, ω_0 is an optical phonon frequency, in the case when acoustic phonons are effective, $\Gamma = g_{\mathrm{ac}}^2/c^2$, where c is the sound velocity.

The process of spontaneous symmetry breaking is the result of infrared divergence of the Feynman diagrams; that is why phonons with wavelengths less than l_{H} are not essential for a qualitative consideration. Corresponding with Wilson's ideas (Wilson and Kogut 1974) the contribution of such phonons can be treated perturbatively, which leads only to a quantative renormalisation of the constants in the effective action. Supposing that this operation has been carried out we cut all ultraviolet divergencies at l_{H}^{-1} instead of at the Brillouin zone boundary.

The holomorphic formalism leads to the usual perturbation theory with propagator, which is the inverse of the operator in the quadratic part of the action (3.8). Comparing (2.16) and (3.8) we get the propagator

$$G_{\mathrm{ab}}^{(0)} = \delta_{\mathrm{ab}} l_{\mathrm{H}}^{-2} \frac{\exp[\zeta\bar{\zeta}'/2]}{\hbar s k \gamma_3 - \omega\gamma_0 + \tilde{m}s^2} \tag{3.9}$$

where γ_i are Dirac matrices; $k \equiv k_z$. The interaction vertex has a Gaussian form

$$-l_{\mathrm{H}}^2 \Gamma \, \mathrm{e}^{2\zeta\bar{\zeta}}. \tag{3.10}$$

There is a small parameter in our theory $-N^{-1}$ for $N > 1$ that allows us to solve this problem for strong interaction Γ. Using a $1/N$ expansion in leading order we have to take into account only the tadpole graph for the mass operator. So, we have an asymptotically exact equation

$$G_{k\omega, ab}^{\alpha\beta}(z, \bar{z}') = G_{k\omega, ab}^{(0)\alpha\beta}(z, \bar{z}') + (-\mathrm{i}) \int \mathrm{d}\zeta \, \mathrm{d}\bar{\zeta} \, G_{k\omega, ac}^{(0)\alpha\gamma}(z, \bar{\zeta}) \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega'}{2\pi} \int_0^{\hbar/l_{\mathrm{H}}} \frac{\mathrm{d}k'}{2\pi} \times T_{cc'd'd}^{\gamma\gamma'\delta'\delta}(k', \omega') G_{k'\omega', c'd}^{\gamma'\delta'}(\zeta, \bar{\zeta}) G_{k\omega, db}^{\delta\beta}(\zeta, \bar{z}') \tag{3.11}$$

where T is the symmetrised vertex

$$T = -\frac{\Gamma}{4! l_{\mathrm{H}}^2} (\delta_{ab}\delta_{cd}\delta_{pq}\delta_{rs} - \delta_{ad}\delta_{bc}\delta_{ps}\delta_{qr}) \tag{3.12}$$

where p, q, r, s are spinor indices.

Taking the trace we have the equation for electron mass

$$M - \tilde{m} = M \int_0^{\hbar/l_{\mathrm{H}}} \frac{\mathrm{d}k}{2\pi} \frac{\Lambda}{l^2 + M^2} + M \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \int_0^{\hbar/l_{\mathrm{H}}} \frac{\mathrm{d}k}{2\pi} \frac{\Gamma}{l_{\mathrm{H}}^2} \frac{N-1}{k^2 - \varepsilon^2 + M^2}. \tag{3.13}$$

The influence of impurities can similarly be taken into account by using the replica trick. Carrying out the integration in (3.11) we have

$$M - \tilde{m} = \frac{\Lambda}{2\pi} \tan^{-1} \frac{\Delta}{M} + \frac{\Gamma(N-1)}{4\pi l_{\mathrm{H}}^2} M \ln \frac{2\Delta}{M} \qquad \Delta = \frac{\hbar}{l_{\mathrm{H}}}. \tag{3.14}$$

The equation (3.14) gives the dependence of the physical mass (gap) M upon the bare mass $\tilde{m}$ which is supposed to be a known function of pressure, temperature, solid-solution components concentration etc (See figure 3).

Let us consider some special cases.

(i) When $\lambda = 0$ (electron–phonon interaction only), we have the equation

$$M - \tilde{m} = \frac{\Gamma N}{4\pi l_{\mathrm{H}}^2} M \ln \frac{2\Delta}{M}. \tag{3.15}$$

