If $H_0$ is directed along the $\{111\}$ axis, then by virtue of the isotropy of the $\{111\}$ plane, there exists in the iron an entire set of spin waves with $0 < \theta_k < \pi/2$. When $P_{||} > 5\, \text{dB}$, spin waves with arbitrary $\theta_k$ are excited at all orientations.

CONCLUSIONS

The experimental results show that the character of the nonlinear relaxation of the spin waves in ferrites depends essentially on the orientation of $H_0$ relative to the crystallographic axes. If $H_0$ is directed along the $\{100\}$ axis, then there exist in the sample, past the stability threshold, two groups of spin waves—primary with $\theta_k = \pi/2$, which are excited directly by the pump, and secondary with $\theta_k = 0$.

The presence of waves with $\theta_k = 0$ is attributed to the existence of a process analogous to saturation of the main resonance, a phenomenon described by Suhl. The gist of this phenomenon is that the primary spin waves act via four-magnon interaction, as a pump for the secondary waves with $\theta_k = 0$, and at a certain threshold amplitude of the primary waves this can lead to parametric generation of waves with $\theta_k = 0$.

Even if the amplitude of the primary waves is below the threshold, the amplitude of the secondary waves is appreciable, since parametric amplification of the waves with $\theta_k = 0$ at the expense of the primary waves is possible, the "signal" for such an amplifier being also primary waves transformed via two-magnon scattering into spin waves with $\theta_k = 0$.

Translated by J. G. Adashko

KINETIC EQUATIONS FOR RELAXATION PROCESSES IN SUPERCONDUCTORS

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Submitted January 1, 1971

Zh. Eksp. Teor. Fiz. 61, 302-307 (July 1971)

We obtain a kinetic equation for the generalized electron density matrix of a superconductor. The relaxation of a system to an equilibrium state described by this equation takes place in two stages. First a fast quasistationary rearrangement of the system occurs; a self-consistent field of the superconducting condensate is formed. The quasi-particle description of the excitations is valid. After that the quasi-particle distribution and the superconducting ordering parameter slowly evolve due to collisions between the excitations.

After a microscopic theory of superconductivity has been constructed\textsuperscript{14}\textsuperscript{15}, various kinetic phenomena in superconductors (such as heat conductivity, ultrasonic absorption, interaction with a weak electromagnetic field) were studied in the framework of the linear functions for the response of the system to a weak external influence (see, e.g.,\textsuperscript{15},\textsuperscript{16}). According to the generalized fluctuation-dissipation theorem\textsuperscript{15} the corresponding kinetic coefficients can be in such case expressed in terms of correlation functions for the equilibrium fluctuations of the system and can be evaluated by applying standard methods of quantum field theory to systems with a large number of degrees of freedom.\textsuperscript{15} A generalization of these methods to the case of essentially non-linear, non-stationary processes in superconductors (such as, e.g., the variable Josephson current,\textsuperscript{13} or resistive states in superconductors of the second kind\textsuperscript{14}) was proposed by Gor'kov and Eliashberg.\textsuperscript{15}

Their procedure was based upon the analytical continuation of the electron Green functions of the superconductor which satisfy the Gor'kov equations,\textsuperscript{15}\textsuperscript{16} to the real frequency region.

The Green functions are a very general theoretical concept which enables us in principle to describe kinetic phenomena in condensed systems even when there is no sharp separation of the time scales (relaxation time) and the usual method of the kinetic equations is inapplicable. However, in contrast to the kinetics of normal metals for a superconductor the choice of the parameters for a kinetic description just as the derivation of the kinetic equations themselves is not at all obvious and must be performed anew starting from first principles of statistical mechanics.

A necessary condition for a microscopic description of non-stationary processes in the case of large deviations from equilibrium is that they proceed slowly in time. Thanks to that condition the statistical operator $\rho$ which satisfies the quasi-Liouville equation

$$\frac{d}{dt}\rho = [\mathcal{H}, \rho]$$

(1) can be synchronized with some set of macroscopic parameters $\rho_k$ of the temporal evolution of which is prescribed by kinetic equations and as a result one can abbreviate the complete mechanical description of the system.\textsuperscript{18} Thus during the lapse of a short time interval $\tau$ the following equations must be satisfied:

$$\frac{d}{dt} \rho_k = \mathcal{L} \rho_k$$

(2)

