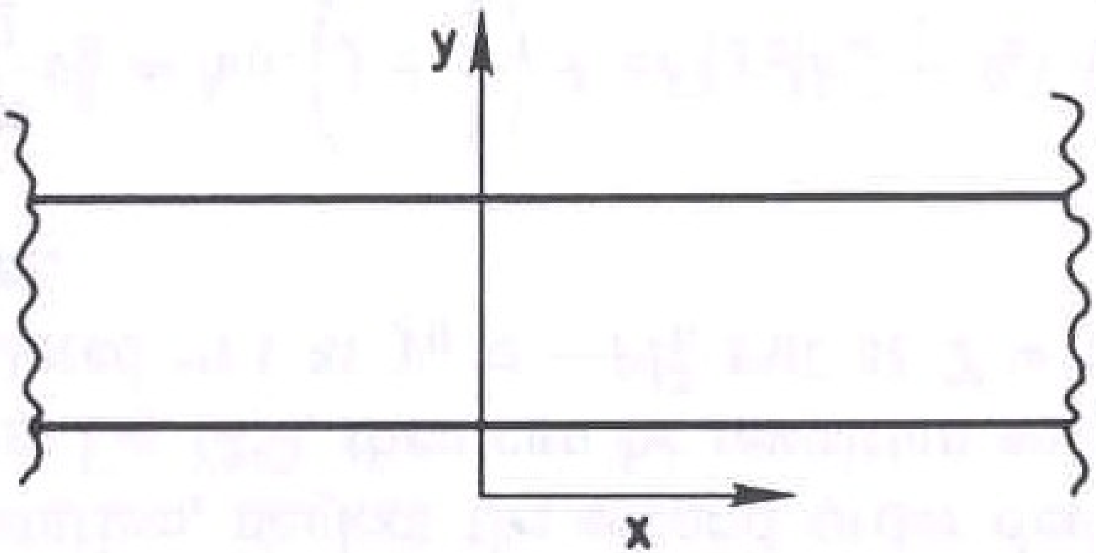
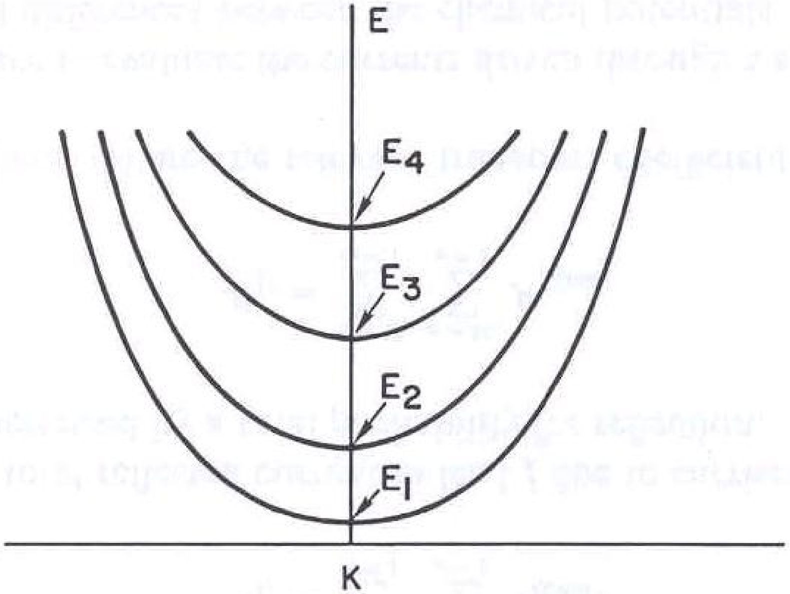
8.513 Lecture 2

Conductance from transmission

Quantum point contact

Multi-channel conductance: leads





asymptotic perfect translation invariant potential

$$V(x,y) = V(y)$$

separable wave function

$$\phi^{\pm}_{\alpha n}(\mathbf{r}, E) = e^{\pm i k_n(E) x}\, \chi_{\alpha n}(y)$$

energy of transverse motion E_n channel threshold

energy for transverse and longitudnial motion

$$E = E_n + \hbar^2 k^2/2m \quad \Longleftrightarrow \quad \text{scattering channel}$$