If the bare mass $\tilde{m}$ is zero, we have a non-trivial solution

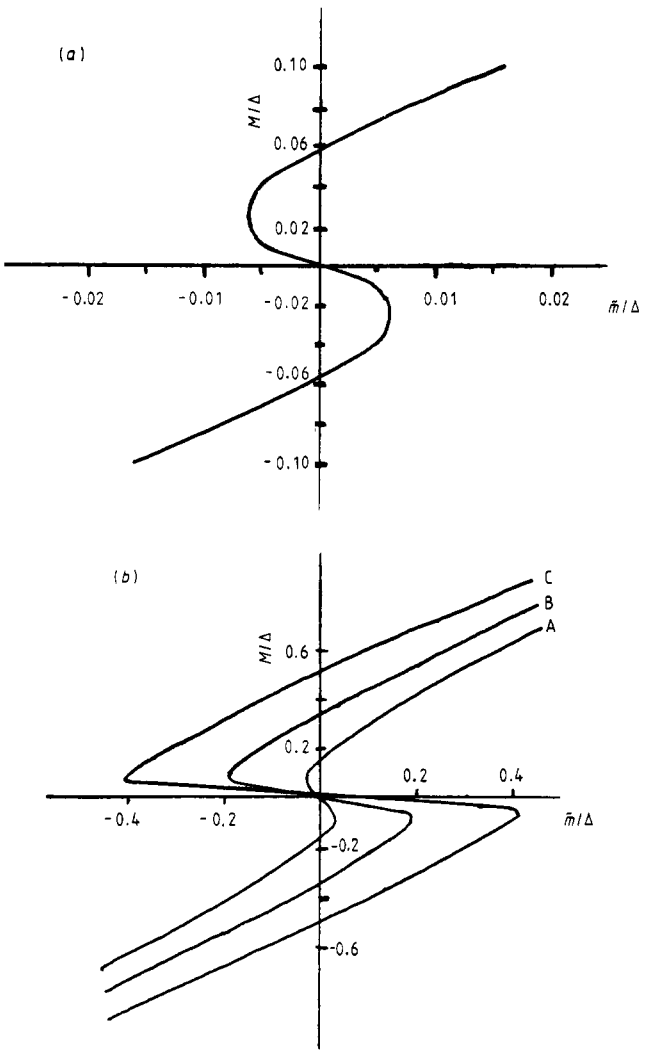

Figure 3. Spontaneous mass generation. (*a*) $\alpha=0$ and (*b*) $\alpha=$A, 0.1(1); B, 0.5(2); C, 1.0. ($\alpha=\lambda/\gamma$ is electron–impurity and electron–phonon coupling constants ratio)

for the spontaneously generated gap

$$E_g^{(0)}=2Ms^2=\frac{4\hbar s}{l_H}\exp\left[-\frac{4\pi l_H^2}{N\Gamma}\right]. \qquad (3.16)$$

(ii) When impurities are taken into account, as well, we also have hysteretic character of the dependence, but with a higher 'coercivity' m_c.

One can see from (3.14) and figure 3 that the physical gap monotonously decreases with the decrease of the bare gap until it reaches the value of spontaneous gap $E_g^{(0)}$ and then it jumps to the value $-E_g^{(0)}$ that corresponds to the valence and conductivity bands inversion. So GS I in a magnetic field becomes unachievable due to the spontaneous mass generation.

An estimation of the order of $E_g^{(0)}$ gives $E_g^{(0)}\sim 1-10$ K. Here we used following conventional values of parameters for IV–VI semiconductors $S\sim 3\times 10^7$ cm s^{-1}, $\omega_0\sim 10^{12}-10^{13}$ s^{-1}, mass density ~ 8 g cm^{-3}, spacing $9\sim 6\times 10^{-8}$ cm and $H\sim 10^4-10^5$ Oe.

The effect of spontaneous-gap generation predicted here can be important for the kinetic phenomena in IV–VI compounds when temperature and chemical potential or photon frequency are less or equal to $E_g^{(0)}$. On the other hand we can have the case when electron DOS is smeared by impurities so that we can neglect the gap $E_g^{(0)}$. We shall meet such a case in §§ 5 and 6.

4. Inversion domains

The 'newly born' ground state is a doubly degenerous normal phase with a positive gap (the normal position of bands) and inversion phase which has similar macroscopic energies. It can lead to the domain-wall-type solutions, linking normal and inversion phases.

The Lagrangian for inhomogenous solution is

$$\mathscr{L}(z)=\sum_{a=1}^{N}\bar{u}_a(z)\left(i\hbar s\gamma_3\frac{\partial}{\partial z}\right)u_a(z) - \tilde{g}\varphi(z)\sum_{a=1}^{N}\bar{u}_a(z)u_a(z)-\tfrac{1}{2}\varphi^2(z) \qquad (4.1)$$

where $\varphi(z)$ is a phonon field, $\tilde{g}$ is the electron–phonon interaction constant. Equivalence of (4.1) and (3.8) can be proved carrying out the integration over ζ, $\bar{\zeta}$ in (3.8), that can be done for the slowly varying fields $u(\zeta)$, $\bar{u}(\bar{\zeta})$ on the scale of $\zeta\sim l_H$ (Ktitorov and Petrov 1986). Resulting Lagrangian corresponds to the Gross–Neveu model (Gross and Neveu 1974). Introducing the auxilliary field $\varphi(z)$ we get (4.1).

Classical equations for the stationary states are the following

$$\left(i\hbar s\gamma_3\frac{\partial}{\partial z}-\tilde{g}\varphi(z)+E\right)u_a(z)=0. \qquad (4.2)$$

Introducing the following new variables

$$U_1^a=\frac{u_1^a+u_2^a}{2} \qquad U_2^a=\frac{u_1^a-u_2^a}{2i} \qquad (4.3)$$

we get the equations

$$i\hbar s\frac{\partial U_1}{\partial z}+\Delta(z)U_2=EU_1 \qquad (4.4)$$

$$-i\hbar s\frac{\partial U_2}{\partial z}+\Delta(z)U_1=EU_2 \qquad (4.5)$$

with the equation of self-consistency

$$\Delta(z)=\frac{-\tilde{g}^2}{2\omega_0^2 l_H^2}\sum_{\substack{\text{occupied}\\ \text{states}}}\left(\bar{U}_1(z)U_2(z)+\bar{U}_2(z)U_1(z)\right). \qquad (4.6)$$

It is easy to see that the homogeneous solution of these equations is given just by (3.16).

Equations (4.4)–(4.6) are analogous to ones that were obtained by a number of authors in the theory of the Peierls systems (Su *et al* 1980, Brazovzskii 1980, Takayama *et al* 1980). These equations have been very well studied, in particular, they have a number of inhomogeneous solutions (Campbell and Bishop 1982).

Most interesting for us are the kinks linking the domains with the normal and inverted band position and polarons corresponding to completely localised states.