Pelefianinskii and Yatsenko\textsuperscript{14} proposed a rather general scheme for deriving kinetic equations in the above-mentioned sense for an extensive class of systems; they consider systems with a Hamiltonian $\mathcal{H}$ which can be split into a "basic" Hamiltonian $\mathcal{H}_0$ guaranteeing the synchronization of the statistical operator with the parameters $\rho_k$ and a relatively weak interaction $V(x) = V(x)\, \mathcal{H}_0$ due to which the parameters \$\rho_k$ relax to their equilibrium values. The choice of the parameters $\rho_k$ is determined by the symmetry properties of the Hamiltonian $\mathcal{H}_0$ and the main condition assumed in\textsuperscript{14} consists in that the corresponding operators $\mathcal{L} \rho_k$ must satisfy the commutation relations:

$$[\mathcal{H}_0, \rho_k] - \sum_{\rho_l} \mathcal{L} \rho_l = 0$$

(4)

The conditions (4) are the natural linear generalization of the usual condition on the order of $\mathcal{H}_0$ and (conservation laws) and enable us to describe the system with a self-consistent field. In particular, as we shall show in the paper, similar relations can be used as a basis to derive kinetic equations also for superconductors since the BCS theory of superconductivity\textsuperscript{14} is a variant of a self-consistent field theory.

1. DERIVATION OF THE KINETIC EQUATIONS

It is well known\textsuperscript{14} that the phenomenology of superconductivity in metals is connected with the fact that thanks to the effective attraction between electrons induced by

\footnotesize{\textsuperscript{14}It is, however, necessary to note that the Gor'kov equations do not describe the relaxation of the electrons with respect to energy, since the electron-electron interaction in them taken into account only to the extent to which it guarantees a zero value of the superconducting ordering parameter.}

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the electron-phonon interaction does not affect the characteristic correlations of electron pairs with antiparallel momenta and spins. The corresponding rearrangement of the one-electron states involves a mirror layer with a width of the order of $T_0$ (the superconducting transition temperature) near the Fermi boundary. It is therefore sufficient for the construction of a kinetic theory of the superconducting state to consider only the dynamics of the electrons in that layer assuming that the fast processes of Fermi-occupancy has essentially already been accomplished.

This observation enables us to simplify considerably the Hamiltonian of the system under consideration. In particular, we can neglect the dispersion law for the electrons to be given and, for the sake of simplicity, we can put $\omega_n = \omega^2 q^2 n_2$. Moreover, as the characteristic frequency determining the interaction of the electrons through the field of the virtual phonons is of the order of the Debye frequency $\omega_D$ in the weak coupling approximation ($T_0 \ll \omega_D$) we can neglect the retardation of this interaction and we can use for the Hamiltonian of the system the Gor’kov formula: $\hat{H} = \hat{H}_F + \hat{H}_I$.

\[ \hat{H}_I = \int d^2 r \int d^2 r' \phi^*(\mathbf{r}) \phi(\mathbf{r}) \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \phi^*(\mathbf{r}') \phi(\mathbf{r}') \]

where $\phi(\mathbf{r})$, $\phi^*(\mathbf{r})$ are the annihilation operators for an electron with a given spin at the point $r$, $g < 0$ is a small constant for the effective attraction of the electrons near the Fermi surface, and $\mathbf{T}$ is a thickness of order $\omega_D$. The vector potential of the electromagnetic field. The potential $a(r)$ includes the scalar potential of the electron-electron interaction with other external fields, impurities, and so on.

Turning now to the condition (4) we note easily that apart from the usual quantities $\sigma_{\uparrow}(r)$, $\sigma_{\downarrow}(r)$ and $\psi_{\uparrow}(r)$, $\psi_{\downarrow}(r)$ from which the single-particle density matrix for normal electrons is constructed, Eqs. (4) are also satisfied by the operators $\phi_{\uparrow}(r)$, $\phi_{\downarrow}(r)$ and $\phi^*_{\uparrow}(r)$, $\phi^*_{\downarrow}(r)$. It is well known that the occurrence of non-zero average values of these quantities is a characteristic property of the superconducting electron gas and reflects the appearance of correlations of electron pairs with antiparallel spin (electro-hole correlations). The more complete set of the quantities $\psi_{\uparrow}(r)$, $\psi_{\downarrow}(r)$, $\phi_{\uparrow}(r)$, $\phi_{\downarrow}(r)$, $\phi^*_{\uparrow}(r)$, $\phi^*_{\downarrow}(r)$ and $\phi_{\uparrow}(r)$, $\phi_{\downarrow}(r)$ must thus be the set of parameters for a superconductor and give us a kinetic description of the superconducting state. In the coordinate representation these quantities are in a natural way combined in the matrix:

\[ \bar{T} = \left( \begin{array}{cc} \psi^*_{\uparrow}(r) & \psi_{\downarrow}(r) \\ \phi^*_{\uparrow}(r) & \phi_{\downarrow}(r) \end{array} \right) \]

