Minimizing the Age of Information in Broadcast Wireless Networks

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Outline

• Age of Information (AoI) and Network Model
• Symmetric Network and the Greedy Policy
• General Network and the Index Policy
• Numerical Results
• Max Throughput vs Min AoI
Age of Information

Example: Single Source
Single Destination

Measuring the freshness of the information
**Age of Information**

**Interdelivery Time:** time elapsed between consecutive packet deliveries.

**Packet Delay:** time elapsed from generation to delivery of a packet.
Age of Information

**AoI**: time elapsed since the most recently delivered packet was generated.

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Age of Information

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Age of Information

**AoI**: time elapsed since the most recently delivered packet was generated.

At time $t$: $\text{AoI} = t - \tau(t)$

$\tau(t)$ is the time stamp of the most recently delivered packet.

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Aol, Delay and Interdelivery time

- **Example:** M/M/1 queue
  
  Controllable arrival rate $\lambda$ and fixed service rate $\mu = 1$ packet per second.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\mathbb{E}[delay]$</th>
<th>$\mathbb{E}[interdel.]$</th>
<th>Average Aol</th>
</tr>
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<tbody>
<tr>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.99</td>
<td></td>
<td></td>
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Source

$\lambda$

$\mu$

Server

Destination

$\mathbb{E}[delay]$ and $\mathbb{E}[interdel.]$ represent the expected delay and interdelivery time, respectively.
Aol, Delay and Interdelivery time

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</tr>
<tr>
<td>0.53</td>
<td>2.13</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>0.99</td>
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Source --- $\lambda$ --- Server ($\mu$) --- Destination
Aol, Delay and Interdelivery time

- Example: M/M/1 queue

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A low Aol is achieved when packets with low delay are delivered regularly.

Minimum throughput requirement DOES NOT guarantee regular deliveries.

Network Model
Network Model

Packet generation at the BS

Timeline:

Generation

Frame 1

1 ... T

Generation

Frame 2

1 ...

New packets replace undelivered packets from the previous frame.
Network Model

**Packet generation at the BS**

- $S_1$
- $S_2$
- $S_M$

**Packet transmissions**

- $p_1$
- $p_2$
- $p_M$

**Timeline:**

- Generation
- Frame 1
- Generation
- Frame 2

**Packet transmission:**

- In a slot, a packet is transmitted to a selected client $i$.
- $p_i$ is the prob. of a successful transm.
- Instantaneous feedback
Network Model

- **Goal**: design a scheduling policy that provides fresh information to the clients.

- **Objective function** is the Expected Weighted Sum AoI:

\[
\text{EWSAoI} = \frac{1}{KT} \mathbb{E} \left\{ \sum_{i=1}^{M} \alpha_i \text{AoI}_i \right\}
\]

where $\alpha_i$ is the client’s weight and $\text{AoI}_i$ is the area under the AoI curve for client $i$. 
Network Model

- **Goal**: design a scheduling policy that provides fresh information to the clients.

- **Equivalent Objective function**:

  \[ J_K^{\pi^*} = \min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{k=1}^{K} \sum_{i=1}^{M} \alpha_i h_{k,i} \right\} \]

  where \( \alpha_i \) is the client’s weight and
  
  \( h_{k,i} \) is the number of frames since the last delivery from client \( i \)

  \( \Pi \) is the class of non-anticipatory policies and \( \pi^* \) is the optimal policy.
Symmetric Network

• **Network with symmetric clients**: $\alpha_i = \alpha$ and $p_i = p$, $\forall i \in \{1, ..., M\}$

• **Greedy Policy** (G): in each slot, select the client with undelivered packet and highest value of $h_{k,i}$.

**Theorem 1**: Optimality of the Greedy Policy.

Among the class of admissible policies $\Pi$, G attains the minimum time average sum AoI.
General Network

• Network with clients having (possibly) different $\alpha_i$ and $p_i$

• Objective Function: 

$$J^*_K = \min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{k=1}^{K} \sum_{i=1}^{M} \alpha_i h_{k,i} \right\}$$

• Policy $\pi$ is a mapping from all possible states in each possible slot to the associated scheduling choice. In general, computing $\pi$ is complex.

• Index Policy: in each slot, select the client with undelivered packet and highest value of $C_i(h_{k,i})$.

• The Index Policy is a low-complexity heuristic that is extensively used in the literature for its strong performance [K. Liu, 2010; R. Weber, 1990; et al.]
General Network: Whittle Index

• For designing the Index Policy, we use the RMAB framework in [16].
  • We relax our problem to the case of a single client, $M = 1$, and add a cost per transmission, $C > 0$.

• The solution to this relaxed problem yields:
  • Condition for indexability;
  • Expression for the Whittle Index, $C_i(h_{k,i})$.

• Challenges:
  • Indexability is often hard to establish
  • Indexable problems might not have closed-form solutions for the Whittle Index.

General Network: Definitions

• Indexability:
  • Consider the relaxed problem with a single client and cost per transmission.
  • Let $\mathcal{P}(C)$ be the set of states for which it is optimal to idle when the cost for transmission is $C$.
  • The problem is indexable if $\mathcal{P}(C)$ increases monotonically from $\emptyset$ to the entire state space as $C$ increases from 0 to $+\infty$.
  • The condition checks if the problem is suited for an Index Policy.