The kink solution reads

$$\Delta(z)=\Delta_0 \tanh \frac{\Delta_0}{\hbar s}(z-z_0) \tag{4.7}$$

$$U_1(z)=-\mathrm{i}\left(\frac{\Delta_0}{4\hbar s}\right)^{1/2} \operatorname{sech} \frac{\Delta_0}{\hbar s}(z-z_0) \tag{4.8}$$

$$U_2(z)=\left(\frac{\Delta_0}{4\hbar s}\right)^{1/2} \operatorname{sech} \frac{\Delta_0}{\hbar s}(z-z_0) \qquad \Delta_0 \equiv E_g^{(0)}. \tag{4.9}$$

The bound electron state in the centre of the gap (as a matter of fact the two-dimensional band) lives on the surface of the kink. Some energy (about $E_g^{(0)}/2$) is necessary to create a kink in a dielectric state but if we have a doped semiconductor an electron can fall from the conductivity band to the two-dimensional band on the surface of the kink that makes this state energetically profitable.

Notwithstanding the mathematically similarity between the Peierls system and GS I, there is a deep difference in the physics. Two possible versions of dimerisation correspond to doubly degenerate states in the trans-polyacetylene, while in our case these are the normal and inverted electron bands.

The solutions found here can be important for galvanomagnetic phenomena in the vicinity of the inversion point. The possibility for electrons to occupy a two-dimensional state on the surface of the kink gives a very interesting method for studying a two-dimensional electron gas in a three-dimensional crystal. In analogy with the trans-polyacetylene one can wait electric current transport by heavy carriers — kinks along the magnetic field direction; a fractional charge can in principle, also be observed.

The possibility of controlling the system parameters by varying the applied magnetic field is an important advantage of this system making the experiment more manageable.

We have to answer the last (but not least) question: if the solutions obtained are stable in a one-dimensional system, why don't fluctuations destroy them? The same question takes place in the Peierls system theory. The answer for that theory is as follows. Interchain interaction stabilises the ground state and suppresses fluctuations. Our system as a matter of fact is also a quasi-one-dimensional one. To see it we transform our action into the representation with definite angular momentum component on the z axis

$$\varphi_{0,m}(r)=\frac{\mathrm{e}^{ik_2 z}}{(2L)^{1/2}}(2^{m+1}\pi m!)^{-1/2} \times \frac{(x+\mathrm{i}y)^m}{l_H^{m+1}} \exp\left(-\frac{x^2+y^2}{4l_H^2}\right)\begin{bmatrix}1\\1\end{bmatrix}. \tag{4.10}$$

Let us expand the field operators ψ and $\bar{\psi}$ on the full orthonormal basis (4.10)

$$\psi=\sum_{m,k_z} C_{m,k_z}\varphi_{0,m} \tag{4.11}$$

$$\bar{\psi}=\sum_{m,k_z} \bar{\varphi}_{0,m}\bar{C}_{m,k_z}. \tag{4.12}$$

The action (3.8) takes the form

$$\mathscr{A}=\mathscr{A}_0+\mathscr{A}_{\mathrm{int}} \tag{4.13}$$

$$\mathscr{A}_0(c,\bar{c})=\int \mathrm{d}z \sum_{m,a}\bar{c}_m^a(z)\left(-\mathrm{i}\hbar s\frac{\partial}{\partial z}+\sigma_3 E\right)c_m^a(z) \tag{4.14}$$

$$\mathscr{A}_{\mathrm{int}}(c,\bar{c})=\int \mathrm{d}z \sum_{m_1,m_2,a}\left(\bar{c}_{m_1}^a(z)c_{m_2}^a(z)\right)^2\Gamma. \tag{4.15}$$

Summing m_1 and m_2 they tend to the maximum value $1/2\pi l_H^2$ $(L=1)$.

The system with action (4.15) can be considered as an array of the chains with interchain interaction that doesn't depend on the number of the chain. So our system has quasi-one-dimensional action. It is known (Stanley 1973) that in the limit $1/2\pi l_H^2 \to \infty$, the saddle-point (quasi-classical) solution is asymptotically exact and it does not depend on m. This proves our supposition that our solutions do not depend on the x and y coordinates, and are stable. As the mean-field theory is asymptotically exact in the limit $L/l_H \to \infty$ the obtained solutions exist in some temperature region $0<T<\Delta$. One could obtain these results in holomorphic representation using the saddle-point method in the corresponding Fock–Bargmann space.

5. The density of states of the gapless impurities semiconductor in a magnetic field

In this section the DOS of the gapless semiconductor in the quantum limit is evaluated. We first describe the procedure for one-dimensionalisation of the Hamiltonian. Then we shall exploit the correspondence between our model and the one-dimensional electron–impurity system without the magnetic field.

Free action in HR has the form

$$\mathscr{A}_0^{\mathrm{hol}}=\int \mathrm{d}k\,\mathrm{d}\omega\,\bar{u}_{k\omega}(\hbar s k\sigma_3-E)u_{k\omega} \tag{5.1}$$

where

$$\sigma_3=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

is a Pauli matrix, $k\equiv k_z$.

The term with interaction after averaging with respect to impurity potential leads to the $\lambda\varphi^4$ theory. Introducing an auxiliary field we have the effective action

$$\mathscr{A}_{\mathrm{int}}^{\mathrm{hol}}=\frac{1}{L}\int \mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}\,\mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}\omega\,\delta(\omega)\,\mathrm{e}^{-\zeta\bar{\zeta}/2} \times \bar{u}_{k_1\omega}(\bar{\zeta})V_{k_1-k_2}(\zeta,\bar{\zeta})u_{k_2\omega}(\zeta) \tag{5.2}$$

where $u_{k\omega}(\zeta)$ are the Grassmann holomorphic variables, $V_{k_1-k_2}(\zeta\bar{\zeta})$ is the Fourier transform of the impurity potential

$$V_{k_1-k_2}(\zeta,\bar{\zeta})=\int \mathrm{d}z\, \mathrm{e}^{\mathrm{i}z(k_2-k_1)}V(z,\zeta,\bar{\zeta}) \tag{5.3}$$