It is convenient in connection with the given matrix description to use what follows the absence spin interaction in $\hat{H}_I$. We thus introduce new canonical annihilation and creation operators of electron holes which formally form vectors, using the equations

\[ \begin{align*}
\psi_{\uparrow}(r) & \rightarrow \psi_{\uparrow}(r) + \frac{1}{\sqrt{2}} \phi_{\uparrow}(r) \\
\phi_{\uparrow}(r) & \rightarrow \phi_{\uparrow}(r) - \frac{1}{\sqrt{2}} \psi_{\uparrow}(r)
\end{align*} \]

In that case $\delta^2 \bar{T}$ can be written as:

\[ \delta^2 \bar{T} = \left( \begin{array}{cc} \phi_{\uparrow}(r) \phi^*_{\uparrow}(r) & \phi_{\uparrow}(r) \psi_{\uparrow}(r) \\
\psi_{\downarrow}(r) \phi^*_{\downarrow}(r) & \psi_{\downarrow}(r) \psi_{\downarrow}(r) \end{array} \right) \]

In the case $\delta^2 \bar{T} = 0$ the Hamiltonian $\hat{H}_F$ can be written as:

\[ \hat{H}_F = \int d^2 r \left[ \left( \delta^2 \bar{T} \right) \hat{N}_F \right] \]

Consequently the Hamiltonians $\hat{H}_F$ and $\hat{H}_I$ of (9) can, apart from a constant, after some simple transformations be written as:

\[ \hat{H}_F = \hat{H}_F^{\prime} + \hat{H}_I^{\prime} \]

where $\hat{H}_F^{\prime}$ is a Hermitian matrix defined by the equations

\[ \begin{align*}
\hat{H}_{\uparrow\uparrow}(r) & = \left( \begin{array}{cc} \phi_{\uparrow}(r) \phi^*_{\uparrow}(r) & \phi_{\uparrow}(r) \psi_{\uparrow}(r) \\
\psi_{\downarrow}(r) \phi^*_{\downarrow}(r) & \psi_{\downarrow}(r) \psi_{\downarrow}(r) \end{array} \right) \\
\hat{H}_{\downarrow\downarrow}(r) & = \left( \begin{array}{cc} \phi_{\downarrow}(r) \phi^*_{\downarrow}(r) & \phi_{\downarrow}(r) \psi_{\downarrow}(r) \\
\psi_{\uparrow}(r) \phi^*_{\uparrow}(r) & \psi_{\uparrow}(r) \psi_{\uparrow}(r) \end{array} \right)
\end{align*} \]

The matrix $\hat{H}_I$ (or, in the usual notation

\[ \hat{H}_I = \int d^2 r \left[ \delta^2 \bar{T} \right] \hat{N}_F \]

In the absence of interactions $\hat{W}(0) = 0$ the Lorentzian equation (1) can then be written in the following form:

\[ \sigma_{\uparrow}(r) = \int d^2 r \left[ \frac{\partial}{\partial x} \hat{H}_I \right] \hat{N}_F \]

The formal solution of this equation with the boundary condition $\sigma(r) = 0$ has the form

\[ \sigma(r) = \int d^2 r \left( \frac{\delta^2 \bar{T}}{\delta x} \right) \hat{N}_F \]

where the integral operator $U$ is determined from the equations

\[ U(r) = \int d^2 r_t \left[ \frac{\delta^2 \bar{T}}{\delta x} \right] \hat{N}_F(r_t) \hat{N}_F(r) \]

or equivalently

\[ \left( \frac{\partial}{\partial x} \right) \sigma(r) = \int d^2 r_t \left[ \frac{\delta^2 \bar{T}}{\delta x} \right] \hat{N}_F(r_t) \hat{N}_F(r) \]

