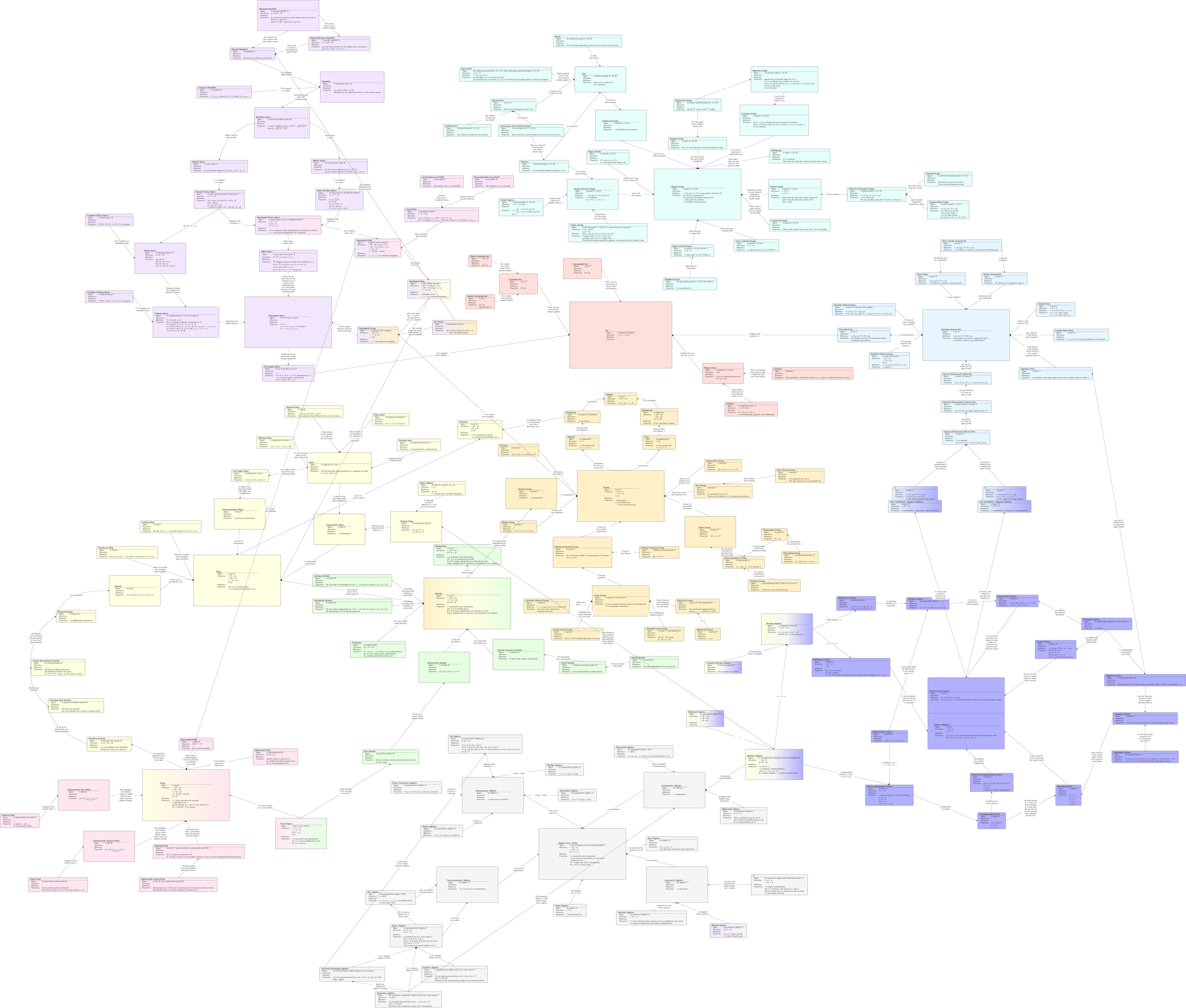

Riemann Manifold
Pseudo Riemann Manifold
Smooth Manifold
Manifold
Compact Manifold
Euclidean Space
Banach Space
Hilbert Space
Normed Vector Space
Inner Product Space
Complete Metric Space
Metric Space
Topological Vector Space
Affine Space
Complete Uniform Space
Uniform Space
Topological Space
Convergent Space
Topological Field
Topological Ring
Topological Group
Lie Group
Tree
Graph
Multigraph
Set
Group
Ring
Field
Module
Vector Space
Lattice

Section 1

Algebra: Basic

Algebra Over a Field

Types:	A set of elements A, with underlying field K
Functions:	$\cdot : A^2 \to A$
	$+ : A^2 \to A$
	$K \times A \to A$
Relations:	
Properties:	$+$ is associative and commutative
	$\cdot$ is not assumed commutative or associative
	$\cdot$ distributes over $+$
	$\times$ by $k \in K$ represents scalar multiplication
	$(A, +, K)$ is a vector space not assumed commutative or associative
	$\cdot$ distributes over $+$

Section 2

Algebra:Commutative Algebras

Commutative Algebra

Types:	An algebra A
Functions:	
Relations:	
Properties:	$\cdot$ is commutative

Differential Algebra

Types:	A commutative algebra A
Functions:	$\partial_i : A \to A$
Relations:	
Properties:	There are finitely many ∂_i over A
	All ∂_i satisfy the Leibniz product rule
	∂_i is a homomorphism of A^+

Polynomial Algebra

Types:	A commutative Algebra $A[X]$
Functions:	$1 \in A[X]$
Relations:	
Properties:	$X = \{x_1, x_2, ...x_n\}$ where x_i is an indeterminate, $n \geq 1$

Heyting Algebra

Types:	A distributive lattice H
Functions:	$\longrightarrow: H^2 \to H$
Relations:	
Properties:	$(c \wedge a) \leq b \Leftrightarrow c \leq (a \longrightarrow b)$
	Monoid $(H, \cdot, 1)$ has operation $\cdot = \wedge$

Boolean Algebra

Types:	A complemented, distributive lattice and algebra B
Functions:	$\neg : B \to B$
	$\oplus : B^2 \to B$
Relations:	
Properties:	$\forall x \in B, x^2 = x$
	$\neg$ is negation (complementation)
	$x \oplus y = (x \vee y) \wedge \neg(x \wedge y)$
	$\oplus$ is algebra addition, $\wedge$ is algebra multiplication

Relational Algebra

Types:	A commutative algebra R
Functions:	$\vee : R^2 \to R$
	$\wedge : R^2 \to R$
	$- : R \to R$
Relations:	
Properties:	

Section 3

Algebra: Associative Algebra

Associative Algebra

Types:	An Algebra A
Functions:	
Relations:	
Properties:	$\cdot$ is associative

Banach Algebra

Types:	An associative algebra A
Functions:	$\| \| : A \to \mathbb{R}$
Relations:	
Properties:	$\forall x, y \in A : \|xy\| \leq \|x\| \|y\|$ A is also a Banach space