where ζ has a value of order l_H. We are interesting by the singularities of the DOS in the limit $E\to 0$, that corresponds to excitations with a wavelength $\lambda \gg l_H$. In this approximation we believe that $u_\omega(k,\zeta)$ and $V_\kappa(\zeta,\bar{\zeta})$ are smooth functions of ζ and $\bar{\zeta}$, Integrating in (5.2) with respect to ζ and $\bar{\zeta}$ the 'sharp' exponential function $\exp(-\zeta\bar{\zeta}/2)$ we finally have

$$a_{\mathrm{int}}^{\mathrm{eff}}=\int \mathrm{d}k_1\, \mathrm{d}k_2\, \bar{u}_\omega(k_1)V(k_1-k_2)u_\omega(k_2)\delta(\omega)\, \mathrm{d}\omega. \tag{5.4}$$

So far we have not considered the matrix structure of the impurity potential. We take into account two kinds of interactions: screened Coulomb impurities and 'chemical' defects with potentials η and ξ, respectively.

$$V(z)=\begin{bmatrix}\eta & \xi\\ \xi & \eta\end{bmatrix}=\eta(z)+\sigma_1\xi(z) \tag{5.5}$$

where

$$\sigma_1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

is a Pauli matrix.

The potential $\eta(z)$ modulates the chemical potential so that the conduction band and valence band extrema are moving in the same direction. The potential $\xi(z)$ modulates the gap so that extrema are moving in opposite directions ('chemical' impurities). In GS I (for example, in solid solutions of the IV–VI semiconductors) iso-electronic substitutional defects stand for 'chemical' impurities, and deviations from stoichiometry stand for Coulombic impurities.

In the coordinate representation (5.1) and (5.4) are given by

$$a_{\mathrm{eff}}=\int \mathrm{d}z\left[\bar{u}(z)\left(-\mathrm{i}\hbar s\sigma_3\frac{\partial}{\partial z}-E\right)u(z) + \bar{u}(z)(\eta(z)+\sigma_1\xi(z))u(z))\right]. \tag{5.6}$$

So in agreement with Wegner's (Wegner 1983) dimensional reduction theorem the problem of the influence of impurities on the electronic states is reduced to the one-dimensional problem of the particle in the random field without a magnetic field (Ovchinnikov and Erikhman 1977, Gredeskul and Pastur 1978). One can see that the action (5.6) doesn't depend on the quantum numbers corresponding to the angular momentum components on the magnetic field direction, so summing up these quantum numbers we get the degeneracy factor $L^2/2\pi l_H^2$.

If the width of the potential is considerably smaller than the distance between impurities, then for states near the special point of the spectrum $E=0$ a specific form of the impurity potential is not important (Liftshitz *et al* 1982). In this case the impurity field can be considered as Gaussian white noise.

We assume

$$\langle\xi(z)\rangle=0 \qquad \langle\eta(z)\rangle=0 \tag{5.7}$$

$$\langle\xi(z)\xi(z')\rangle=2D_1\delta(z-z') \tag{5.8}$$

$$\langle\eta(z)\eta(z')\rangle=2D_2\delta(z-z'). \tag{5.9}$$

Variation of a_{eff} with respect to the field $\bar{u}$

$$\frac{\delta\mathscr{A}_{\mathrm{eff}}}{\delta\bar{u}}=0 \tag{5.10}$$

yields the system of two first-order equations. These equations define the eigen-states of the electron near $E=0$

$$\begin{aligned}-\mathrm{i}\hbar s\frac{\partial u_1}{\partial z}+\eta(z)u_1+\xi(z)u_2&=Eu_1\\ \mathrm{i}\hbar s\frac{\partial u_2}{\partial z}+\eta(z)u_2+\xi(z)u_1&=Eu_2.\end{aligned} \tag{5.11}$$

The correspondence between the problem of three-dimensional electrons DOS in a magnetic field taking into account only the first Landau level and one-dimensional problem without a magnetic field follows from a comparison of (5.11) and (5.7–5.9) and equations in above-mentioned paper of Ovchinnikov and Erikhman.

The correct ground state is defined by introduction of the real vectors f and φ:

$$u_1=\frac{f_1+\varphi_2}{2}+\mathrm{i}\frac{f_2-\varphi_1}{2} \tag{5.12}$$

$$u_2=\frac{f_1-\varphi_2}{2}-\mathrm{i}\frac{f_2+\varphi_1}{2}. \tag{5.13}$$

Substituting (5.12) and (5.13) in (5.11) we can show that the equations for f and φ coincide and correspond to the spectrum (5.11); what is more we can see that the function $\rho_H(E)$ is symmetric relatively the point of intersection of the bands $E=0$. Therefore we can consider the case $E\geqslant 0$.

Following Liftschitz *et al* (1982) we obtained the Fokker–Planck equation for distribution $P(z,x)=\langle\delta(x-x(z))\rangle$ of the phase of $x(z)=f_2/f_1P(z,x)$ is the functional of the random fields $\eta(z)$ and $\xi(z)$. We have

$$\frac{\partial P}{\partial z}+\frac{\partial}{\partial x}\{[E(1+x^2)+2D(x^2-1)x]P\} - D\frac{\partial^2}{\partial x^2}[(x^2-1)^2P]=0 \tag{5.14}$$

The number of states in energy range $(0, E)$ is obtained from the oscillation theorem (Ovchinnikov and Erikhman 1977) and coincide with the poles of the function $x(z)$ in the region $(0, L)$, $L\to\infty$.