According to the basic assumption (2) about the synchronization, the solution (21) must satisfy the following asymptotic relation:

\[ \sigma(r) = \int d^2 r_t \left( \frac{\delta^2 \bar{T}}{\delta x} \right) \hat{N}_F(r_t) \hat{N}_F(r) \]

and replacing in Eq. (22) $\sigma(r) = \sigma(r)$ we obtain finally the following boundary condition for Eq. (17):

\[ \sigma(r) = \int d^2 r_t \left( \frac{\delta^2 \bar{T}}{\delta x} \right) \hat{N}_F(r_t) \hat{N}_F(r) \]

In connection with the definitions (20) and (22) we note the transformation law of the quantity $\bar{T}$ under the action of the operator $U$:

\[ U(r) = \hat{T}_{\uparrow}(r) \hat{T}_{\downarrow}(r) \]

where by virtue of Eqs. (22) and (16) the unitary matrix $\hat{T}$ is determined by equations analogous to:

\[ \hat{T}_{\uparrow}(r) = \hat{T}_{\downarrow}(r) \hat{T}_{\uparrow}(r) \]

and

\[ \hat{T}_{\downarrow}(r) = \hat{T}_{\downarrow}(r) \hat{T}_{\downarrow}(r) \]

where, according to Eqs. (19) and (20) the following relation is satisfied:

\[ \hat{T}_{\uparrow}(r) = \hat{T}_{\downarrow}(r) \hat{T}_{\uparrow}(r) \]

From (27), (28), and (29) we obtain the transformation law for the operator $\hat{W}(0)$ of (24):

\[ \sigma(r) = \int d^2 r_t \left( \frac{\delta^2 \bar{T}}{\delta x} \right) \hat{N}_F(r_t) \hat{N}_F(r) \]

To solve Eq. (17), we change it into an integral equation using the boundary condition (26). Making the sub
The form of the "collision integral" \( I^{(m)}(y) \) of (38) in Eq. (41) remains formally as before. It is only necessary to replace in Eq. (22), which define the unitary operator \( U(\mathbf{r}, i\beta) \), the Hamiltonian \( H^{(0)} \) of (18) by the following:

\[
H^{(0)}(\mathbf{r}) = \nabla \cdot \mathbf{L}(\mathbf{r}, \gamma).
\]

The last changes of the matrix \( \gamma \) are now according to (41) described by the following equation:

\[
\delta \gamma_{ij}(\mathbf{r}) = \nabla \cdot \mathbf{L}(\mathbf{r}, \gamma).
\]

We get in zeroth approximation the equation:

\[
\gamma_{ij}(\mathbf{r}) = 0.
\]

The matrix \( \gamma \) is the solution of this equation. In the next approximation Eq. (14) the small correction \( \gamma' \) to the matrix \( \gamma \) looks like:

\[
\gamma_{ij}(\mathbf{r}) = \nabla \cdot \mathbf{L}(\mathbf{r}, \gamma) = 0.
\]

From the condition that this equation is solvable we get

\[
\mathbf{L} \cdot \nabla \mathbf{g} = 0,
\]

or, by virtue of Eqs. (47) and (48), alternatively

\[
\mathbf{L} \cdot \nabla \mathbf{g} = 0.
\]

Using Eqs. (30), (40), (42), (45), (47), and (48) we can find the explicit form of the collision integral \( I_{ij}(\mathbf{r}) \) of (52). First of all, since the matrix \( \gamma \) in the form (47) is a "stationary point" of Eq. (46), we may assume Eq. (47), and then \( \mathbf{L} \cdot \nabla \mathbf{g} = 0 \) for the operator \( \mathbf{U}(\mathbf{r}, \gamma) \), since according to the definition (20) in that case \( \mathbf{L} \cdot \nabla \mathbf{g} = 0 \). Hence it follows that

\[
\mathbf{U}(\mathbf{r}, \gamma) = \exp(-\mathbf{L}(\mathbf{r}, \gamma)),
\]

We expand the operators \( \mathbf{U}(\mathbf{r}) \) and \( \nabla \cdot \mathbf{L}(\mathbf{r}, \gamma) \) in terms of the single-particle states \( \{ \mathbf{r}, \alpha \} \) and get

\[
\mathbf{U}(\mathbf{r}, \gamma) = \sum_{\alpha} \mathbf{U}(\mathbf{r}, \alpha) \mathbf{a}_\alpha^\dagger(\mathbf{r}) \mathbf{a}_\alpha(\mathbf{r}) \mathbf{b}_\alpha^\dagger(\mathbf{r}) \mathbf{b}_\alpha(\mathbf{r}),
\]