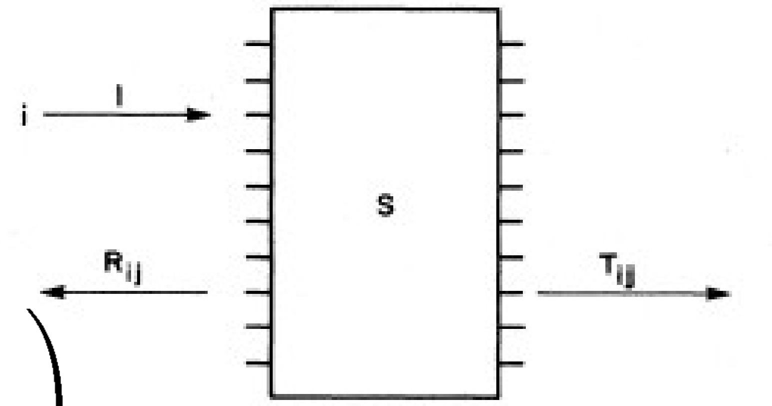
Multi-channel conductor: scattering matrix

$s_{\alpha\alpha,mn} \Longrightarrow s_{\alpha\alpha}$,

$s_{\beta\alpha,mn} \Longrightarrow s_{\beta\alpha}$

Block matrices

lead indices channel indices



$$s = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \qquad s = \begin{pmatrix} s_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix}$$

orthogonal unitary

Incident current in channel n

$$I_{in} = (e/h)eV$$

reflection transmission

$$R_{\alpha\alpha,nm} = |s_{\alpha\alpha,nm}|^2 \ , \ T_{\beta\alpha,mn} = |s_{\beta\alpha,mn}|^2 \ ,$$

Multi-channel conductance, kT = 0, two terminal

$$I = (e/h)eV \sum_{mn} T_{RL,mn} = (e^2/h)VT \Longrightarrow G = \frac{e^2}{h}T$$

Total transmission probability T

Eigen channels

$$T = \sum_{mn} T_{\beta\alpha,mn} = \sum_{mn} |s_{\beta\alpha,mn}|^2 = Tr[s^\dagger_{\alpha\beta} s_{\alpha\beta}] = Tr[t^\dagger t]$$

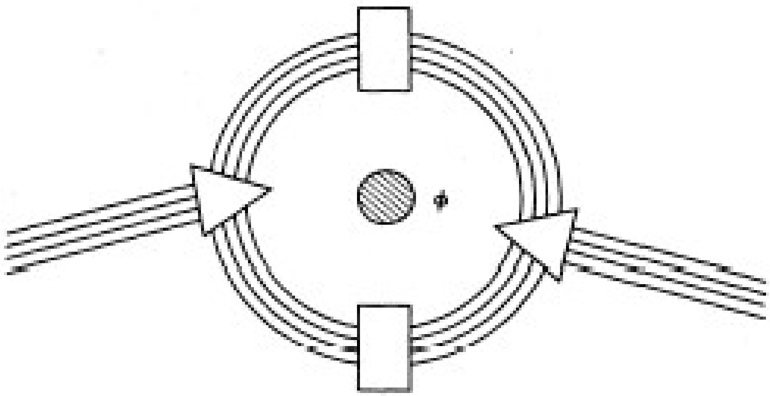
$t^\dagger t$ hermitian matrix; real eigenvalues T_n

$r^\dagger r$ hermitian matrix; real eigenvalues R_n

$$T = Tr[t^\dagger t] = \sum_n T_n$$

$$G = \frac{e^2}{h} \sum_n T_n$$

T_n are the genetic code of mesoscopic conductors !!