Finally we represent some appropriate results for DOS in the GS I quantum limit

$$\rho_H(0)=\rho_{0H}\frac{[(D_1+D_2)/D_2]}{F^2[\frac{1}{2},\frac{1}{2},1,D_1/(D_1+D_2)]} \tag{5.15}$$

where F is a degenerate hypergeometric function.

In the absence of the 'chemical' defects ($D_1=0$) the expression (5.15) is equal to the free DOS ρ_{0H}. For $D_2\to 0$, as can be seen from (5.15) DOS has a singularity in limit $E\to 0$

$$\rho_H=\frac{\rho_{0H}}{\pi^2}\frac{(D_1/D_2)^{1/2}}{\ln^2 D_1/D_2}. \tag{5.16}$$

Calculating only the Coulomb potential gives rise to a finite value of the DOS even for $E_g\neq 0$

$$\rho_H=\rho_{0H}/I_0^2\left(\frac{E_g\hbar s}{2D}\right) \tag{5.17}$$

where I_0 is Bessel function of the imaginary argument. The DOS (5.17) for $E_g=0$ is equal to unperturbed one. If we take into account only 'chemical' defects the number of states is equal to

$$N(E)=4D/\pi^2\hbar s\left[J_\nu^2\left(\frac{E\hbar s}{2D}\right)+N_\nu^2\left(\frac{E\hbar s}{2D}\right)\right] \tag{5.18}$$

where $J_\nu(x)$, $N_\nu(x)$ are Bessel and Neuman functions, $\upsilon=E_g\hbar s/2D$. It can be shown that the DOS $\rho_H(E)$ has the Dyson's singularity for $E\to 0$

$$\rho_H(E)=-\rho_{0H}\frac{2\pi D}{\hbar sE\ln^3(\hbar sE/4D)}. \tag{5.19}$$

For $E_g=D$ we obtain from (5.18) the DOS corresponding to the free one (2.5). This is a rather surprising result: we find that the energy gap is closed. If $E_g\gg 2D$ we have usual energy dependence of DOS in the second Landau sub-band

$$\rho_H(E)=\rho_{0H}E\left[E^2-\frac{2\hbar^2 s^2}{l_H^2}\right]^{-1/2} \tag{5.20}$$

The results obtained can be useful for a discussion of the experimental data in narrow-gap Kane-type semiconductors. In GS II where near the intersection point there is additional heavy parabolic band except linear ones our results can be useful if the contribution of the heavy band can be separated.

Note that Dyson's singularity leads to the delocalisation of the states near $E=0$ so that localisation length $l_{loc}\sim(E\rho(E)/N(E))^{-1}\to\infty$ when $E\to 0$. It can be essential for the galvanomagnetic phenomena. For finite gap $l_{loc}\sim(Dm_0s^2)^{-1}$.

6. Two-dimensional electronic gapless states in magnetic field

Two-dimensional electronic GS on the surface of a semiconductors were shown to exist in the works of Volkov and Pinsker (1977) and Djakonov and Haetsky (1981). Another possibility for two-dimensional GS to appear was suggested by Volkov and Pankratov (1985). It is shown that electrons living in the interface between IV–VI semiconductors with normal and inverted bands comprise the gapless fermionic branch. Corresponding electron states can be described by two-dimensional Dirac equations. These states are really influenced by lattice irregularities, impurities etc. The electron density of states (DOS) is particularly sensitive to the irregularities while a strong magnetic field perpendicular to the (interface) surface is applied. Such a problem in the case of one-component (Schrödinger) electron state was exactly solved by Wegner (1983) and then using a more elegant method by Brezin *et al* (1984).

Our purpose here is to generalise the above works on the case of the two-component (Dirac) electron states.

The Hamiltonian of two-dimensional electrons in the strong magnetic field in the presence of the random potential reads

$$\mathscr{H}=\mathscr{H}_0+V(r) \tag{6.1}$$

$$V(r)=V_1(r)+\sigma_1 V_2(r)$$

$$\langle V_i\rangle=0 \qquad \langle V_i(r)V_j(r')\rangle=\lambda_i\delta(r-r')\delta_{ij} \tag{6.2}$$

where V_1 is the screened Coulomb potential modulating the chemical potential and V_2 is the essentially short-range potential modulating electron mass (gap).

Calculating the Green function and averaging it over all configurations of the random potential we get

$$\langle G\rangle=\int P[V]G(x,x';V(x))\mathrm{D}V \tag{6.3}$$

$$G_{\alpha j,\alpha' j'1.1}(E+\mathrm{i}\delta,r^1)=-\mathrm{i}\left(\int \mathrm{D}\varphi\mathrm{D}\bar\varphi\varphi_{\alpha j}(r)\bar\varphi_{\alpha'\delta'}(r')\right.$$

$$\left.\times\exp\left[\mathrm{i}\int \mathrm{d}^2r''\bar\varphi(E-\mathscr{H}+\mathrm{i}\delta)\varphi\right]\right)$$

$$\times\left(\int \mathrm{D}\varphi\mathrm{D}\bar\varphi\exp\left[\mathrm{i}\int \mathrm{d}^2r''\bar\varphi)E-\mathscr{H}+\mathrm{i}\delta)\varphi\right]\right)^{-1} \tag{6.4}$$

and substituting the result (6.3) into the general formula for DOS

$$\rho(E)=-\frac{1}{\pi}\,\mathrm{Im\,Tr}\,\langle G(E+\mathrm{i}\delta,x,x)\rangle \tag{6.5}$$

we can calculate the electronic DOS. Making use of (6.4) we can write

$$G_{ii}(r,r)=-\mathrm{i}\int \mathrm{D}u\mathrm{D}\bar u\mathrm{D}v\mathrm{D}\bar v u_i(\zeta)\bar u_i(\bar\zeta)\mathrm{e}^{-|\zeta|^2/2}\mathrm{e}^{\mathrm{i}\mathscr{A}} \tag{6.6}$$

$$\mathscr{A}[u,\bar u,v,\bar v]=\mathrm{i}E\int \mathrm{d}\zeta\,\mathrm{d}\bar\zeta\,\mathrm{e}^{-|\zeta|^2/2}(\bar u u+\bar v v)$$

$$-\mathrm{i}\int \mathrm{d}\zeta\,\mathrm{d}\bar\zeta\,\mathrm{e}^{-|\zeta|^2/2}(\bar u V u+\bar v V v) \tag{6.7}$$

where u and v are the commuting and anti-commuting (Grassmannian) spinor fields, respectively.

