Boltzmann kinetic equation

1. Angular harmonics of the distribution function.

Consider degenerate two-dimensional electrons moving in the presence of a constant magnetic field. The system is described by the Boltzmann equation

$$(\partial_t + v\nabla + (e\mathbf{E} + e\mathbf{v}\times\mathbf{B})\nabla_p)f(t,r,p) = \int d\theta_{p'} w(\theta_p - \theta_{p'})(f(t,r,p') - f(t,r,p))$$

a) For a spatially uniform system, and in the absence of electric field, show that the Boltzmann equation can be brought to the form

$$(\partial_t + \omega_c\partial_\theta)f(t,\theta) = \int d\theta' w(\theta - \theta)(f(t,\theta') - f(t,\theta)), \quad \omega_c = eB/m \tag{1}$$

where the angle $\theta = \theta_p$ parameterizes the constant energy surface $\epsilon(p) = \epsilon_0$.

(ii) By writing the distribution function as Fourier series in θ, and solving Eq.(1), find the rate of relaxation of different angular harmonics. (Note that the term $\omega_c\partial_\theta$ couples the $\cos m\theta$ and $\sin m\theta$ harmonics.) Compare the results at $B = 0$ and at finite B.

2. Velocity autocorrelation.

Consider a two-dimensional electron gas in a disorder potential with long-range correlations, in which scattering is of a small-angle character, and in a finite magnetic field. In this case the Boltzmann equation describes diffusion of velocity along the constant energy surface:

$$(\partial_t + \omega_c\partial_\theta)f(t,\theta) = D\partial_\theta^2 f(t,\theta)$$

(true for a spatially homogeneous system).

a) Find the Green's function for this equation, $g(t,\theta-\theta')$, which satisfies the equation $(\partial_t + \omega_c\partial_\theta - D\partial_\theta^2)g(t,\theta-\theta') = \delta(t)\delta(\theta-\theta')$. (Hint: use the angular harmonic decomposition $f(t,\theta) = \sum_m \tilde{f}_m(t)e^{im\theta}$ and solve separately for each $\tilde{f}_m(t)$.)

b) Using the Green's function found in part a) or otherwise, analyze the velocity autocorrelation function $\langle v_\alpha(t)v_\beta(0)\rangle$. Use it to find the diffusion tensor components, and the longitudinal and Hall resistivities ρ_{xx} and ρ_{xy}

3. Weiss oscillations.

Consider a two-dimensional electron gas in a magnetic field in the presence of a weak external potential $U(r) = U_0\cos 2\pi y/a$, where $U_0 \ll E_F$ and $a \gg \lambda_F$.

a) Show that the resistivity of such a system will oscillate as a function of magnetic field. Which component of the resistivity tensor, ρ_{xx}, ρ_{yy}, or both of them, will exhibit oscillations? Find the condition for the maxima and minima of the resistance.

b) Find the magnitude of the oscillatory part of the resistivity as a function of the magnetic field and the period a.

See: C. W. J. Beenakker and H. van Houten review cond-mat/0412664 "Quantum transport in semiconductor nanostructures."