Minimizing the Age of Information in Wireless Networks with Stochastic Arrivals

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Outline

• Age of Information

• Network Model

• Scheduling Policies and Performance Guarantees
  • Stationary Randomized Policy
  • Max-Weight Policy

• Numerical Results

• Final Remarks & Current Work: from theory to practice (short video)
Age of Information

Delivery of packets to the destination
Packet generation at the source

Source
Single packet queue
≡ LIFO queue
Channel
Destination
Age of Information

Delivery of packets to the destination

Packet generation at the source

Source

LIFO queue

Channel

Destination

What is the system time of the HoL packet at the queue?

How old is the information at the destination?
Age of Information

Delivery of packets to the destination
Packet generation at the source

Source → LIFO queue → Channel → Destination

$z(t)$

$\tau(t)$
Age of Information

Delivery of packets to the destination
Packet generation at the source

Source
LIFO queue
Channel
Destination

$z(t)$
$h(t)$
Age of Information

Delivery of packets to the destination
Packet generation at the source

Source
LIFO queue
Channel
Destination

\[ I[1] \rightarrow \text{Interdelivery Time} \]

\[ z[1] \rightarrow \text{Delay of the 1st and 2nd packets} \]

\[ z(t) \]

\[ h(t) \]

\[ z[1] \]

\[ z[2] \]
Network Model

Packet arrivals from each stream

Unreliable transmissions on a shared medium

Scheduling Policy at the Base Station attempts to minimize the average AoI in the network.
## Literature

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<tr>
<th>Papers</th>
<th>Packet Arrivals</th>
<th>Channel Reliability</th>
<th>Queueing Discipline</th>
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*Note: The table entries marked with an 'X' indicate the presence of a particular characteristic.*
Network Model - Two Streams Example

Scheduler  BS

\[ z_1(t) \quad z_2(t) \]

\[ h_1(t) \quad h_2(t) \]

- Active Source
- Reliable Channel

Decision time
Network Model - Two Streams Example

Scheduler

\[ z_1(t) \quad h_1(t) \quad h_2(t) \quad z_2(t) \]

\[ \lambda_1 \quad \lambda_2 \]

- Packet Arrivals
- Reliable Channel

Decision time

Scheduler

1 2

\[ h_1(t) \quad z_1(t) \]

\[ h_2(t) \quad z_2(t) \]

time

- Packet Arrivals
- Reliable Channel

Decision time
Network Model - Two Streams Example

Scheduler BS

\( \lambda_1 \) \( \lambda_2 \)

\( z_1(t) \) \( z_2(t) \)

\( h_1(t) \) \( h_2(t) \)

\( p_1 \) \( p_2 \)

1 \( p_1 \) 2 \( p_2 \)

- Packet Arrivals
- Unreliable Channel

\( p_1 \) is good
\( p_2 \) is bad

Decision time
Network Model

Bernoulli packet arrivals with rate, $\lambda_i$

$\lambda_1, z_1(t)$ 1

$\lambda_2, z_2(t)$ 2

...$\lambda_N, z_N(t)$ N

Probability of a successful packet transmission, $p_i$

$1 \rightarrow p_1 \rightarrow h_1(t), w_1$

$2 \rightarrow p_2 \rightarrow h_2(t), w_2$

...$N \rightarrow p_N \rightarrow h_N(t), w_N$

Weights $w_i$ represent priority

Values of $N, \lambda_i, w_i, p_i$ are fixed and known. Values of $h_i(t)$ and $z_i(t)$ are known by the BS.
Network Model - Scheduling Policy \( \pi \)

During slot \( t \):

1) A **new packet arrives** to the queue of stream \( i \) w.p. \( \lambda_i, \forall i \in \{1, \ldots, N\} \) \[ a_i(t) = 1 \]

2) BS runs the transmission scheduling policy \( \pi \) and **selects a single stream** \( i \) \[ u_i(t) = 1 \]

3) HoL packet of stream \( i \) is successfully **delivered** to destination \( i \) w.p. \( p_i \) \[ d_i(t) = 1 \]

Class of non-anticipative policies \( \Pi \). **Arbitrary policy** \( \pi \in \Pi \).
Network Model - Objective Function

• Expected Weighted Sum AoI when policy $\pi$ is employed:

$$\mathbb{E}[J^\pi] = \lim_{T \to \infty} \frac{1}{TN} \sum_{t=1}^{T} \sum_{i=1}^{N} w_i \mathbb{E}[h_i^\pi(t)],$$

where $h_i^\pi(t)$ is the AoI of stream $i$ at the beginning of slot $t$ and $w_i$ is the positive weight.

