Homework problems on graphene lectures

Problem 1: (*massive Dirac particles in graphene*) Consider a tight-binding model on a honeycomb lattice with on-site potential[1] different on the sublattices A and B.

a) Develop a general solution of this problem in terms of plane wave states, expressing the spectrum of excitations in terms of the nearest neighbor hopping amplitude t and the asymmetry parameter $\Delta = \frac{1}{2}(V_A - V_B)$.

b) Linearize the solution found in part a) near the points K and K' treating the asymmetry Δ as a small perturbation, and find how the massless Dirac picture is altered. Show that Δ generates a finite mass of Dirac quasiparticles.

Problem 2: (*chiral dynamics*)

a) For massless Dirac paticles moving in a general potential $V(x)$ (no y-dependence) show that at normal incidence, $k_y = 0$, there is perfect transmission.

b) For a step-like potential $V(x) = V_0 \,\mathrm{sign} x$ find transmission as a function of the incidence angle. For simplicity, consider the case of zero particle Fermi energy (zero doping).

c) (optional) Qualitatively, discuss transmission at a nonzero energy.

Problem 3: (*relativistic Landau levels and parity anomaly*) a) Consider 2D massless Dirac particles in external uniform magnetic field B. Find the Landau level (LL) energy spectrum. For each LL, determine the valley and spin degeneracy.

b) Generalize the solution of part a) to the case of massive Dirac particles constructed in Problem 1. Find the Landau level spectrum and its valley and spin degeneracy. For each of the Dirac points K and K', how does the energy of the $n = 0$ LL depend on Δ?

c) Show that the contributions of the states at K and K' to Hall conductivity σ_{xy} *are not equal.* Show that at zero ϵ_F (zero doping) there is a nonzero Hall conductivity[2] for both K and K' due to the $n = 0$ level. Find σ_{xy}^K and $\sigma_{xy}^{K'}$.

We shall discuss solutions in class on Tuesday and Thursday. If you'll have any questions, ask in person or by email (levitov@mit.edu)

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[1] It is believed that this describes epitaxial graphene on SiC substrate. It was recently shown (Mattausch, Pankratov, Phys. Rev. Lett. 99, 076802) that all electrons in the first carbon layer are covalently bound to the substrate, leaving no π electron band and no conducting states with low energy. Atoms of the second graphene layer are sitting on top the first layer, shifted so that it provides a modulation $V_A \neq V_B$. Thus the π band of the second layer is described by the standard graphene tight-binding Hamiltonian with a small sublattice modulation.

[2] The net Hall conductivity at zero ϵ_F vanishes: $\sigma_{xy}^K + \sigma_{xy}^{K'} = 0$.