An associative algebra and locally finite poset, A $\delta : A \to A$
$* : A^2 \to A$ $*$ is algebra multiplication
Any $a \in A$ behaves such that $[b, c] \mapsto a(b, c)$
$a(b, c)$ is taken from a commutative ring of scalars
δ is the identity function

Operator Algebra

Types:	An associative Algebra A
Functions:	$\circ : A^2 \to A$
Relations:	
Properties:	$a \in A$ are continuous linear operators over a topological vector space $\circ$ is operator composition (and algebra multiplication)

Section 4

Algebra: Noncommutative Algebra

Noncommutative Algebra

Types:	An algebra A
Functions:	
Relations:	
Properties:	$\cdot$ in A is need not be commutative

Free Algebra

Types:	A noncommutative algebra $A[X]$
Functions:	$1 \in A[X]$
Relations:	
Properties:	$X = \{x_1, x_2, ..., x_n\}$ where x_i is an indeterminate from an alphabet
	1 is the empty word

Tensor Algebra

Types:	A noncommutative Algebra T
Functions:	$\oplus : T^2 \to T$
	$\otimes : T^2 \to T$
Relations:	
Properties:	T is defined over any vector space V
	$T^k V = V \otimes_1 V \otimes_2 ... \otimes_k V$
	$T(V) = C \oplus V \oplus (V \otimes V) \oplus (V \otimes V \otimes V) \oplus ....$
	$T^k V \otimes T^l V = T^{k+l} V$
	$T(V)$ is the most general algebra over V

Universal Enveloping Algebra

Types:	A unital associative Algebra $\mathcal{U}(\mathfrak{g})$ of a Lie Group $\mathfrak{g}$
Functions:	
Relations:	
Properties:	$\mathcal{I}$ is the ideal generated from $x \otimes y - y \otimes x - [x, y] \forall x, y \in T(\mathfrak{g})$
	$\mathcal{U}(\mathfrak{g}) = \otimes \mathfrak{g}) / \mathcal{I}$

Symmetric Algebra

Types:	An associative, commutative algebra $\bigvee(V)$ of a vector space V
Functions:	$1 \in \bigvee(V)$
Relations:	
Properties:	$\mathcal{I}$ is the ideal generated from $w \otimes v - v \otimes w, \forall w, v \in V$
	$\bigvee(V) = T(V)/\mathcal{I}$
	Division by the commutator makes $\bigvee(V)$ commutative

Exterior Algebra

Types:	A nonassociative algebra $\bigwedge(V)$ of a vector space V
Functions:	
Relations:	
Properties:	$\mathcal{I}$ is the ideal generated from $w \otimes v + v \otimes w, \forall w, v \in V$
	$\bigwedge(V) = T(V)/\mathcal{I}$
	Division by the anticommutator makes $\bigvee(V)$ anticommutative

Section 5

Algebra: Nonassociative Algebra

Nonassocative Algebra

Types:	An algebra A
Functions:	
Relations:	
Properties:	$\cdot$ is need not be associative

Jordan Algebra

Types:	A nonassociative algebra A
Functions:	
Relations:	
Properties:	$\forall x, y \in A : (xy)(xx) = (x(y(xx))$

Power Associative Algebra

Types:	A nonassocative Algebra A
Functions:	
Relations:	
Properties:	$\forall x, y \in A : x(x(xx)) = (xx)(xx) = (x(xx))x$

Flexible Algebra

Types:	A nonassociative Algebra A
Functions:	
Relations:	
Properties:	$\forall x, y \in A : x(yx) = (xy)x$

Alternative Algebra

Types:	A nonassociative Algebra A
Functions:	
Relations:	
Properties:	$\forall x, y \in A : xx(y) = x(xy)$

Lie Algebra

Types:	A nonassociative Algebra $\mathfrak{g}$
Functions:	$[\,] : \mathfrak{g}^2 \to \mathfrak{g}$
Relations:	
Properties:	$\forall x, y \in \mathfrak{g} : [x, y] = -[y, x]$
	$\forall x, y, z \in \mathfrak{g} : [x, [y, z]] + [y[z, x]] + [z, [x, y]] = 0$
	$\forall x, y, z \in \mathfrak{g} : [ax + by, z] = a[x, z] + b[y, z]$ and $[x, ay + bz] = a[x, y] + b[x, z]$
	$[x, x] = 0$

Section 6

Algebra: Not further categorized

Zero Algebra

Types:	An algebra A
Functions:	
Relations:	
Properties:	$\forall u, v \in A, uv = 0$
	A is inherently associative and commutative

Unital Algebra

Types:	An algebra A
Functions:	$1 \in A$
Relations:	
Properties:	1 is the identity for $\cdot$

Section 7

Fields: Basic

Field

Types:	A ring K
Functions:	$+ : K^2 \to K$
	$\times : K^2 \to K$
	$-_+^{-1} : K \to K$
	$-_\times^{-1} : K/\{0\} \to K/\{0\}$
	$0, 1 \in K$
Relations:	
Properties:	$+, \times$ both associate and commute
	$\times$ distributes over $+$
	0 is the identity for $+$ and 1 is the identity for $\times$
	$(K, +)$ and $(K, \times)$ are groups

Section 8

Fields: Topological + related

Topological Field

Types:	A topological ring F
Functions:	$\cdot : R^2 \to R : (x, y) \mapsto x \cdot y$
	$+ : R^2 \to R : (x, y) \mapsto x + y$
	$-_+^{-1} : R \to R$
	$-_\times^{-1} : F/\{0\} \to F/\{0\}$
Relations:	
Properties:	$+, \cdot, -_+^{-1}, -_\times^{-1}$ are continuous mappings

Local Field

Types:	A topological Field K
Functions:	$\lvert\,\rvert : K \to \mathbb{R}_{\geq 0}$
Relations:	
Properties:	$\forall u, v \in K, \exists \mathcal{U}, \mathcal{V} \in \tau : u \in \mathcal{U}, v \in \mathcal{V}, \mathcal{U} \cap \mathcal{V} = \varnothing$
	if $C \subseteq \tau$ satisfies $X = \bigcup_{x \in C} x$, then there exists $F \subseteq C, F$ finite $X = \bigcup_{x \in F} x$

Archimedian Local Field

Types:	A local field K
Functions:	
Relations:	
Properties:	The sequence $\lvert n\rvert_{n\geq 1}$ is unbounded

Nonarchimedian Local Field

Types:	A local field K
Functions:	
Relations:	
Properties:	The sequence $\lvert n\rvert_{n\geq 1}$ is bounded