One sees easily from Eqs. (9), (40), and (47) that the distribution function \( \phi(\mathbf{r}, \gamma) \) is equal to

\[
\phi(\mathbf{r}, \gamma) = \mathbf{b}_\alpha(\mathbf{r}) \mathbf{a}_\alpha(\mathbf{r}) \mathbf{U}(\mathbf{r}, \gamma).
\]

Substituting expansion (54) into Eqs. (9) and (45) we get

\[
\mathbf{U}(\mathbf{r}, \gamma) = \sum_{\alpha} \mathbf{U}(\mathbf{r}, \alpha) \mathbf{a}_\alpha(\mathbf{r}) \mathbf{b}_\alpha(\mathbf{r}),
\]

where \( \mathbf{M}_\alpha \mathbf{b}_\alpha \mathbf{a}_\alpha \) is the matrix element of the interaction.

\[
\mathbf{M}_{\alpha \beta} = \int \mathbf{L}(\mathbf{r}, \gamma) \mathbf{a}_\beta^\dagger(\mathbf{r}) \mathbf{b}_\alpha(\mathbf{r}),
\]

Substitution of Eqs. (55), (56), and (57) into Eq. (39) gives the following result for the collision integral \( \mathbf{L}^{(m)}(\mathbf{r}) \) of (53):

\[
\mathbf{L}^{(m)}(\mathbf{r}) = - \int \mathbf{L}(\mathbf{r}, \gamma) \mathbf{a}_\beta^\dagger(\mathbf{r}) \mathbf{b}_\alpha(\mathbf{r}),
\]

where \( \mathbf{M}_{\alpha \beta}(\mathbf{r}) \) differs from \( \mathbf{M}_{\alpha \beta} \) by the replacement of the matrix element \( \mathbf{M}_{\alpha \beta} \) by

\[
\mathbf{M}_{\beta \alpha}(\mathbf{r}) = \mathbf{M}_{\alpha \beta}(\mathbf{r}) \exp(-\mathbf{L}(\mathbf{r}, \gamma)) \mathbf{a}_\beta^\dagger(\mathbf{r}) \mathbf{b}_\alpha(\mathbf{r}),
\]

The remaining calculations in Eq. (58) reduce to the standard calculation of commutators and the subsequent averaging using Wick's theorem. The final result for the collision integral looks as follows:

\[
\mathbf{L}^{(m)}(\mathbf{r}) = - \int \mathbf{L}(\mathbf{r}, \gamma) \mathbf{a}_\beta^\dagger(\mathbf{r}) \mathbf{b}_\alpha(\mathbf{r}),
\]
For what follows it is sufficient to consider the asymptotic behavior of the solutions of Eqs. (53) and (64) as $t \to +\infty$, and verify the relaxation process described by these equations. The following relations for the derivatives $\Delta u_{11}$ and $\Delta u_{10}$ follow from Eqs. (53) and (64):

$$
\Delta \frac{\partial u_{11}}{\partial t} = -\frac{4}{3} \sum \text{Tr}(u_{11}) \Delta u_{11} - \frac{4}{3} \sum \text{Tr}(u_{10}) \Delta u_{10},
$$

$$
\Delta \frac{\partial u_{10}}{\partial t} = -\frac{4}{3} \sum \text{Tr}(u_{10}) \Delta u_{10} - \frac{4}{3} \sum \text{Tr}(u_{11}) \Delta u_{11}.
$$

By virtue of the definition (8) of the matrix $\gamma$, the equilibrium distribution function determined by this collision integral $(\partial u_{11} / \partial t = 0)$ is clearly equal to

$$
\gamma = 1 + (t^{-1}, T_{1}), \quad (t^{-1} = (t^{-1} + 1)^{-1}).
$$

We must then note that because of the used definition (7) of electrons and holes the constant correction to the energy $\gamma$, Eq. (60) postulated by the integral (59) would destroy a separation of the Fermi surfaces for electrons and holes respectively. The true chemical potential connected with the conservation of the number of electrons (see Eq. (44)) is contained in the phase of the ordering parameter $\chi$ and is the equilibrium state we see from Eq. (42) that we have $3/2 \pi T \approx 2\mu$ apart from a small renormalization.