The corresponding free equation reads

$$\mathscr{H}_0\Phi=0 \qquad \Phi=\begin{pmatrix}u\\ v\end{pmatrix}. \tag{6.8}$$

Therefore, there exists at least one zero eigenvalue of the problem (in the absence of impurities), that is a necessary

condition for supersymmetry (ss) to exist. Note, that the action used in the work by Brezin *et al* (1984) is not supersymmetric. We show here that taking into account the 'chemical' impurities (V_2-potential) does not violate this intrinsic ss. It is necessary not to confuse it with the extrinsic ss used here for the averaging of connected G-functions.

Averaging (6.6) with the distribution $P[V(r)]$ we get

$$\langle G_{ii}(r)\rangle = -\mathrm{i}\,\mathrm{e}^{-|\zeta|^2/2}\int \mathrm{D}u\ldots \mathrm{D}\bar{v}u_i(\zeta)\bar{u}_i(\bar{\zeta})\,\mathrm{e}^{\mathrm{i}a_{\mathrm{eff}}} \tag{6.9}$$

$$a_{\mathrm{eff}} = E\int \mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}\,\mathrm{e}^{-|\zeta|^2/2}(\bar{u}u+\bar{v}v) + \int \mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}\,g[\mathrm{e}^{-|\zeta|^2/2}(\bar{u}Vu+\bar{v}Vv)] \tag{6.10}$$

where

$$g(\alpha) = \ln\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\alpha V}P[V]\mathrm{D}V. \tag{6.11}$$

One can see that (6.10) is invariant under magnetic translations with a Jacobian that is equal to unity since the fermionic part is the inverse of the bosonic contribution. Let us write the Greens function in the manifest ss invariant form

$$\langle G_{ii}(r,r)\rangle = -\mathrm{i}\,\mathrm{e}^{-|\zeta|^2/2}\langle u_i(\zeta)\bar{u}_i(\bar{\zeta})\rangle \tag{6.12}$$

where $\langle u_i(\zeta)\bar{u}_i(\bar{\zeta})\rangle = c_{ii}\mathrm{e}^{|\zeta|^2/2}$ with ζ and $\bar{\zeta}$ independent factors c_{ii}. Our goal now is to find these factors. Following Brezin *et al* (1984) we introduce superfields

$$\Phi_i(\zeta,\vartheta) = u_i(\zeta) + (\sqrt{\tfrac{1}{2}})\vartheta v_i(\zeta) \tag{6.13}$$

$$\bar{\Phi}_2(\bar{\zeta},\bar{\zeta}) = \bar{u}_i(\bar{\zeta}) + (\sqrt{\tfrac{1}{2}})\bar{v}_i(\bar{\zeta})\bar{\vartheta}_i. \tag{6.14}$$

Let us fix our normalisations by listing the basic formulae

$$\int \mathrm{d}\vartheta\,\mathrm{d}\bar{\vartheta}(1,\text{ or }\vartheta,\text{ or }\bar{\vartheta}) = 0 \tag{6.15}$$

$$\int \mathrm{d}\vartheta\,\mathrm{d}\bar{\vartheta}\,\vartheta\vartheta = \frac{1}{\pi}.$$

For effective action (6.9) we have

$$a = \mathrm{i}2\pi l_{\mathrm{H}}^2 E\int \mathrm{d}\vartheta\,\mathrm{d}\bar{\vartheta}\,\mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}\,\mathrm{e}^{-(1/2)(\zeta\bar{\zeta}+\vartheta\bar{\vartheta})}\bar{\Phi}_i\Phi_i + 2\pi l_{\mathrm{H}}^2\int \mathrm{d}\vartheta\,\mathrm{d}\bar{\vartheta}\,\mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}\,h[\mathrm{e}^{(1/2)(\zeta z\bar{\zeta}'\vartheta\bar{\vartheta})}\bar{\Phi}_i\Phi_i] \tag{6.16}$$

where

$$h(\alpha) = \int_0^{\alpha}\frac{\mathrm{d}\beta}{\beta}\,g(\beta). \tag{6.17}$$

This action is ss invariant

$$\Phi_i(z,\vartheta) = \tilde{\Phi}_i(z-a,\vartheta-\omega)\exp[\tfrac{1}{2}(z\bar{a}+\vartheta\bar{\omega})-\tfrac{1}{4}(|a|^2+|\omega|^2)]$$
$$\delta z = \bar{\omega}\vartheta \qquad \delta\vartheta = \omega z. \tag{6.18}$$

The problem of the Green function calculation is reduced to the calculation of the average value

$$\exp[-\tfrac{1}{2}(\zeta\bar{\zeta}+\vartheta\bar{\vartheta})]\langle\Phi_i(\zeta,\vartheta)\bar{\Phi}_i(\bar{\zeta},\bar{\vartheta})\rangle = C_{ii}. \tag{6.19}$$