• AoI-optimal policy achieves:

$$\mathbb{E}[J^*] = \min_{\pi \in \Pi} \mathbb{E}[J^\pi]$$
Stationary Randomized Policies

- **Policy R**: in each slot $t$, select stream $i$ with probability $\mu_i \in (0,1]$
- Sequence of transmission schedules from stream $i$ is a renewal process
Stationary Randomized Policies

- **Policy** $R$: in each slot $t$, select stream $i$ with probability $\mu_i \in (0,1]$
- Sequence of transmission schedules from stream $i$ is a renewal process
- Evolution of AoI is NOT STOCHASTICALLY RENEWED after every packet delivery
- Expression for time-average $\mathbb{E}[h_i^R(t)]$?
Stationary Randomized Policies

- **Policy** $R$: in each slot $t$, select stream $i$ with probability $\mu_i \in (0,1]$

- Under policy $R$, $(h_i(t), z_i(t))$ evolves as a 2-dim MC with countably-infinite state space
Stationary Randomized Policies

- **Policy R**: in each slot $t$, select stream $i$ with probability $\mu_i \in (0,1)$

- Under policy $\mathbf{R}$, $(h_i(t), z_i(t))$ evolves as a 2-dim MC with countably-infinite state space

- Analyzing this MC, we obtain:

\[
\mathbb{P}(h) = \lambda_i p_i \mu_i \left[ \sum_{n=0}^{h-1} (1 - \lambda_i)^{h-1-n} (1 - p_i \mu_i)^n \right]
\]

\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[h_i^R(t)] = \frac{1}{p_i \mu_i} + \frac{1}{\lambda_i} - 1
\]
Stationary Randomized Policies

• Network with parameters \((N, p_i, \lambda_i, w_i)\) and **LIFO queues**:

![Optimal Randomized policy for Single packet queues](image)

- **Theorem**: the optimal scheduling probabilities are: \(\mu_i \sim \sqrt{w_i / p_i}, \forall i\) and the performance of the optimal policy is such that:

\[
\mathbb{E}[J^*] \leq \mathbb{E}\left[J^{RS}\right] \leq 4 \mathbb{E}[J^*]
\]
Stationary Randomized Policies

• Network with parameters \((N, p_i, \lambda_i, w_i)\) and **FIFO queues**:

\[ E[J^R] = \min_{R \in \Pi_R} \left\{ \sum_{i=1}^{N} \frac{w_i}{N} \left[ \frac{1}{p_i \mu_i} + \frac{1}{\lambda_i} + \left[ \frac{\lambda_i}{p_i \mu_i} \right]^2 \frac{1 - p_i \mu_i}{p_i \mu_i - \lambda_i} \right] \right\} \]

s.t. \( \sum_{i=1}^{N} \mu_i \leq 1 \);

\( p_i \mu_i > \lambda_i, \forall i \).

• **Theorem**: the optimal scheduling probabilities are given by Algorithm 2 which uses the *bisection method* to find the set of \( \mu_i^* \). [details are omitted]
Max-Weight Policy for any queueing discipline

- Lyapunov Function: $L(t)$ high value at undesirable states

- Lyapunov Drift: $\Delta(t) = \mathbb{E}\{L(t + 1) - L(t)\}$

- Max-Weight policy attempts to minimize $\Delta(t)$ at every slot $t$
Max-Weight Policy for any queueing discipline

• Lyapunov Function: \[ L(t) = \frac{1}{N} \sum_{i=1}^{N} \beta_i h_i(t) \], where \( \beta_i > 0 \) is a constant

• Lyapunov Drift: \[ \Delta(t) = \mathbb{E}\{ L(t + 1) - L(t) | h_i(t), z_i(t) \} \]

• Substituting the expression for the evolution of \( h_i(t + 1) \) into the drift:

\[
\Delta(t) = -\frac{1}{N} \sum_{i=1}^{N} \beta_i p_i \left( h_i(t) - z_i(t) \right) \mathbb{E}[u_i(t)|h_i(t), z_i(t)] + \frac{1}{N} \sum_{i=1}^{N} \beta_i
\]

• **MW policy**: in slot t, schedule stream \( (u_i(t) = 1) \) with highest value of:

\[
\beta_i p_i \left( h_i(t) - z_i(t) \right)
\]
Max-Weight Policy for any queueing discipline

• **MW policy:** in slot $t$, schedule stream $(u_i(t) = 1)$ with highest value of:

$$\beta_i p_i (h_i(t) - z_i(t))$$

• For different queueing disciplines substitute the corresponding $z_i(t)$

• $p_i (h_i(t) - z_i(t))$ represents the expected AoI reduction from selecting $i$

• **Theorem:** consider a network employing LIFO queues. The performance of MW policy when $\beta_i = \sqrt{w_i/p_i}, \forall i$ is such that:

$$\mathbb{E} [J^{MW}] \leq \mathbb{E} [J^{RS}]$$
Numerical Results

- Metric:
  - Expected Weighted Sum AoI: $\mathbb{E}[ J^{\pi} ]$

- Network setup with $N = 4$ streams. Stream $i$ has:
  - channel reliability $p_i = i/N$
  - arrival rate $\lambda_i = \lambda \times (N + 1 - i)/N$
  - weights: $w_1 = w_2 = 4$ and $w_3 = w_4 = 1$

- Arrival rate in the range $\lambda \in \{0.01, 0.02, \ldots, 0.35\}$
- Each simulation runs for $T = 2 \times 10^6$ slots
- Each data point is an average over 10 simulations
Video of Testbed

Current work on the AoI scheduling problem.

Testbed implementation is not complete. There are a few missing parts/tests.

Video shows a short demo.

Network Setup:
- Two sources generating packets
  - Bernoulli arrivals
  - Single packet queue discipline
- Sources send packets to Base station according to Greedy Policy on $h_i(t)$
Final Remarks

In this presentation:

• Developed scheduling policies for wireless networks with stochastic arrivals, unreliable channels and different queueing disciplines

• Described performance guarantees

• Conclusion: Max-Weight with LIFO queues has superior performance.

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