Section 9

Fields: Characteristics

Characteristic Zero Field

Types:	A field K
Functions:	
Relations:	
Properties:	$\nexists n \in \mathbb{N} : \underbrace{1 + 1 + \ldots + 1}_{n} = 0$

Ordered Field

Types:	A characteristic zero field K
Functions:	
Relations:	$\leq$
Properties:	$\leq$ respects $+$ and $\times$
	K is necessarily infinite

Characteristic Nonzero Field

Types:	A field K
Functions:	
Relations:	
Properties:	$\exists n \in \mathbb{N} : \underbrace{1 + 1 + \ldots + 1}_{n} = 0$

Finite Field

Types:	A characteristic nonzero field K
Functions:	
Relations:	
Properties:	K has a finite number of elements
	The order of some $k \in K$ is p^n for some p prime, $n \in \mathbb{N}$

Section 10

Fields: Not further categorized

Vector Space

Types:	A set V over a field K
Functions:	$+ : V^2 \to V$
	$-_{+}^{-1} : V \to V$
	$K \times V \to V$
	$0 \in V$
Relations:	
Properties:	$+$ is associative and commutative
	$K \times V$ is scalar multiplication by K
	$(V, +)$ is a group

Exponential Field

Types:	A field K
Functions:	$\Phi : K^+ \to K^\times$
Relations:	
Properties:	Φ is a homomorphism

Differential Field

Types:	A polynomial field K
Functions:	$\partial_i : K \to K$
Relations:	
Properties:	Finitely many ∂_i exist over K
	∂_i satisfies the Leibniz product rule
	∂_i is a homomorphism of K^+

Section 11

Fields: Algebraic

Splitting Field

Types:	A polynomial field K
Functions:	
Relations:	
Properties:	K is extended so that some polynomial (or set of polynomials) $k \in K$ can be written as the product n linear factors, where n is the degree of the polynomial

Algebraically Closed Field

Types:	A field K and a polynomial ring $K[X]$
Functions:	
Relations:	
Properties:	All polynomials $x \in K[X]$ can be decomposed as the product of linear factors Equivalently, there is no proper algebraic extension of K

Section 12

Graphs: Basic

Simple Graph

Types:	A tuple $G = (V, E)$, V, E are sets
Functions:	
Relations:	E
Properties:	E is composed of unordred pairs of V E is irreflexive and symmetric

Section 13

Graphs: Undirected Graphs

Undirected graph

Types:	A graph $G = (V, E)$
Functions:	
Relations:	
Properties:	$\sim$ is irreflexive and symmetric

Tree

Types:	A undirectd graph $G = (V, E)$
Functions:	
Relations:	
Properties:	There are no cycles in G G is connected

Forest

Types:	An undirected graph $G = (V, E)$
Functions:	
Relations:	
Properties:	G's connected component are trees, but G need not be connected

Rooted Tree

Types:	A tree G
Functions:	
Relations:	
Properties:	One vertex is designated as the root

Line Graph

Types:	An undirected graph $L(G) = (V^*, E^*)$, with underlying undirected graph $G = (V, E)$
Functions:	$f : E \to V^*$
Relations:	$\sim^* = \{\{x, y\} \in E^*\}\vert$
Properties:	f maps edges in G to vertexes in $L(G)$ An unordered pair of vertices $\{v_i^*, v_j^*\} \in E^*$ iff their corresponding edges in G share an endpoi

Section 14

Graphs: Directed Graphs

Simple Directed Graph

Types:	A graph $G = (V, E)$
Functions:	
Relations:	$\rightarrow = \{(x, y) \in V \times V \mid (x, y) \in E\}$
Properties:	$\rightarrow$ is irreflexive
	$\forall u, v \in V, u \rightarrow v \nRightarrow v \rightarrow u$

Polytree

Types:	A directed graph $G = (V, E)$
Functions:	
Relations:	
Properties:	G's underlying undirected graph is a tree

Ordered Tree

Types:	A directed graph $G = (V, E)$
Functions:	
Relations:	
Properties:	The children of nodes in G are ordered

Arboresence and Antiarboresence

Types:	A rooted polytree, $G = (V, E)$
Functions:	
Relations:	
Properties:	Every node has a directed path away from or toward the root

Cayley Graph

Types:	A directed graph $\Gamma = \Gamma(G, S)$, S a generating set for group G
Functions:	$f : G \rightarrow V(G)$
	$g : S \rightarrow C_S$
Relations:	$E(\Gamma) = \{(g, gs) \in G \times G \mid \forall g \in G, \forall s \in S\}$
Properties:	f assigns each $g \in G$ to a vertex in Γ
	g assigns each $s \in S$ to a unique color
	The identity element generally is ignored, so the graph does not contain loops

Graph Algebra

Types:	
Functions:	$\cdot : V^2 \to V \cup \{0\}$
Relations:	
Properties:	$x \cdot y : \begin{cases} x & x, y \in V, (x,y) \in E \\ 0 & x, y \in V \cup \{0\}, (x,y) \notin E \end{cases}$
	$0 \notin V$

Section 15

Graphs: K-Partite Graphs

K-partite Graph

Types:	A graph $G = (V, E)$
Functions:	
Relations:	
Properties:	$V_1, V_2, ..., V_n$ are disjoint sets that partition G's vertices
	All $e_i \in E$ can be written as $\{v_i, v_j\}$ where $v_i \in V_i, v_j \in V_j$ and $i \neq j$
	G is k-colorable

Bipartite Graph

Types:	A k-partite graph $G = (V, E)$
Functions:	
Relations:	
Properties:	Special case of k-partite graph for $k = 2$
	V_1, V_2 are disjoint sets partition G's vertices
	All $e_i \in E$ can be written as $\{v_i, v_j\}$ for some $v_i \in V_1$ and $v_j in V_2$
	G has no odd cycles
	G is 2-colorable

Hypercube Graph

Types:	A regular, bipartite graph $Q_n = (X, E)$
Functions:	
Relations:	
Properties:	Q_n has 2^n vertices and $2^{n-1}n$ edges

Section 16

Graphs: Perfect Graph

Perfect Graph

Types:	A graph $G = (X, E)$
Functions:	$\omega : G \mapsto n$
	$\chi : G \mapsto n$
Relations:	
Properties:	$\omega(G)$ is the the size of the largest clique in G
	$\chi(X)$ is the the chromatic number of G
	$\omega(G) = \chi(G)$

Perfectly Orderable Graphs

Types:	A perfect graph $G = (V, E)$
Functions:	
Relations:	$<= \{(x, y) \in V \times V \mid x < y\}$
Properties:	$<$ is a total order
	For any chordless path $abcd \in G$ and $a < b$, then d $\nless$ c

Chordal Graph

Types:	A perfectly orderable graph $G = (V, E)$
Functions:	
Relations:	
Properties:	All induced cycles are of order 3
	G has a perfect elimination order