It is clear that in order to find (6.19) we have to calculate the Gaussian integrals in the superspace (superdeterminants). But these superdeterminants are unities due to mutual compensation of bosonic and fermionic ones. The problem of the calculation of the perturbative corrections to the Green function is reduced to the calculation of the symmetry coefficients of proper graphs. These coefficients can be calculated with a derivative function that gives us the expression for the sum of the graphs

$$C_{ii} = -\mathrm{i}\int \mathrm{d}\varphi\,\mathrm{d}\bar{\varphi}\,\varphi_i\bar{\varphi}_i\exp[2\pi l_{\mathrm{H}}^2(\mathrm{i}\varepsilon\bar{\varphi}\varphi + h[\bar{\varphi}\varphi])] \times\left(\int \mathrm{d}\varphi\,\mathrm{d}\bar{\varphi}\exp[2\pi l_{\mathrm{H}}^2(\mathrm{i}\varepsilon\bar{\varphi}\varphi + h[\bar{\varphi}\varphi])]\right)^{-1}$$
$$\equiv \frac{1}{2\pi l_{\mathrm{H}}^2}\frac{\partial}{\partial\varepsilon}\ln\int \mathrm{d}\varphi\,\mathrm{d}\bar{\varphi}\exp[2\pi l_{\mathrm{H}}^2(\mathrm{i}\varepsilon\bar{\varphi}\varphi + h[\bar{\varphi}\varphi])] \tag{6.20}$$

where $\varepsilon = E + \mathrm{i}\delta$.

Making use of (6.5) and taking a trace over the spinor indices we get

$$\rho(E) = \frac{1}{\alpha\pi}\,\mathrm{Im}\,\frac{\partial}{\partial\varepsilon}\ln\int_0^{\infty}\mathrm{d}x\,\mathrm{d}y\exp[\mathrm{i}\alpha\varepsilon(x+y) - \alpha\lambda_1(x+y)^2 - \alpha\lambda_2(x-y)^2] \tag{6.21}$$

where $\alpha = 2\pi l_{\mathrm{H}}^2$, $x = |u_1|^2$, $y = |u_2|^2$, u_i are spinor components. As it can be derived from (6.21) the full number of free states is twice the one in the one-band case (Schrödinger case)

$$\int_{-\infty}^{\infty}\rho_{0\mathrm{H}}(E)\,\mathrm{d}E = \int_{-\infty}^{\infty}\frac{1}{2\pi l_{\mathrm{H}}^2}\,2\delta(E)\,\mathrm{d}E = (\pi l_{\mathrm{H}}^2)^{-1}. \tag{6.22}$$

Anomalous behaviour of the DOS $\rho(E)$ is connected with impurities potential modulating the gap (Ktitorov and Petrov 1986). Therefore we may restrict our study by the case $\lambda_1 \ll \lambda_2$. Carrying out the integration in (6.21) we have in the limit $\lambda_1 = 0$, $\lambda_2 \neq 0$

$$\rho(E) = \left(\frac{\pi}{2\lambda_2}\right)^{1/2}\frac{1}{l_{\mathrm{H}}\pi^2} \times\left(\frac{\pi}{2}\delta(x) + \frac{\dfrac{1}{\sqrt{\pi}}\mathrm{e}^{x^2}}{1+\left(\dfrac{2}{\sqrt{\pi}}\mathrm{e}^{x^2}F(x)\right)^2}\right) \tag{6.23}$$

where

$$x = \frac{E}{2}\left(\frac{\alpha}{\lambda_2}\right)^{1/2} \qquad F(x) = \mathrm{e}^{-x^2}\int_0^{x}\mathrm{e}^{u^2}\,\mathrm{d}u$$

is a Doson integral. As can be shown from (6.21) there is a singularity of DOS in the centre of the band spectrum.

From (6.21) we have the following expression for $\lambda_1 \ll \lambda_2$

$$\rho(E)=\frac{1}{2\pi l_H^2}\delta(E)+\left(\frac{2}{\lambda_1}\right)^{1/2}\frac{1}{2\pi^2 l_H}\frac{\pi}{4}\left\{e^{-x^2/\lambda^2}+e^{x^2}\left[F\left(\frac{x}{\lambda}\right)F(x)\left(1-\frac{\lambda^2}{2x^2}\right)-\frac{\lambda}{2x}\left(F(x)-\lambda F\left(\frac{x}{\lambda}\right)\right)\right]\right\}$$
$$\times\left[\left(\frac{\sqrt{\pi}}{2}e^{-x^2/\lambda^2}-\frac{1}{\sqrt{\pi}}\frac{\lambda}{x}e^{x^2}F(x)\right)^2+F^2\left(\frac{x}{\lambda}\right)\right]^{-1} \tag{6.24}$$

where $\lambda=(\lambda_1/\lambda_2)^{1/2}$.

The corresponding DOS is depicted in figure 4. For E near the centre of the bands we have the following asymptotic of the DOS

$$\frac{\rho(E)}{\left[\frac{1}{2\pi l_H^2}\left(\frac{2}{\lambda_1}\right)^{1/2}\right]}=\begin{cases}\left\{e^{x^2/\lambda^2}\left[1+\frac{2\lambda}{\pi}e^{x^2/\lambda^2}\left(2\frac{x}{\lambda}F\left(\frac{x}{\lambda}\right)-1\right)\right]\right\}\left\{\left[1-\frac{2\lambda}{\pi}e^{x^2/\lambda^2}\right]^2+\left[\frac{2}{\sqrt{\pi}}e^{x^2/\lambda^2}F\left(\frac{x}{\lambda}\right)\right]^2\right\}^{-1} & x<\lambda\\ e^{x^2/\lambda^2}\left\{1+\left[\frac{2}{\sqrt{\pi}}e^{x^2/\lambda^2}F\left(\frac{x}{\lambda}\right)\right]^2\right\}^{-1} & x\sim\lambda\\ \left[\lambda e^{x^2}\left(1-\frac{F(x)}{x}\right)\right]\times\left[1+\left(\frac{2}{\sqrt{\pi}}e^{x^2}F(x)\right)^2\right]^{-1} & x>\lambda.\end{cases} \tag{6.25}$$

