1. Universal conductance fluctuations

Consider conductance of a cube of size L in a d-dimensional space, such that transport in this cube is diffusive, $L \gg \ell \gg \lambda_F$. In such a system, conductance $G = L^{d-2}\sigma$ is a function of realization of the disorder potential, fluctuating about its average value $G_{\rm ave} = \langle G \rangle_{\rm disorder}$. By a dimensional analysis of the diagram shown in Fig.1, show that when the system is fully coherent, $L \ll L_\phi = \sqrt{D\tau_\phi}$, the conductance variance $\delta G^2 = \langle (G - G_{\rm ave})^2 \rangle_{\rm disorder}$

a) is independent of the mean free path ℓ, and

b) is independent of the system size L (see Rammer, Chap.11 posted as PDF on 8513 webpage)..

c) Argue that for macroscopic systems conductance fluctuations are below the scale of UCF: for system size $L \gg L_\phi$, the phase-breaking length, the fluctuating part of conductance scales as $\delta G = G - \langle G \rangle \approx \frac{e^2}{h}(L_\phi/L)^\alpha$, because conductances of the L_ϕ-blocks add classically. Find the value of α and check that it is in accord with the central limit theorem.

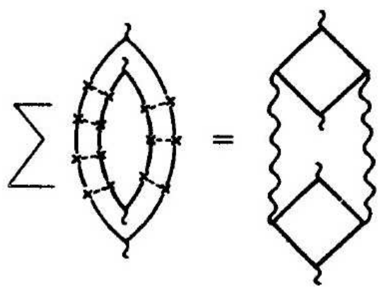

Figure 1: One of the diagrams contributing to mesoscopic conductance fluctuations

2. Scaling theory for Anderson localization

Consider the renormalization group (RG) equation for dimensionless conductance $g = G/(e^2/h)$,

$$\frac{d \log g}{d\xi} = \beta(g), \quad \xi = \log \frac{L}{\ell}, \tag{1}$$

which describes the dependence of conductance of a quantum system on the spatial scale (or, system size).

a) Solve the RG equation for $\beta(g) = (d-2) - a/g$, an expression inferred from weak localization correction. Show that the dependence $g(L)$ for $d = 1, 2, 3$ agrees with the results found in Problem 1 b) of PS#8.

b) Suppose the beta function has a zero at $g = g_c$ and behaves linearly near this zero, $\beta(g) \approx s \ln g/g_c$, where $|g - g_c| \ll g_c$. This is the case, for example, in three dimensions. In the work by Abrahams, Anderson, Licciardello, and Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979), the RG equation (1) was used to derive scaling behavior for localization length below the transition and conductivity above the transition:

$$L_*(\epsilon < 0) \propto |\epsilon|^{-1/s}, \quad \sigma(\epsilon > 0) \propto \epsilon^{(d-2)/s}$$

where ϵ is an external parameter that controls the transition ($g = g_c$ at $\epsilon = 0$). Reproduce the argument of AALR leading to these results.