Comparability Graph

Types:	A perfect graph $C = (S, \perp)$
Functions:	
Relations:	$\perp = \{(x, y) \in S \mid x < y\}$
Properties:	S is the set of vertices
	S is a poset under $<$
	$\perp$ is the edge relation
	$(x, y), (y, z) \in E \implies (x, z) \in E$

Section 17

Graphs: Not further categorized

Hyper Graph

Types:	A tuple $H = (V, E)$
Functions:	
Relations:	
Properties:	$E = \{e_i \vert i \in I_e, e_i \subseteq V\}$
	I_e is an index set for edges in H

Connected Graph

Types:	A graph $G = (V, E)$
Functions:	
Relations:	
Properties:	There exists a path between any two $v \in V$ in the graph

Multigraph

Types:	
Functions:	
Relations:	
Properties:	E is a multiset
	More than one edge may connect the same two vertices

Vertex Labeled Graph

Types:	A graph $G = (V, E)$
Functions:	$f : V \to L$
Relations:	
Properties:	f maps vertexes to a set of labels L

Edge Labeled Graph

Types:	A graph $G = (V, E)$ and set L
Functions:	$f : E \to L$
Relations:	
Properties:	f maps edges to a set of labels L

Weighted Graph

Types:	An edge labeled graph $G = (V, E)$ and labels L
Functions:	
Relations:	
Properties:	L is an ordered set

Regular Graph

Types:	A graph $G = (V, E)$
Functions:	
Relations:	
Properties:	All $v \in V$ are connected to the same number of edges

Section 18

Groups: Prelims

Magma

Types:	A set M of elements
Functions:	$\cdot : M^2 \to M$
Relations:	
Properties:	$\forall u, v \in M : u \cdot v \in M$

Semigroup

Types:	A magma S of elements
Functions:	
Relations:	
Properties:	$\cdot$ is associative

Monoid

Types:	A semigroup S
Functions:	$1 \in S$
Relations:	
Properties:	1 is the identity for $\cdot$ $1 \cdot\ u = u \cdot 1 = u$

Quasigroup

Types:	A magma Q
Functions:	$\diagdown: Q^2 \to Q$
	$\diagup: Q^2 \to Q$
	$1_R, 1_L \in Q$
Relations:	
Properties:	Q is a cancellative magma

Loop

Types:	A quasigroup Q
Functions:	$1 \in Q$
Relations:	
Properties:	1 is the identity for $\cdot$
	$1 \cdot x = x \cdot 1 = x$

Group

Types:	A set G
Functions:	$\cdot : G^2 \to G$
	$-^{-1} : G \to G$
	$1 \in G$
Relations:	
Properties:	$\cdot$ is associative
	1 is the identity for $\cdot$
	$-^{-1}$ is the inverse map for $\cdot$

Section 19

Groups: Random/Uncatagorized

P Group

Types:	A group G
Functions:	
Relations:	
Properties:	$\forall g \in G, \lvert g \rvert = p^n, p$ prime, $n \in \mathbb{N}$

Torsion Free Group

Types:	A group G
Functions:	
Relations:	
Properties:	$\nexists g \in G, g \neq 1 : g^n = 1, n \in \mathbb{N}$

Simple Group

Types:	A group G
Functions:	
Relations:	
Properties:	$N \lhd G \Longrightarrow N = \{1\}$ or G

Topological group

Types:	A group G
Functions:	
Relations:	
Properties:	$-^{-1}$ and $\times$ are continuous maps

Lie Group

Types:	A topological group G
Functions:	
Relations:	
Properties:	$\forall p \in G, \exists \mathcal{U}_p \cong B_r(p)$, for some $r > 0$ $-^{-1}$ and $\times$ are smooth maps

Section 20

Groups: Finite Groups

Finite Group

Types:	A group G
Functions:	
Relations:	
Properties:	$\lvert G \rvert = n, n \in \mathbb{N}$

Fintie Symmteric Group

Types:	A finite group S_n
Functions:	
Relations:	
Properties:	$S_n = \{\sigma \vert \sigma : X \to X$ is bijective $\}$

Permutation Group

Types:	A group P_n
Functions:	
Relations:	
Properties:	$P_n \leq S_p$

Primitive Group

Types:	A permutation group G that acts on a set X
Functions:	
Relations:	
Properties:	G preserves only trivial partitions

Alternating Group

Types:	A permutation group A_n
Functions:	
Relations:	
Properties:	$A_n = \{\sigma \vert \sigma \in S_n \wedge \sigma$ even $\}$

Section 21

Groups: Free Groups

Free Group

Types:	A group F
Functions:	
Relations:	
Properties:	F is generated by a set S
	There are no relations on F_S (beyond group axioms)

Free Abelian Group

Types:	A group F
Functions:	
Relations:	
Properties:	F is generated by a set S
	The only relation on F is commutativity

Section 22

Groups: Abelian Groups

Abelian Group

Types:	A group G
Functions:	
Relations:	
Properties:	$\cdot$ is commutative

Section 23

Groups: Group Presentations

Finitely Generated Group

Types:	A group G
Functions:	
Relations:	
Properties:	G is of the form $\langle S \vert R \rangle$, S a generating set, R relators
	$\vert S \vert = n, n \in \mathbb{N}$

Finitely Presented Group

Types:	A finitely generated group G
Functions:	
Relations:	
Properties:	$\vert R \vert = n, n \in \mathbb{N}$

Section 24

Groups: Cyclic Groups

Cyclic Group

Types:	A group G with element a
Functions:	
Relations:	
Properties:	G can be generated from a and a set of relations R G is necessarily commutative

Virtually Cyclic Group

Types:	A group V with subgroup H
Functions:	
Relations:	
Properties:	$\exists H \leq V : H$ is cyclic $\lvert V : H\rvert = n, n \in \mathbb{N}$

Locally Cyclic Group

Types:	A group L
Functions:	
Relations:	
Properties:	$\forall H \leq L$, if H is finitely generated, it is cyclic

Cyclically Ordered Group

Types:	A group C
Functions:	
Relations:	$[\ ,\ ,\ ] = \{(a,b,c) \in C \times C \times C \vert [a,b,c]\}$
Properties:	$[a,b,c]$ is cyclic, asymmetric, transitive and total

Polycyclic Group

Types:	A group P with subgroups H_i
Functions:	
Relations:	
Properties:	P is necessarily finitely presented $\{H_i\}_{i \in \{1,2,..,n-1\}} : H_i/H_{i+1}$ is cyclic

Metacyclic Group	
Types:	A group G
Functions:	
Relations:	
Properties:	$n \leq 2$

Section 25

Modules: Basic

Module	
Types:	A set M
Functions:	$+ : M^2 \to M$
	$-_{+}^{-1} : M \to M$
	$R \times M \to R$
Relations:	
Properties:	$+$ is associative and commutative
	$(M, +)$ is an abelian group
	$R \times M$ is scalar multiplication by elements in a ring
	Scalar multiplication is associative and distributes over addition