Mulichannel = parallel conductance of many single channel conductors

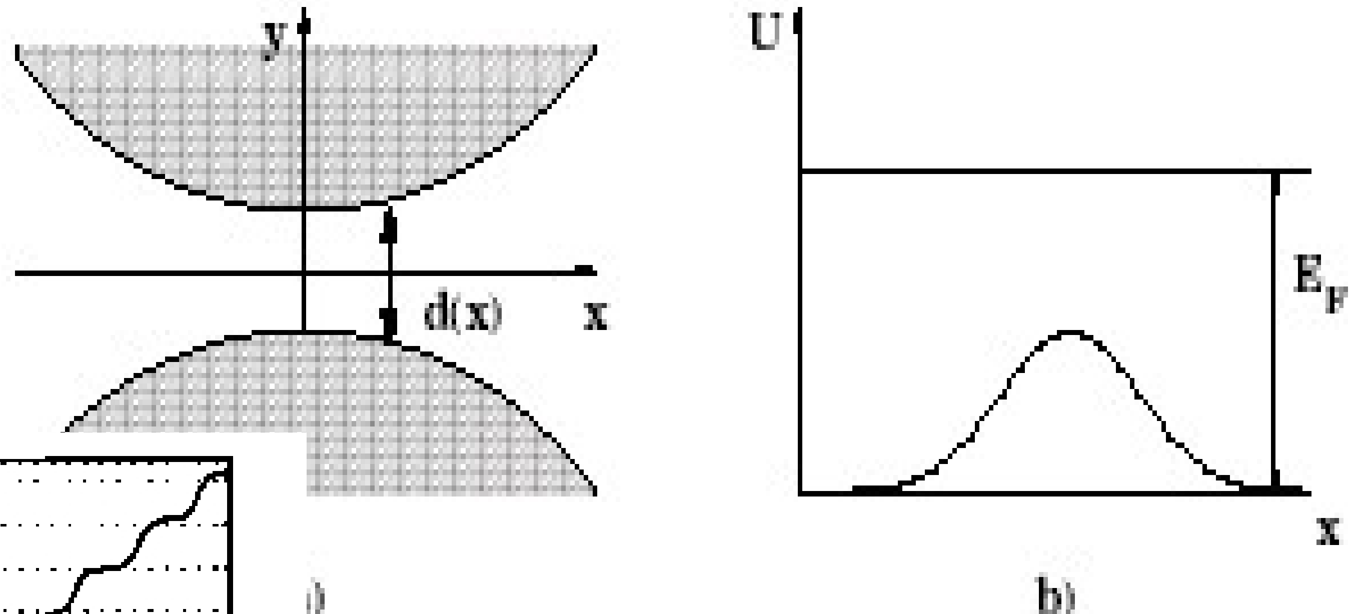
Quantum point contact

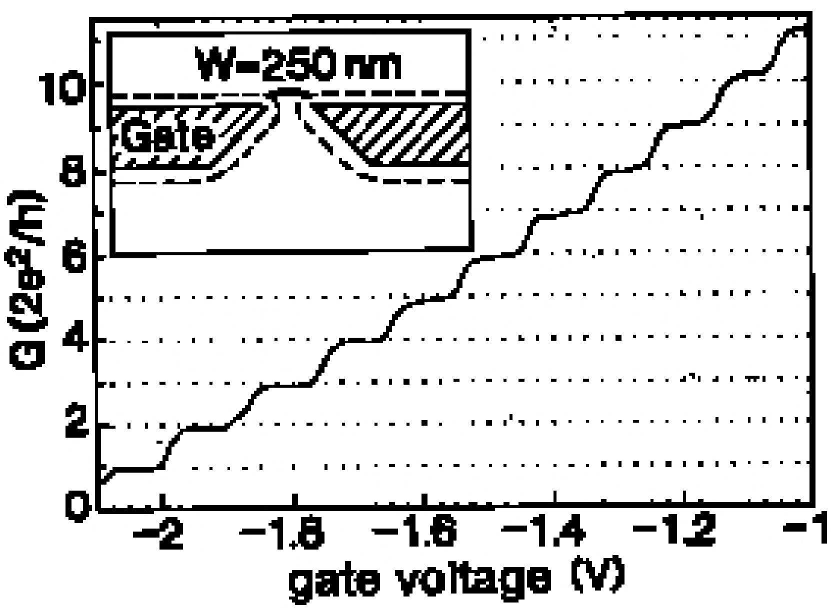
van Wees et al., PRL 60, 848 (1988)

Wharam et al, J. Phys. C 21, L209 (1988)

gate

2D-electron gas





Quantized conductance: saddle

Buttiker, Phys. Rev. B41, 7906 (1990)

