1. Spectral statistics and level repulsion for random matrices.
   a) Consider $2 \times 2$ hermitian matrices $H$ drawn from a Gaussian ensemble,
   \[ H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}, \quad h_{21} = h_{12}^*, \quad dP(H) = c e^{-a \text{Tr} H^2} dh_{11} dh_{22} dh_{12}' dh_{12}'' \]
   where $h_{12} = h_{12}' + ih_{12}''$. Find the joint distribution function of the eigenvalues of $H$.

   b) Same as in a) but for $2 \times 2$ real-valued hermitian matrices (i.e. symmetric real matrices, $\beta = 1$). Compare with the result found in part a) for nearly coinciding energy levels, and show that level repulsion is weaker than for hermitian matrices, $\beta = 2$.

2. Density of states for Gaussian matrices.
   Consider the density of states for a Gaussian ensemble of $N \times N$ hermitian matrices, $P(H) \propto e^{-a \text{Tr} H^2}$.
   a) By using a self-consistent Born approximation for the Greens function $G(\epsilon) = 1/(\epsilon - H + i\delta)$, or otherwise, show that the ensemble-averaged density of states at large $N$ is given by a semicircle $\rho(\epsilon) = \Delta^{-1} \sqrt{1 - \epsilon^2/\epsilon_0^2}$.

   b) Relate the values of $\Delta$ and $\epsilon_0$ to $N$ and $a$.

3. Spectral correlation function at large $N$ (hermitian ensemble).
   a) Consider the ground state $\psi(x_1, \ldots, x_N)$ of $N$ noninteracting fermions moving in a one-dimensional parabolic potential. Show that the curvature of the potential can be chosen so that the probability $|\psi(x_1, \ldots, x_N)|^2$ coincides with the eigenvalue distribution in a Gaussian ensemble of hermitian random matrices, $\beta = 2$, whereby the Pauli exclusion principle mimics level repulsion of random matrices.

   b) Consider a one-dimensional ideal Fermi gas with a constant, spatially uniform density $n$. Find the two-point density correlator $\langle \rho(x) \rho(x') \rangle = \langle \psi^\dagger(x) \psi(x) \psi^\dagger(x') \psi(x') \rangle$. Use this result to obtain the pair correlation function of energy levels for the hermitian ensemble.