Section 26

Modules: Not further categorized

Semimodule	
Types:	A set M
Functions:	$+ : M^2 \to M$
	$R \times M \to M$
Relations:	
Properties:	$+$ is associative and commutative
	$(M, +)$ is an commutative monoid
	$R \times M$ is scalar multiplication by elements in a ring
	Scalar multiplication is associative and distributes over addition

Simple Module

Types:	A module M
Functions:	
Relations:	
Properties:	The only submodules of M are 0 and M

D Module

Types:	A module $M[X]$
Functions:	$\partial_{R_i} : M \to M$
Relations:	
Properties:	$X = \{x_1, x_2, ..., x_n\}$ where x_i is an indeterminant ∂_{R_i} is in the ring of scalar multiplication ∂_{R_i} satisfies the Leibniz product rule

Section 27

Modules: Chains

Noetherian Module

Types:	A module M
Functions:	
Relations:	
Properties:	For any chain of submodules $S_0 \subseteq S_1 \subseteq ... \subseteq S_k$ there is some $k \geq 0$ s.t. $S_k = S_{k+1}$ All submodules of M are finitely generated

Artinian Module

Types:	A module M
Functions:	
Relations:	
Properties:	For any chain of submodules $S_0 \supseteq S_1 \supseteq ... \supseteq S_k$ there is some $k \geq 0$ s.t. $S_k = S_{k+1}$

Section 28

Modules: Finitely Generated

Finitely Generated Module

Types:	A module M
Functions:	
Relations:	
Properties:	M has a finite number of generators

Cyclic Module

Types:	A finitely generated module M
Functions:	
Relations:	
Properties:	M is generated from a single element

Section 29

Modules: Toward Vector Space

Torsion Free Module

Types:	A module M
Functions:	
Relations:	
Properties:	$\nexists m \in M, r, m \neq 0 : r \cdot m = 0$

Flat Module

Types:	A torsion Free module M
Functions:	
Relations:	
Properties:	Taking the tensor product over M preserves exact sequences

Projective Module

Types:	A flat module M
Functions:	
Relations:	
Properties:	

Free Module

Types:	A projective module M
Functions:	
Relations:	
Properties:	M has no further relations beyond module axioms
	M has a basis

Fintie DimensionalVector Space

Types:	A set V over a field K
Functions:	$+ : V^2 \to V$
	$-_{+}^{-1} : V \to V$
	$K \times V \to V$
	$0 \in V$
Relations:	
Properties:	$+$ is associative and commutative
	$K \times V$ is scalar multiplication by K
	$(V, +)$ is a group

Section 30

Posets and Lattices: Basics

Set

Types:	a collection of objects
Functions:	$\subset, \cap, \cup, \setminus, \times, \wp, \shortparallel$
Relations:	$\in$
Properties:	

Preordered Set

Types:	A set P
Functions:	
Relations:	$\leq = \{(x, y) \in P \times P \vert x \leq y\}$
Properties:	All elements need not be comparable under $\leq$
	$\leq$ transitive and reflexive

Partially Ordered Set

Types:	A set P
Functions:	
Relations:	$\leq = \{(x,y) \in P \times P \| x \leq y\}$
Properties:	All elements need not be comparable under $\leq$
	$\leq$ transitive, reflexive and antisymmteric

Section 31

Posets and Lattices: Posets not Further Catagorized

Partially Ordered Space

Types:	A poset X endowed with topology τ
Functions:	
Relations:	$\leq = \{(x,y) \in X \times X \| x \leq y\}$
Properties:	$\forall x, y \in X, x \nleq y : \exists \mathcal{U}, \mathcal{V} \subset \tau, x \in \mathcal{U}, y \in \mathcal{V} : u \nleq v, \forall u \in \mathcal{U}, \forall v \in \mathcal{V}$

Locally Finite Poset

Types:	A poset P
Functions:	
Relations:	
Properties:	$\forall x, y \in P, x \leq y : [x, y]$ has finitely many elements

Partially Ordered group

Types:	A poset and group G
Functions:	$+ : G^2 \to G$
	$-_{+}^{-1} : G \to G$
	$0 \in G$
Relations:	$\leq = \{(x,y) \in G \times G \mid 0 \leq -x + y\}$
Properties:	$\leq$ respects $+$

Section 32

Posets and Lattices: Strict/Total Posets

Strict Poset

Types:	A poset P
Functions:	
Relations:	$<= \{(x,y) \in P^2 \mid x < y\}$
Properties:	$<$ is irreflexive, transitive, and asymmetric

Totally Ordered Set

Types:	A poset P
Functions:	
Relations:	
Properties:	All elements are comparable under $\leq$

Strict Totally Ordered Set

Types:	A poset P
Functions:	
Relations:	$<= \{(x,y) \in P \times P \mid x < y\}$
Properties:	$<$ is irreflexive, transitive, asymmetric and trichotomous

Section 33

Posets and Lattices: Graded Stuff

Graded Poset

Types:	A poset P
Functions:	$\rho : P \to \mathbb{N}$
Relations:	$\lessdot = \{(x,y) \in P \mid \nexists z : x < z < y$
Properties:	$x < y \implies \rho(x) < \rho(y)$
	$x \lessdot y \implies \rho(y) = \rho(x) + 1$

Eulerian Poset

Types:	
Functions:	
Relations:	
Properties:	

Section 34

Posets and Lattices: Linked + Related

Upward (Downward) Linked Set

Types:	A subset S of poset P
Functions:	
Relations:	
Properties:	$\forall x, y \in S : \exists z \in P$ s.t. $x \le (\ge) z$ and $y \le (\ge) z$

Upwards (Downwards) Centered Set

Types:	A linked subset S of poset P
Functions:	
Relations:	
Properties:	$\forall Z \subseteq P, Z$ has an upper (lower) bound $\in P$

Upward (Downward) Directed Set

Types:	A preset P
Functions:	
Relations:	
Properties:	$\le$ is a preorder $\forall x, y \in P : \exists z$ s.t. $x \le (\ge) z$ and $y \le (\ge) z$

Algebraic Poset

Types:	A poset P
Functions:	
Relations:	
Properties:	each element is the least upper bound of the compact elements below it

Section 35

Posets and Lattices: Semilattices

Meet Semilatttice: Order Theoretic Edition

Types:	A poset P
Functions:	
Relations:	$\leq = \{(x,y) \in P \times P \mid x \leq y\}$
Properties:	$\forall x, y \in S, \exists c : c \leq x, c \leq y$
	c is the unique greatest lower bound

Meet Semilattice: Algebraic Edition

Types:	A poset P
Functions:	$\wedge : P^2 \to P$
Relations:	
Properties:	$\wedge$ is associative, commutative, and idempotent