It follows from Eqs. (47), (60), and (61) that the equilibrium density matrix is equal to

$$
\gamma = 1 + (t^{-1}, T_{1}).
$$

Using the expansion

$$
1 - (t^{-1}, T_{1}) = \sum \frac{\exp(-\alpha)}{\alpha} \alpha + \cdot \cdot \cdot
$$

we see easily that the equilibrium density matrix (61) which we have thus been determined in terms of the temperature-dependent Green function:

$$
\gamma(t) = (1 + t^{-1}, T_{1}) = 1 + \text{Tr}(\gamma(t)).
$$

Using the definition (42) we can verify that Eq. (62) together with the self-consistency equations (43) and (44) is equivalent to Gorkov's equations

$$
\gamma(t) = 1 + \text{Tr}(\gamma(t)),
$$

and it is written in a matrix notation.

In connection with the fact that the matrix $\gamma$ is non-diagonal in the "isoptic spin" in the electron-hole space it is of interest to study in more detail the relaxation processes leading to the formation of the superconducting condensate, i.e., to the appearance of the self-consistent field of the parameter $\Delta$ and the quasiparticle description of the excitations. In its pure form this process appears when there are no external fields in the spatially uniform case, when

$$
\text{Tr}(\gamma(t), n) = \frac{1}{2} \sum \exp(-\alpha/\gamma(t), n),
$$

where $\nu$ is the normalization volume while Eqs. (42) to (45) take the form

$$
\nu = \text{Tr}(\gamma(t), n),
$$

$\alpha = \text{Tr}(\gamma(t), n).$

The equation $\Delta u_{11} / \Delta t = 0$ is proved analogously.

The equation obtained are clearly a consequence of the fact that the ordering parameter $\Delta$ is a self-averaging quantity. We showed that all physical quantities can be expressed in terms of some sums of matrices $\gamma_{p}$ over $p$. After the lapse of a sufficiently long time $t \gg 1 / T_{c}$ the oscillating terms in (60) for the matrix $\gamma_{p}$ therefore do not contribute to these quantities and the matrix $\gamma_{p}$ is asymptotically equivalent to the following one (compare Eq. (47)):

$$
\gamma_{p} \approx \frac{1}{2} \sum \text{Tr}(\gamma_{p}).
$$

The function $\gamma_{p}$ in (63) is clearly of the same order of magnitude as the electron energy reckoned from the Fermi boundary.

Using Equations (67) we can evaluate the matrix element (57) in the collision integral (59) and find the equation for $a_{1}$. Introducing the slow change in the distribution function $\phi_{1}$

$$
\phi_{1} = \sum \text{Tr}(u_{11}) \frac{1}{2} \sum \text{Tr}(u_{10}) \frac{1}{2} \sum \text{Tr}(u_{11})
$$

and by means of Eq. (60) we obtain the following approximation:

$$
\phi_{1} \approx \frac{1}{2} \sum \text{Tr}(u_{11}) \frac{1}{2} \sum \text{Tr}(u_{10}) \frac{1}{2} \sum \text{Tr}(u_{11})
$$

Effective use can be made of the asymptotic behavior of the visual equation for $\Delta u_{11}$ (57) in the short time $t \gg 1 / T_{c}$.

The solution of this equation by a Laplace transformation leads to the following expression:

$$
\phi_{1} = \frac{1}{2} \sum \text{Tr}(u_{11}) \frac{1}{2} \sum \text{Tr}(u_{10}) \frac{1}{2} \sum \text{Tr}(u_{11})
$$

where $\gamma(t)$ is the initial value of the perturbation. The solution of this equation is found by the method of the Green's function $\phi_{1}$.

The self-consistent equation for the relaxation time $\tau_{c}$, the equation for $\phi_{1}$ takes the form:

$$
\phi_{1} = \frac{1}{2} \sum \text{Tr}(u_{11}) \frac{1}{2} \sum \text{Tr}(u_{10}) \frac{1}{2} \sum \text{Tr}(u_{11})
$$

The real root of this equation occurs when $T \leq T_{c}$. We have then asymptotically

$$
\phi_{1} = \frac{1}{2} \sum \text{Tr}(u_{11}) \frac{1}{2} \sum \text{Tr}(u_{10}) \frac{1}{2} \sum \text{Tr}(u_{11})
$$

where $T_{c}$ is the critical temperature for superconducting. In particular, using perturbation theory we can find the linear responses, for instance, to a weak electric field. One can show that the corresponding results in this case are the same as the well-known ones.$^{12}$

8 This equation enables us moreover to study the problem of the temperature-dependence of the Landau-Landau equation, and the viscous tensor coefficients in the hydrodynamic equations of a superfluid Fermi liquid. We have investigated different non-linear effects in superconductors. We hope to consider a number of such problems in the near future.