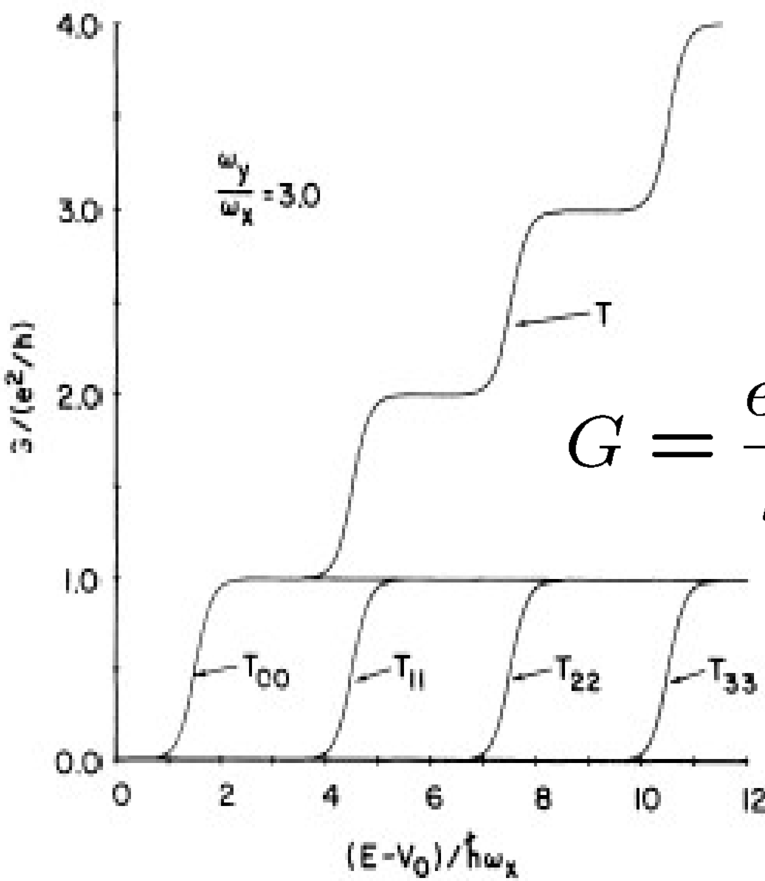
Saddle-point

$$V(x,y) = V_0 - (1/2)m\omega_x^2 x^2 + (1/2)m\omega_y^2 y^2 + \ldots$$

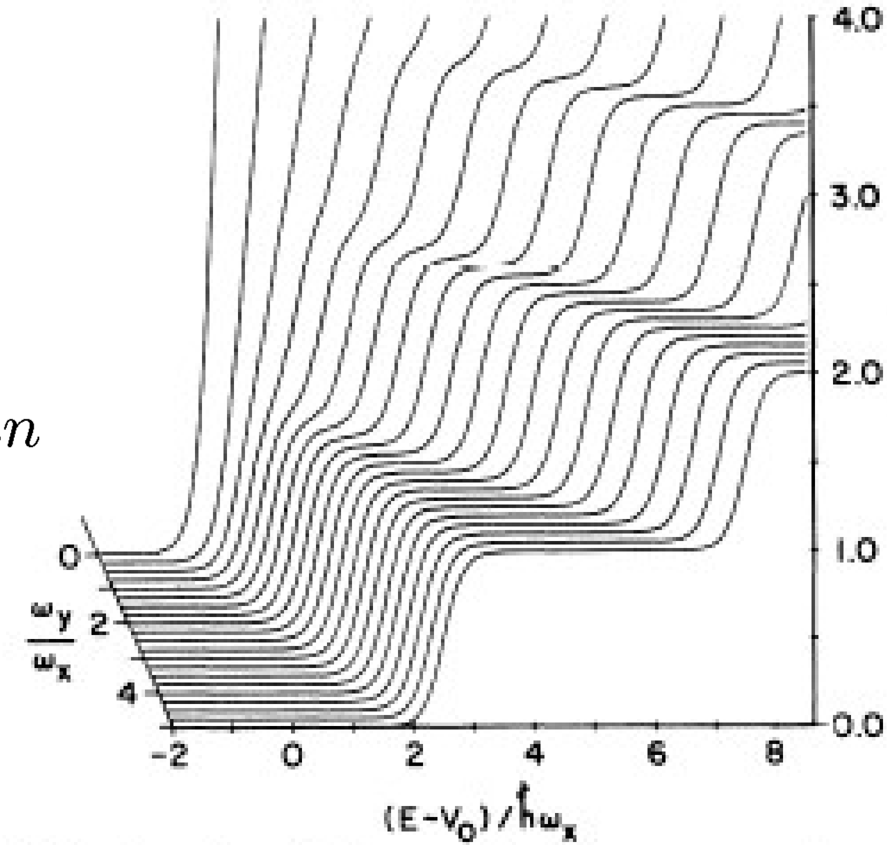
Transmission probability

Solve 1d Schrodinger eqn for an inverted parabola (Kemble 1937)