One can see that the singularity (6.24) at $E=0$ is smeared over the range $\sim(2\lambda_1/\alpha)^{1/2}$ due to the influence of Coulomb impurities. In the case of the absence of 'chemical' impurities the singularity is also absent

$$\rho(E)=\frac{(2/\lambda_1)^{1/2}}{4\pi^2 l_H}\frac{e^{x^2}}{x^2+\left(\frac{1}{\sqrt{\pi}}e^{x^2}\frac{d}{dx}F(x)\right)^2} \tag{6.26}$$

$$x=\frac{E}{2}\left(\frac{\alpha}{\lambda_1}\right)^{1/2}.$$

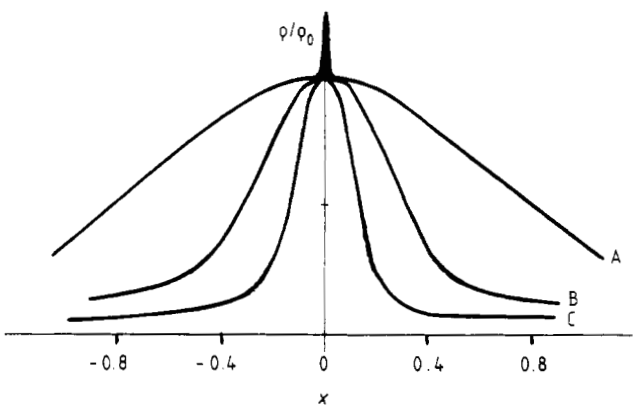

Figure 4. Two-dimensional density of states (6.24). A, $\lambda=0.5$; B, 0.2; C, 0.1.

Note, that in the case $\lambda_1=\lambda_2=w/8$ we have the result coinciding with one obtained by Affleck (1984) up to a factor of two.

$$\rho(E)=\left(\frac{2}{w}\right)^{1/2}\frac{2}{\pi^2 l_H}e^{\nu^2}\left[1+\left(\frac{2}{\sqrt{\pi}}\int_0^{\nu}e^{u^2}du\right)^2\right]^{-1}$$
$$\nu=E\left(\frac{\alpha}{w}\right)^{1/2}. \tag{6.27}$$

At large energies for $E^2\gg 4\lambda_2/\alpha\gg 4\lambda_1/\alpha$ we have

$$\rho(E)=\frac{(\sqrt{2})l_H E^2}{4\lambda_2\sqrt{\lambda_2}}\exp\left(-\frac{\pi l_H^2 E^2}{2\lambda_2}\right) \tag{6.28}$$

and the DOS coincides with the results obtained by Brezin *et al* (1984) and Wegner (1983). Equation (6.28) also was obtained by Affleck (1984) in the semi-classical approximation.

A narrow peak appears in disordered GS I when impurities have a Lorentzian distribution. Indeed after averaging we have $h(x)=-\lambda|x|$. The effective action has the form of the free one. This case leads to the generalisation of the well known Lloyd model (Lloyd 1969) for two-dimensional gapless states. The DOS has the form

$$\rho(E)=\frac{1}{2\pi^2 l_H^2}\left(\frac{\Lambda_+}{\Lambda_+^2+E^2}+\frac{\Lambda_-}{\Lambda_-+E^2}\right) \tag{6.29}$$

where $\Lambda_\pm=\lambda_1\pm\lambda_2$.

From (6.29) it follows that for close intensities of the correlators ($\Lambda_-\to 0$) in $E=0$ the DOS have a singularity. Note, that in the work by Brezin *et al* (1984) the singularity of the DOS appears only in the case of the Poisson distribution.

7. Conclusion

The basic aim of the present work was to investigate the energy spectrum and density of states in gapless semiconductors of type I in the presence of a strong magnetic field, random impurities and electron–phonon interaction. We used a two-band semiconductor model in which the electronic states and spectrum are described by quasi-relativistic Dirac equations. In the quantum limit there is a GS I in the absence of interaction leads to the appearance of gapless collective excitations of the Bose type.

The electronic states are described in terms of holomorphic representations.

We investigated the effect of the phonon and the impurities on the energy spectrum of three-dimensional GS I. The results are very surprising. We found a spontaneous gap generation phenomenon. As a result, GS I absolutely unachievable. In material with an inverted band structure the conduction band and valence band have to invert without crossing zero while the gap is finite. When the impurities are taken into account we also have a

hysteretic character of the dependence but with an higher 'coercivity' m_c.

Thus the 'newly born' ground state is doubly degenerate. In this situation we obtain the Bogoliubov–de Gennes-type equations. The corresponding solution leads to the domain-wall-type solitons, linking normal and inversion phases. Two-dimensional electrons are localised in the domain walls. The possible implications of these results for semiconductors IV–VI will be described separately. They may be important in the galvanomagnetic effects.

We have considered the problem of a non-interacting electrons in the presence of impurities with δ-correlated distributions. To calculate the three-dimensional DOS we reduced the problem to the one-dimensional one without a magnetic field. From the basic Fokker–Planck equation the number of electronic states is evaluated. The corresponding DOS is singular at $E=0$. We have shown that there is Dyson singularity in the middle of the band, as a consequence the states on the centre of the band remain extended.

For two-dimensional electrons the δ-function singularity of DOS is found in the case of gap-modulating impurities that is a result of the supersymmetry being preserved in this case. 'Relativistic' two-dimensional electrons can be found on the IV–VI semiconductor surfaces, corresponding to the 'supersymmetric' interfaces (Volkov and Pankratov 1985) and in domain walls, as predicted in this work. The electron DOS singularities can be observed in galvanomagnetic, optical and tunnel spectroscopy experiments.

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