WITH APOLOGIES TO CASIMIR

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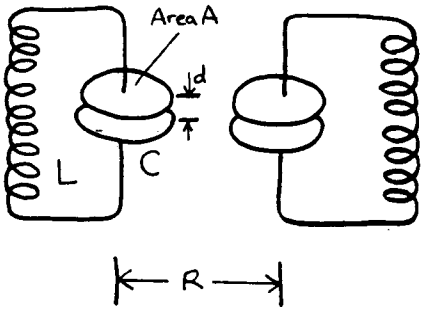
The Casimir effect, in case you have forgotten, is an attraction between conducting planes that arises from the vacuum energy in the intervening space. The force is too weak to command great attention—in fact, it is barely detectable—but Hendrik Casimir's line of reasoning has nevertheless been extraordinarily productive. By encouraging a rather literal view of vacuum fluctuations, his approach unifies and simplifies phenomena such as the van der Waals force, resonance dispersion and the effects quantum electrodynamics produces in a cavity, such as inhibited spontaneous emission and vacuum Rabi oscillations. On the cosmic scale of things these might seem like small potatoes, but Casimir's thinking also underlies the most plausible explanation yet for Creation—namely, generation of the universe from vacuum fluctuations.

Casimir's papers on vacuum forces and the van der Waals interaction[1] are not easy reading because the mode structure of an extended system is a trifle complicated mathematically. However, the physically important results practically pop out if one limits the system to two modes. So, here—with apologies to Casimir for doing crudely today what he did elegantly more than 40 years ago—is a matchbook calculation of the van der Waals force and some of its cousins.

Consider a system composed of two harmonic oscillators—identical LC circuits will do nicely. The energy of each oscillator is given by $(n + \frac{1}{2})\hbar\omega_0$, where n is its occupation number and $\omega_0 = 1/\sqrt{LC}$. The oscillators are in their ground states, located so far apart that any interaction is negligible. Their energy is $E_0 = \frac{1}{2}\hbar\omega_0 + \frac{1}{2}\hbar\omega_0$. Suppose next that the oscillators are arranged as shown in the figure below, separated by some distance R and interacting via their electric dipole fields. The degeneracy



of the two modes is split, and the natural frequencies become $\omega_\pm = \omega_0\sqrt{1 \pm \kappa}$, where a simple calculation shows that the coupling coefficient κ has the value $Ad/4\pi R^3$. (Out of respect for Casimir, who does not much care for the SI system, I am using Gaussian units.) The system's energy is now $E' = \frac{1}{2}\hbar\omega_- + \frac{1}{2}\hbar\omega_-$. The interaction energy is

$$\Delta E = E' - E_0$$
$$= \frac{1}{2}\hbar\omega_0(\sqrt{1+\kappa} - 1)$$
$$+ \frac{1}{2}\hbar\omega_0(\sqrt{1-\kappa} - 1) \qquad (1)$$

Evaluating this result to lowest order in κ yields

$$\Delta E = -\frac{\hbar\omega_0}{8}\kappa^2 = -\frac{\hbar\omega_0}{8}\frac{\alpha^2}{R^6} \qquad (2)$$

The symbol α, whose value equals the polarizability of the capacitor, is introduced to emphasize the similarity of the LC circuit to an atomic system. Equation 2 has the same form as the usual expression for the van der Waals interaction of two atoms.

The van der Waals interaction is generally described in terms of a correlation between the instantaneous dipoles of two atoms or molecules. However, it is evident that one can just as easily portray it as the result of a change in vacuum energy due to an alteration in the mode structure of the system. The two descriptions, though they appear to have nothing in common, are both correct. Incidentally, although we considered identical oscillators, the result also holds for nonidentical oscillators, with suitable modifications of the constants.

So what have we learned? First, that one must take the vacuum seriously; second, that intermolecular forces and a number of slightly bizarre radiative phenomena, and possibly even Creation, have something in common; and third, that Casimir is pretty smart.

Reference

1. H. B. G. Casimir, D. Polder, Phys. Rev. **73**, 360 (1948). H. B. G. Casimir, J. Chim. Phys. **47**, 407 (1949). ■

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