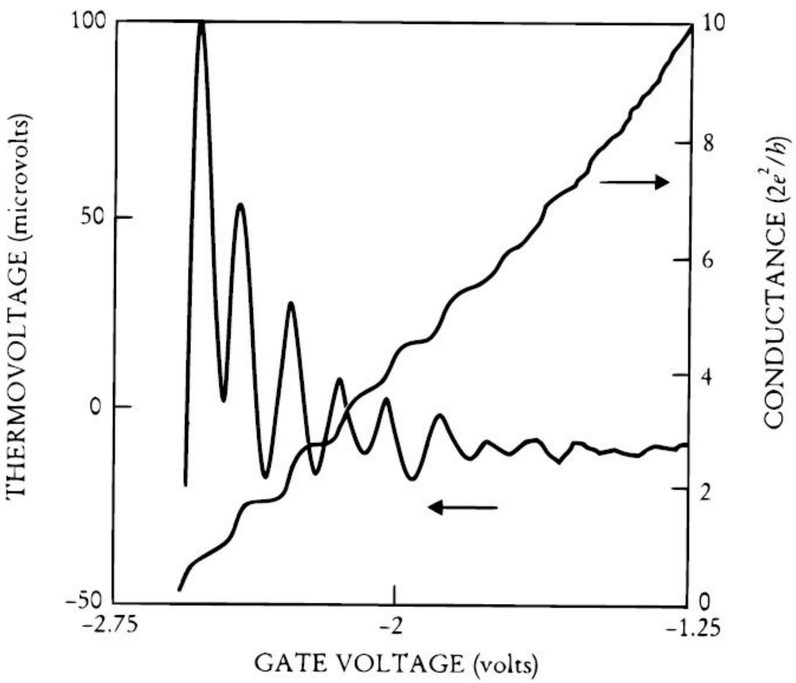
$$T_{nn} = \delta_{mn}\frac{1}{1+e^{-\pi\epsilon_n}} \; ; \epsilon_n = 2[E - V_0 - \hbar\omega_y(n+1/2)]/\hbar\omega_x$$



$$G = \frac{e^2}{h}\sum_n T_{nn}$$



Thermo-electric effects in QPC



A small temperature difference δT

$$\begin{pmatrix} \text{electrical current} \\ \text{heat current} \end{pmatrix} = \begin{pmatrix} G & L \\ L' & K \end{pmatrix} \cdot \begin{pmatrix} -V \\ \delta T \end{pmatrix}.$$