Join Semilatttice: Order Theoretic Edition

Types:	A poset P
Functions:	
Relations:	$\leq = \{(x,y) \in P \times P \mid x \leq y\}$
Properties:	$\forall x, y \in P, \exists c : x \leq c, y \leq c$
	c is the unique least upper bound

Join Semilattice: Algebraic Edition

Types:	A poset P
Functions:	$\vee : P^2 \to P$
Relations:	
Properties:	$\vee$ is associative, commutative, and idempotent

Section 36

Posets and Lattices: Lattices

Lattice (order theory)

Types:	A set L
Functions:	
Relations:	$\leq = \{(x,y) \in L \times \mid x \leq y\}$
Properties:	$\forall x, y \in L, \exists c, d : c$ is the greatest lower bound and d is the least upper bound

Lattice (Algebra)

Types:	A set L
Functions:	$\vee : L^2 \to L$
	$\wedge : L^2 \to L$
Relations:	
Properties:	$(L, \vee), (L, \wedge)$ are both semilattices connected by absorption laws
	$\forall a, b \in L : a \vee (a \wedge b) = a, a \wedge (a \vee b) = a$

Complete Lattice

Types:	A lattice L
Functions:	
Relations:	
Properties:	$\forall A \subseteq L, A$ has a greatest lower bound and least upper bound

Continuous Lattice

Types:	A complete lattice A
Functions:	
Relations:	$\ll = \{(x, y) \in L \times L \mid \forall D \subseteq L : y \leq \sup D, \exists x \in L, \exists d \in D \text{ s.t. } x \leq d\}$
Properties:	$\forall x \in P, \{a \mid a \ll x\}$ is directed and has least upper bound x

Algebraic Lattice

Types:	A continuous lattice A
Functions:	
Relations:	
Properties:	Each element is the least of compact elements below it: those x who satisfy $x \ll a$

Section 37
Posets and Lattices: A Big Web of Lattices

Relatively Complemented Lattice

Types:	A lattice L
Functions:	
Relations:	
Properties:	$\forall c \forall d \geq c, [c,d] : \forall a \in [c,d], \exists b$ s.t.:
	$a \vee b = d$
	$a \wedge b = c$
	a and b are relative complements

Bounded Lattice

Types:	A lattice L
Functions:	
Relations:	
Properties:	$\forall x \in L :$
	$x \wedge 1 = x$
	$x \vee 1 = 1$
	$x \vee 0 = x$
	$x \wedge 0 = 0$

Complemented Lattice

Types:	A lattice L
Functions:	
Relations:	
Properties:	$\forall a \in L, \exists b \in L :$
	$a \vee b = 1$
	$a \wedge b = 0$

Residuated Lattice

Types:	A lattice L
Functions:	$\cdot : L^2 \to L$
	$1 \in L$
Relations:	
Properties:	$(L, \cdot, 1)$ is a monoid
	$(L, \leq)$ is a lattice
	$\forall z : \forall x$, there exists a greatest y and $\forall y$, there exists a greatest x s.t. $x \cdot y \leq z$

Atomic Lattice

Types:	A lattice L
Functions:	
Relations:	$\lessdot = \{(x, y) \in P \mid \nexists z : x < z < y\}$
Properties:	$\forall b \in L : 0 \wedge b = 0$
	$\exists a_i \in L : 0 \lessdot a_i$
	$\forall b \neq a_i, \exists a_i : 0 < a_i < b$
	$\{a_i\}_{i \in I}$ is the set of atoms in L

Semimodular Lattice

Types:	A semimodular lattice L
Functions:	
Relations:	
Properties:	$a \leq c \implies a \vee (b \wedge c) = (a \vee b) \wedge c$

Modular Lattice

Types:	A semimodular lattice L
Functions:	
Relations:	
Properties:	$a \leq c \implies a \vee (b \wedge c) = (a \vee b) \wedge c$

Distributive Lattice

Types:	A modular lattice L
Functions:	
Relations:	
Properties:	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
	$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Orthomodular Lattice

Types:	A lattice L
Functions:	
Relations:	
Properties:	$a \leq b \implies b = a \vee (b \wedge a^{\perp})$

Orthocomplemented Lattice

Types:	A lattice L
Functions:	$\perp: L \to L$
Relations:	
Properties:	$\perp$ maps an element a to its complement
	$a \vee a^{\perp} = 1$
	$a \wedge a^{\perp} = 0$
	$(a^{\perp})^{\perp} = a$
	$a \leq b \implies a^{\perp} \geq b^{\perp}$

Geometric Lattice

Types:	A semimodular, atomic, algebraic Lattice L
Functions:	
Relations:	
Properties:	L is finite

Heyting Algebra

Types:	A distributive lattice H
Functions:	$\longrightarrow: H^2 \to H$
Relations:	
Properties:	$(c \wedge a) \leq b \Leftrightarrow c \leq (a \longrightarrow b)$
	Monoid $(H, \cdot, 1)$ has operation $\cdot = \wedge$

Complete Heyting Algebra

Types:	A heyting algebra H
Functions:	
Relations:	
Properties:	H is complete as a lattice

Boolean Algebra

Types:	A complemented, distributive lattice and algebra B
Functions:	$\neg : B \to B$
	$\oplus : B^2 \to B$
Relations:	
Properties:	$\forall x \in B, x^2 = x$
	$\neg$ is negation (complementation)
	$x \oplus y = (x \vee y) \wedge \neg(x \wedge y)$
	$\oplus$ is algebra addition, $\wedge$ is algebra multiplication

Section 38

Rings: Basics

Ring

Types:	A set R
Functions:	$\cdot : R^2 \to R$
	$+ : R^2 \to R$
	$-_{+}^{-1} : R \to R$
	$0 \in R$
Relations:	
Properties:	$(R, +)$ is an abelian group
	$\cdot$ is a monoid and distributes over $+$

Commutative Ring

Types:	A ring R
Functions:	
Relations:	
Properties:	$\cdot$ is commutative

Noncommutative Ring

Types:	A ring R
Functions:	
Relations:	
Properties:	$\cdot$ need not be commutative

Section 39

Rings: Ideals + Related

Left (right) Ideal

Types:	A subgroup of a ring, I
Functions:	
Relations:	
Properties:	$\forall x \in I, \forall r \in R : rx \ (xr) \in I$

Ideal

Types:	A subgroup of a ring, I
Functions:	
Relations:	
Properties:	The left and right ideals generated by a subgroup are equal $\forall x \in I, \forall r \in R : rx \in I$

Maximal ideal

Types:	A subgroup of elements, I
Functions:	
Relations:	
Properties:	$I \subseteq J \implies I = J$ or $J = R$