The author of this paper is grateful to B. V. Peletminskii for discussions of this paper.


$^{12}$ N. N. Bogolyubov, V. V. Tomashchewsky, and D. V. Shirkov, Noyzii metotវវ for spherography and spherowski (A New Method in the Theory of Superconductivity), Ned. Akad.

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CRITICAL CURRENT OF AN IDEAL TYPE II SUPERCONDUCTOR IN THE MIXED STATE

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Submitted January 20, 1971

Zh. Eksp. Teor. Fiz. 61, 398–413 (July, 1971)

We consider a perfectly uniform film of a type II superconductor (x > 1, 0 ≤ d ≤ δs, d is the film thickness and δs the penetration depth) and a perfectly uniform plate (d ≈ δs). We show that in both cases, when the external magnetic field Hext is in parallel to the applied field, the critical state is a triangular vortex lattice which is uniform in density, similar to the lattice of a bulk sample. The interaction between the vortex lattice and the Meissner currents causes its stability against small displacements, i.e., pinning. We find the pinning force and the critical current at right angles to the field, which violates the stability of the vortex lattice. In the case of a film the current is determined by Eq. (20) and in the case of a plate by Eq. (30). The latter formula is similar to the one proposed by Campbell, Evetts, and Daw-Hughes.\(^{11}\)

We have found earlier\(^{11}\) the dependence of the critical current on the external magnetic field for a perfectly uniform film of a type II superconductor when the external magnetic field which is parallel to the surface of the film and at right angles to the current changes from zero to \(H_{c1}(d)\), the first critical field of the film.\(^{11}\)

In the present paper, we calculate the critical field of a perfectly uniform superconductor in the mixed state. We consider two limiting cases: the case of a thin film, the thickness of which satisfies the inequalities 0 ≤ d ≤ δs, where δs is the penetration depth, and the case of a thick film (plate), the thickness of which d ≈ δs.

We consider first the structure of the mixed state of the film when the external magnetic field \(H_{ext} = (H_{c1}(d) ≤ H_{c2} ≤ H_{c1})\) is parallel to the film surface while there is no transport current. It turns out that in a film, as in a bulk superconductor, the vortices are distributed with a constant density. However, the surfaces of the film are by their very nature pinning centers\(^{10}\) and this leads to the instability of the vortex structure with respect to a transverse transport current. When a transverse transport current is switched on the whole vortex structure moves as a unit in the direction of the Lorentz force over a distance proportional to the transport current.

All these results turn out to be valid also for the mixed state of a plate.

We call the transport current for which the vortex instability develops the critical one. In the case of a film the critical current turned out to be independent of the external magnetic field.

1. STRUCTURE OF THE MIXED STATE OF A FILM WHEN THERE IS NO TRANSPORT CURRENT

Let us start with defining more precisely the problem. We consider a perfectly uniform film of thickness \(d (x^2 ≤ d ≤ 1)\) of a type II superconductor, \(x ≈ 1\). Here and below we use the units introduced in the GL paper\(^{11}\): the unit length in the quantity \(b/d(T)\) and the magnetic field quantity \(2H_{c1}\) where \(2H_{c1}\) is the critical thermodynamic field.

\[\mathbf{F} = \mathbf{F}_s - 2 \int H \cdot \mathbf{d}H_dV\]

The system of our theory in terms of the parameters \(\mathbf{R}\) and \(\mathbf{x}\) and then minimize it with respect to these parameters.

The free energy of a film unit layer along the \(x\)-axis is equal to\(^{9}\)

\[\mathbf{F} = \mathbf{F}_s - 2 \int H \cdot \mathbf{d}H_dV\]

where \(B_{r,m}\) is the field produced by the whole vortex system (and only by the vortex system) at the center of the vortex with index \(l = m\), where \(l\) is the number of the rows in which the vortex is situated, and \(m\) the number of the vortex in that row. We take as the zero