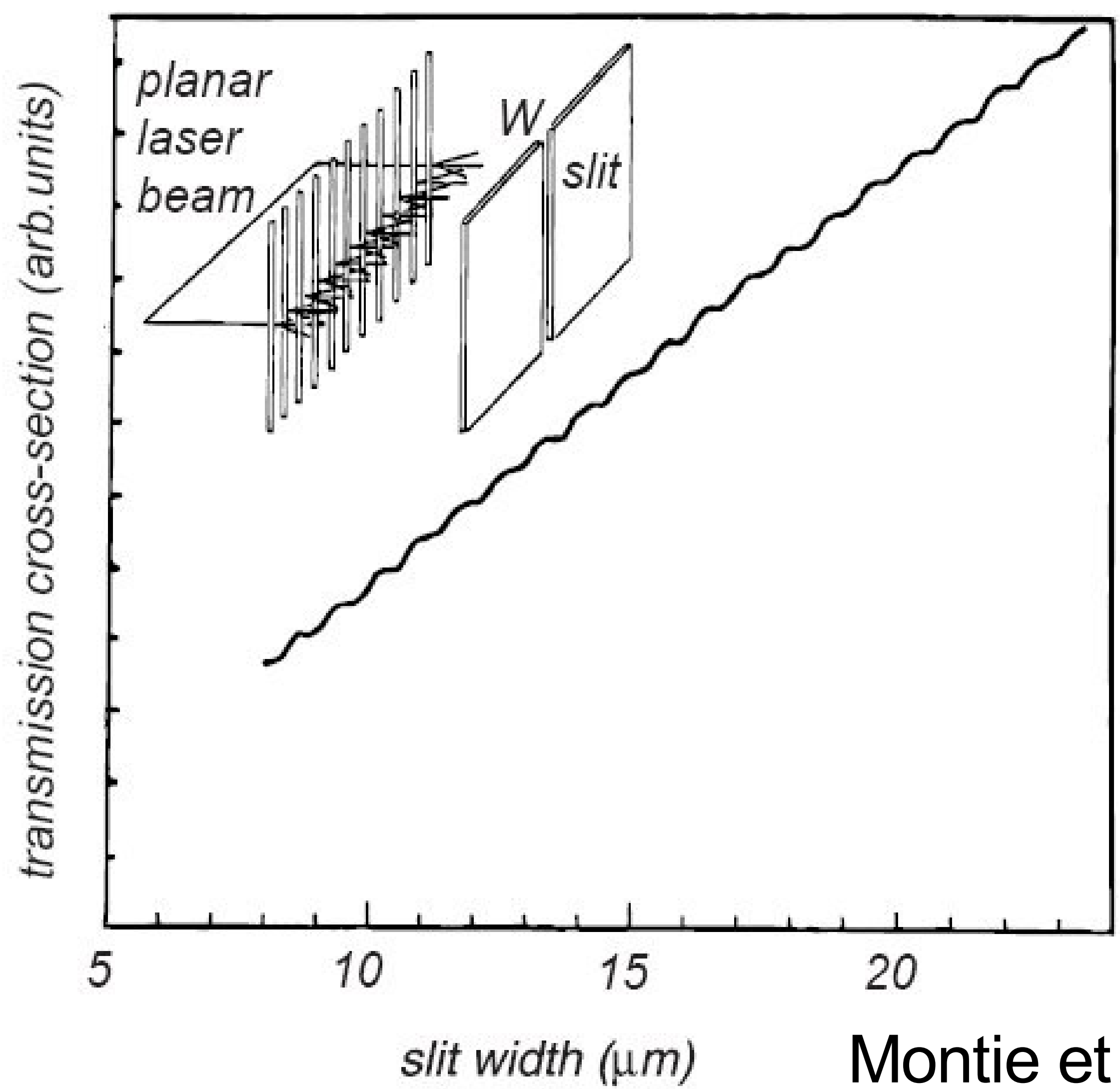
The thermal conductance K time-reversal symmetry

The thermo-electric coefficients L and $L' = -LT$

$K \propto t$ and $L \propto dt/dE_{\mathrm{F}}$, where $t = \sum_n t_n$

Molenkamp et al. 1990

Optical analog of a QPC: transmission through a slit



Periodicity λ/2

2D isotropic illumination by shining light from a diode laser λ=1.55μm

Montie et al et al. 1991

Transmission coefficient for inverted parabola

from QM by Landau & Lifshitz

Construct the scattering state of S.E. in a quasiclassical form at large |x|:

$$U(x) = -kx^2/2 \qquad p = \sqrt{2m\left(E + \frac{1}{2}kx^2\right)} \approx x\sqrt{mk} + E\sqrt{\frac{m}{k}}\frac{1}{x}$$

Use the asymptotic form of the wavefunction

$$\psi = \text{const}\cdot\xi^{\pm i\varepsilon - 1/2}\exp(\pm i\xi^2/2),$$

$$\xi = x\left(\frac{mk}{\hbar^2}\right)^{1/4}, \quad \varepsilon = \frac{E}{\hbar}\sqrt{\frac{m}{k}}.$$

Particle incident from the left-hand side is described by

$$\psi = B\xi^{i\varepsilon - 1/2}\exp(i\xi^2/2) \qquad x \to \infty,$$ transmitted wave

$$\psi = (-\xi)^{-i\varepsilon - 1/2}\exp(-i\xi^2/2) + A(-\xi)^{i\varepsilon - 1/2}\exp(i\xi^2/2) \qquad x \to -\infty.$$ Incident and reflected wave

Extend the solution of S.E. to complex ξ (b/c its an analytic function) and continue it from positive to negative ξ through the upper half-plane, over a large circle of radius ρ, $0<\phi<\pi$

$$\xi = \rho e^{i\varphi}, \quad i\xi^2 = \rho^2(-\sin 2\varphi + i\cos 2\varphi),$$