Principal ideal

Types:	A subgroup of elements, I
Functions:	
Relations:	
Properties:	I is generated by a single element

Prime Ideal

Types:	A subgroup of elements I
Functions:	
Relations:	
Properties:	$ab \in I \implies a \in I$ or $b \in I$

Quotient Ring

Types:	A ring Q
Functions:	
Relations:	$\sim = \{(x, y\} \in R \times R \mid x - y \in I\}$
Properties:	Q is constructed by dividing R with one of its ideals

Section 40

Rings: Booleans + Related

Boolean Ring

Types:	A commutative ring R
Functions:	
Relations:	
Properties:	$\forall x \in R : x^2 = x$
	$\forall x \in R : 2x = 0$

Boolean Algebra

Types:	A complemented, distributive lattice and algebra B
Functions:	$\neg : B \to B$
	$\oplus : B^2 \to B$
Relations:	
Properties:	$\forall x \in B, x^2 = x$
	$\neg$ is negation (complementation)
	$x \oplus y = (x \vee y) \wedge \neg(x \wedge y)$
	$\oplus$ is algebra addition, $\wedge$ is algebra multiplication

Sigma Algebra

Types:	A subset Σ, of $\wp(X)$, X a set
Functions:	$\cap, \cup, \setminus$
Relations:	
Properties:	$X \in \Sigma$
	$\cap, \cup$ closed under countable operations

Section 41

Rings: Randoms/Not further catagorized

Semiring

Types:	A set R
Functions:	$\cdot : R^2 \to R$
	$+R^2 \to R$
	$0 \in R$
Relations:	
Properties:	$+$ is a commutative monoid
	$\cdot$ is a monoid and distributes over $+$

Noetherian Ring

Types:	A ring R
Functions:	
Relations:	
Properties:	For any $I_0 \subseteq I_1 \subseteq ... \subseteq I_k$ there is some $k \geq 0$ s.t. $I_k = I_{k+1}$

Artinian Ring

Types:	A ring R
Functions:	
Relations:	
Properties:	For any $I_0 \supseteq I_1 \supseteq ... \supseteq I_k$ there is some $k \geq 0$ s.t. $I_k = I_{k+1}$

Topological Ring

Types:	A ring R with topology τ
Functions:	$\cdot : R^2 \to R : (x,y) \mapsto x \cdot y$
	$+ : R^2 \to R : (x,y) \mapsto x + y$
	$-_+^{-1} : R \to R$
Relations:	
Properties:	$\cdot$ distributes over $+$
	$+, \cdot, -_+^{-1}$ are continuous mappings

Section 42

Rings: Valuation bs

Domain

Types:	A ring R
Functions:	
Relations:	
Properties:	$\forall a, b \in R, ab = 0 \implies a = 0$ or $b = 0$

Integral Domain

Types:	A ring R
Functions:	
Relations:	
Properties:	$\cdot$ is additionally commutative

Unique Factorization Domain

Types:	An integral domain R
Functions:	
Relations:	
Properties:	All ideals are finitely generated
	All irreducible elements are prime
	$\forall x \in R, x \neq 0, x = up_1p_2...p_n$ for u unit, p_i prime

Principal Ideal Domain

Types:	A unique factorization domain R
Functions:	
Relations:	
Properties:	All ideals are principal
	Any two elements have a greatet common divisor

Euclidean Domain

Types:	A principal idea domain R
Functions:	$f_i : R \setminus \{0\} \to R$
Relations:	
Properties:	f_i is the euclidean (gcf) algorithim
	R may have many f_i or just one

Section 43

Rings: Budget versions of fields

Field

Types:	A ring K
Functions:	$+ : K^2 \to K$
	$\times K^2 \to K$
	$-_{+}^{-1} : K \to K$
	$-_{\times}^{-1} : K/\{0\} \to K/\{0\}$
	$0, 1 \in K$
Relations:	
Properties:	$+, \times$ both associate and commute
	$\times$ distributes over $+$

Module

Types:	An abelian group $(M, +)$
Functions:	$\cdot : r \times M \to M$
	$\cdot : M \times r \to M$
Relations:	
Properties:	$\cdot$ is scalar multiplication by elements in a ring, R
	$\cdot$ associates and distributes over addition

Vector Space

Types:	A set V over a field K
Functions:	$+ : V^2 \to V$
	$-_{+}^{-1} : V \to V$
	$K \times V \to V$
	$0 \in V$
Relations:	
Properties:	$+$ is associative and commutative
	$K \times V$ is scalar multiplication by K
	$(V, +)$ is a group

Finite Field

Types:	A characteristic nonzero field K
Functions:	
Relations:	
Properties:	K has a finite number of elements
	The order of some $k \in K$ is p^n for some p prime and $n \in \mathbb{N}$

Section 44

Sets: Basics

Set	
Types:	a collection of objects
Functions:	$\subset, \cap, \cup, \setminus, \times, \wp, \vert\,\vert$
Relations:	$\in$
Properties:	

Section 45

Sets: Countable

Countable Set	
Types:	A set S
Functions:	
Relations:	
Properties:	$\vert S\vert \leq \aleph_0$

Infinite Countable Set	
Types:	A set S
Functions:	
Relations:	
Properties:	$\vert S\vert = \aleph_0$ bijection into $\mathbb{N}$

Finite Countable Set	
Types:	A set S
Functions:	
Relations:	
Properties:	$\vert S\vert < \aleph_0$

Section 46

Sets: Uncountable

Uncountable Set

Types:	A set S
Functions:	
Relations:	
Properties:	$\lvert S\rvert > \aleph_0$

Section 47

Sets: Classes

Proper Class

Types:	a collection, C of sets
Functions:	$\varphi(x)$
Relations:	
Properties:	$\varphi(x)$ is a membership function
	C is not a set

Ordinal

Types:	A collection of sets, X_i
Functions:	$\times, +, X^n, S(n)$
Relations:	$>$
Properties:	$X_k = \{0, 1, 2, ...X_{k-1}\}$
	$>$ is trichotomous, transitive, and wellfounded

Cardinal

Types:	Ordinals κ
Functions:	
Relations:	
Properties:	The cardinality κ is the least ordinal, α, s.t. there is a bijection between S and α

Section 48

Topologies: Basics

Convergent Space

Types:	A set S and filters on $S, \mathcal{F}$
Functions:	
Relations:	$\to$
Properties:	$\forall F \in \mathcal{F}, x \in S, F \to x \Leftrightarrow F$ convergences to X
	$\to$ is centered, isotone, and directed
	$\mathcal{N}(x) = \bigcap\{F \in \mathcal{F} \mid F \to x\}$

Topological Space

Types:	A set X and $\tau \subseteq \wp(X)$
Functions:	
Relations:	
Properties:	$X, \varnothing \in \tau$
	$x_i \in \tau \implies \bigcap_{i \in I} x_i \in \tau$ for finite I
	$x_i \in \tau \implies \bigcup_{j \in J} x_j \in \tau$

Section 49

Topologies: Algebraic Topologies

Topological Group

Types:	A group G with topology τ
Functions:	$\cdot : G^2 \to G : (x, y) \mapsto x \cdot y$
	$-^{-1} : G \to G$
	$1 \in G$
Relations:	
Properties:	$\cdot, -^{-1}$ are continuous mappings

Topological Ring

Types:	A ring R with topology τ
Functions:	$\cdot : R^2 \to R : (x, y) \mapsto x \cdot y$
	$+ : R^2 \to R : (x, y) \mapsto x + y$
	$-_{+}^{-1} : R \to R$
Relations:	
Properties:	$\cdot$ distributes over $+$
	$+, \cdot, -_{+}^{-1}$ are continuous mappings

Topological Field

Types:	A topological ring K
Functions:	A field K with topology τ
Relations:	$\cdot : K^2 \to K : (x, y) \mapsto x \cdot y$
	$+ : K^2 \to K : (x, y) \mapsto x + y$
	$-_{+}^{-1} : K \to R$
	$-_{\times}^{-1} : K/\{0\} \to K/\{0\}$
Properties:	

$+, \cdot, -_{+}^{-1}, -_{\cdot}^{-1}$ are continuous mappings

Topological Vector Space

Types:	A vector space V over a topological field K
Functions:	$+ : V^2 \to V$
	$k \times V \to V$
	$-_{+}^{-1} : V \to V$
Relations:	
Properties:	$k \times V$ represents scalar multiplication by elements in a field K
	$+, -_{+}^{-1}$ and scalar multiplication are continuous

Section 50

Topologies: Web I

Uniform Space

Types:	A topological space U and entourages Φ
Functions:	
Relations:	$U = \{(x,y)\mid x \approx_u y\}$
Properties:	Φ is a nonempty collection of relations on S
	$U \in \Phi$ and $U \subseteq V \subseteq X \times X \implies V \in \Phi$
	$\forall U \in \Phi, \exists V \in \Phi, \forall x,y,z : V \circ V \subseteq \Phi : x \approx_V y, y \approx_V z \implies x \approx_U z$
	$\forall U \in \Phi, \exists V \in \Phi, \forall x,y : V \circ V \subseteq \Phi : y \approx_V x \implies x \approx_U y$
	$U,V \in \Phi \implies U \cap V \in \Phi$

Complete Uniform Space

Types:	A uniform Space U
Functions:	
Relations:	
Properties:	$\forall F \forall U : \exists A \in F : A \times A \subseteq U, F$ converges

Manifold

Types:	A topological space, M
Functions:	
Relations:	
Properties:	$\forall p \in M, \exists u \in \mathcal{U}(p) : u \cong \mathbb{R}^n$
	Manifolds are not inherently metric or inner product spaces

Compact Manifold

Types:	A manifold M
Functions:	
Relations:	
Properties:	$X = \bigcup_{x \in C} x$, then for $F \subseteq C, F$ finite, $X = \bigcup_{x \in F} x$

Smooth Manifold

Types:	A manifold M
Functions:	
Relations:	
Properties:	Derivatives of arbitrary orders exist

Riemann Manifold

Types:	A smooth manifold M
Functions:	$g_p : T_pM^2 \to \mathbb{R}$
Relations:	
Properties:	g_p is the inner product on the tangent space of a point p
	$g_p(X,Y) = g_p(Y,X)$
	$g_p(aX+Y,Z) = ag_p(X,Z) + g_p(Y,Z)$

Pseudo Riemann Manifold

Types:	A smooth manifold M
Functions:	$g_p : T_pM^2 \to \mathbb{R}$
Relations:	
Properties:	g_p is the inner product on the tangent space of a point p
	$g_p(X,Y) = 0\ \forall Y \implies X = 0$

Affine Space

Types:	A set A and vector space $\overrightarrow{A}$
Functions:	$A \times \overrightarrow{A} \to A : (a,v) \mapsto a+v$
Relations:	
Properties:	$\overrightarrow{A}$'s additive group acts freely and transitively on A
	$\forall v,w \in \overrightarrow{A}, \forall a \in A, (a+w)+u = a+(w+u)$
	$\forall a,b \in A, \exists v \in \overrightarrow{R} : b = a+v$
	$\forall a \in A, \overrightarrow{A} \to A : v \mapsto a+v$ is bijective

Section 51

Topologies: Web II

Metric Space

Types:	A topological space M
Functions:	$d : M^2 \to \mathbb{R}$
Relations:	
Properties:	$d(x,y) \geq 0$
	$d(x,y) = 0 \Leftrightarrow x = y$
	$d(x,y) = d(y,x)$
	$d(x,z) \leq d(x,y) + d(y,z)$

Complete Metric Space

Types:	A metric space M
Functions:	
Relations:	
Properties:	$\forall F \forall U : \exists A \in F : A \times A \subseteq U, F$ converges

Normed Vector Space

Types:	A metric and topological vector space V
Functions:	$\Vert\ \Vert : V \to \mathbb{R}$
Relations:	
Properties:	$\Vert x \Vert > 0$ for $x \neq 0$ and $\Vert x \Vert = 0$ iff $x = 0$
	$\Vert \alpha x \Vert = \vert\alpha\vert \Vert x \Vert$
	$\Vert x + y \Vert \leq \Vert x \Vert + \Vert y \Vert$
	$\Vert\ \Vert$ induces a metric on $V : d(x, y) = \Vert x - y \Vert$

Banach Space

Types:	A vector space V
Functions:	
Relations:	
Properties:	for any Cauchy Sequence $\{x_n\} lim_{x\to\infty} \Vert x_n - x \Vert = 0$

Inner Product Space

Types:	A vector space V and field of scalars F
Functions:	$\langle \cdot, \cdot \rangle : V^2 \to F$
Relations:	
Properties:	$\langle x, y \rangle = \overline{\langle y, x \rangle}$
	$\langle x, x \rangle \geq 0$
	$\langle x, x \rangle = 0$ iff $x = 0$
	$\langle ax, y \rangle = a \langle x, y \rangle$
	$\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$

Hilbert Space

Types:	An inner product space H
Functions:	
Relations:	
Properties:	A norm may be defined as $\Vert x \Vert = \sqrt{\langle x, x \rangle}$
	for any Cauchy Squence $\{x_n\}, lim_{x\to\infty} \Vert x_n - x \Vert = 0$

Euclidean Space

Types:	A banach and hilbert space $\mathbb{R}^n$
Functions:	
Relations:	
Properties:	A norm is defined as $\|x\| = \sqrt{\langle x, x\rangle} = \sqrt{\sum_{i=1}^{n}(x_i)^2}$ $d(x,y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$