Upon doing this, transmitted wave turns into reflected wave such that

$$A = B\left(e^{i\pi}\right)^{i\varepsilon - 1/2} = -iBe^{-\pi\varepsilon};$$

NB: for $\pi/2 < \varphi < \pi$ the modulus $|\exp(i\xi^2/2)|$ Is exp. large, 1^{st} term lost

Combine with the unitarity relation $|A|^2 + |B|^2 = 1.$

Obtain the transmission and reflection probabilities

$$T = |B|^2 = \frac{1}{1 + e^{-2\pi\epsilon}}.$$

$$R = 1\text{-}T = \frac{1}{1 + e^{2\pi\epsilon}}$$

Finite temperature

Conductance: finite temperature

$$dI_{inc,L} = \frac{e}{h} dE\, f_L(E) \quad \Longrightarrow$$

current of left movers

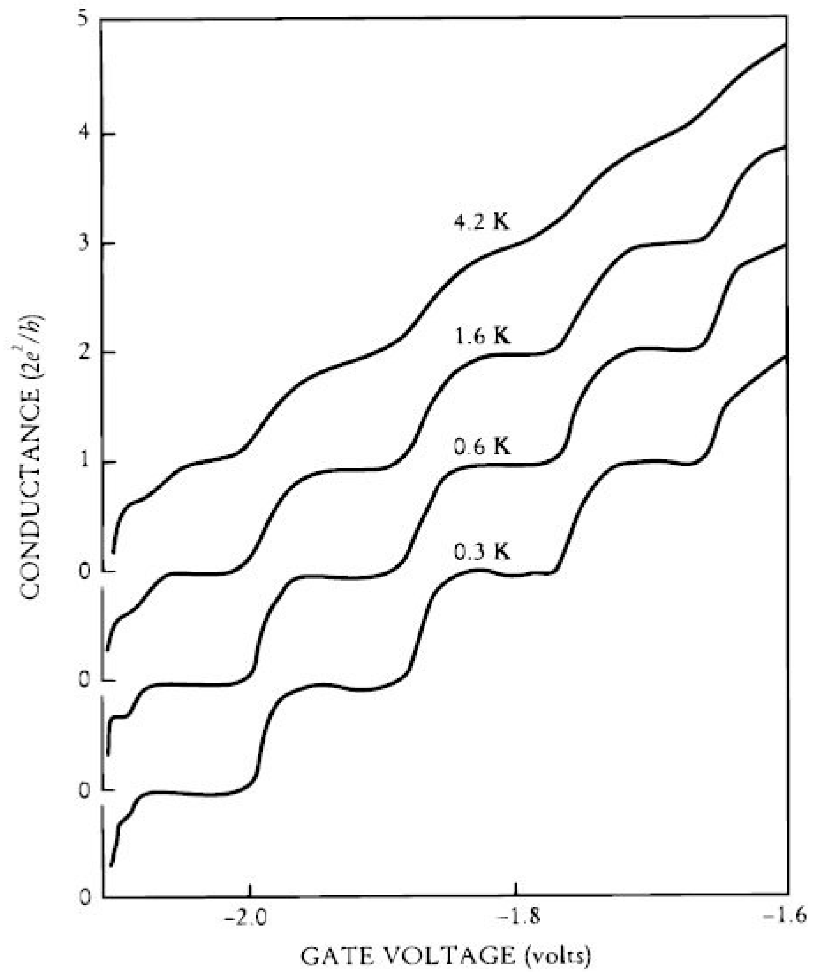
$$I_L = \frac{e}{h} \int dE\, T(E)\, f_L(E)$$

current of right movers

$$I_R = -\frac{e}{h} \int dE\, T(E)\, f_R(E)$$

net current

$$I = \frac{e}{h} \int dE\, T(E)\, (f_L(E) - f_R(E))$$



linear response

$$f_L(\mu_L) = f(\mu_0) - (df/dE)eV_L + .. \qquad V = V_L - V_R \quad \Longrightarrow$$

conductance

Transmission probability evaluated in the equilibrium potential

$$G = I/V = \frac{e^2}{h} \int dE\, T(E)\, (-df(E)/dE)$$

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