• Whittle Index:
  • Given indexability, $C(h)$ is the infimum cost $C$ that makes both scheduling decisions equally desirable in state $h$.
  • $C(h)$ represents how valuable is to transmit a client in state $h$.

General Network: Index Policy

• We establish that the problem is indexable and find a closed-form solution for the Whittle Index.

• Index Policy: in each slot, select the client with undelivered packet and highest value of $C_i(h_{k,i})$, where:

$$C_i(h_i) = \frac{T\alpha_i}{2} p_i h_i \left[ h_i + \frac{1 + (1 - p_i)^T}{1 - (1 - p_i)^T} \right]$$

• Observe that the client ordering imposed by the Index Policy is the same as the one imposed by the Greedy Policy for the case of symmetric networks. Thus, the Index Policy is OPTIMAL.
Numerical Results

• Metric is the normalized AoI: $\frac{EWSAoI}{MT}$

• Comparison:
  • Greedy Policy Simulation (each point is an average over 1k runs)
  • Index Policy Simulation (each point is an average over 1k runs)
  • Optimal Policy Computation (using Dynamic Programming)

• Two settings:
  • Symmetric Network
  • General Network
Symmetric Network

\[ M = 2 \]
\[ T = 5 \]
\[ K = 150 \]
\[ h_{1,i} = 1 \]
\[ \alpha_i = 1 \]
\[ p_i = p \in \left\{ \frac{1}{15}, \frac{2}{15}, \ldots, \frac{14}{15} \right\} \]

Corroborates our result:

Greedy \equiv \text{Index} \equiv \text{Optimal} for symmetric networks
Index is close to the optimal performance.

General Network

- Computation of the DP
- Simulation of Index Policy
- Simulation of Greedy Policy

Parameters:
- $M = 2$
- $T \in \{1, 2, \ldots, 10\}$
- $K = 200$
- $h_{1,i} = 1$
- $\alpha_i = 1$
- $p_1 = \frac{2}{3}$
- $p_2 = \frac{1}{10}$
Minimum Delivery Ratio Constraint

- In our network setting, undelivered packets are replaced.
- Consider the problem of finding the scheduling policy \( \eta \in \Pi \) that satisfies:

\[
P(\hat{q}^\eta_i \geq q_i) = 1, \ \forall i
\]

where \( q_i \) is the minimum delivery ratio requirement of client \( i \) and \( \hat{q}^\eta_i \) is:

\[
\hat{q}^\eta_i \triangleq \lim_{K \to \infty} \inf \frac{1}{K} \sum_{k=1}^{K} e_i(k), \text{ where } e_i(k) = \begin{cases} 
1, & \text{if delivery } (k, i) \\
0, & \text{otherwise}
\end{cases}
\]

- An equivalent problem is to find the \( \eta \) that maximizes the Expected Weighted Sum Throughput (next slide).

Max Throughput vs Min AoI

• The **Throughput maximization** metric is given by:

\[
\text{EWST} = \frac{1}{K} \mathbb{E} \left\{ \sum_{k=1}^{K} \sum_{i=1}^{M} \alpha_i e_i(k) \right\}, \text{ where } e_i(k) = \begin{cases} 1, & \text{if delivery (k, i)} \\ 0, & \text{otherwise} \end{cases}
\]

• The **AoI minimization** metric is:

\[
\text{EWSAoI} = \frac{1}{K} \mathbb{E} \left\{ \sum_{k=1}^{K} \sum_{i=1}^{M} \alpha_i \left( \frac{T^2}{2} + T^2 h_{k,i} \right) \right\}, \text{ where } h_{k+1,i} = \begin{cases} 1, & \text{if delivery (k, i)} \\ h_{k,i} + 1, & \text{otherwise} \end{cases}
\]

• For comparing the scheduling policies that result from each problem, we consider their DP solutions.
Max Throughput vs Min AoI

- For a fixed vector of client weights $\tilde{\alpha}$, the Dynamic Programs yield:

  - We sweep $\tilde{\alpha}$ and plot the results next:
    - **Red** is for metrics associated with $\pi^*$.
    - **Blue** is associated with $\eta^*$.

```plaintext
Max Throughput vs Min AoI

- For a fixed vector of client weights $\tilde{\alpha}$, the Dynamic Programs yield:

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    - **Red** is for metrics associated with $\pi^*$.
    - **Blue** is associated with $\eta^*$.
```
Policies that OPTIMIZE AoI

The AoI ASSOCIATED with Throughput optimal policies

M = 2
T = 5
K = 120
p_1 = 1/5
p_2 = 1/3
α varies
The throughput ASSOCIATED with AoI optimal policies

Throughput Comparison

Policies that OPTIMIZE throughput

M = 2
T = 5
K = 120
p_1 = 1/5
p_2 = 1/3
α varies
Max Throughput vs Min AoI

• The conclusion illustrated by the numerical results holds in general:

<table>
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<tr>
<th>Thr Optimal Policies ⇒ AoI Optimal</th>
</tr>
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<td>AoI Optimal Policies ⇒ Thr. Optimal$^1$</td>
</tr>
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</table>

$^1$ Pareto optimal

• By minimizing the AoI, we are assured to achieve maximum throughput.

• However, it is still not known how to design a scheduling policy that achieves a **given throughput** with minimum AoI.
AoI, Throughput and Interdelivery times

• **Proposition:** Consider the network over an infinite horizon, namely $K \to \infty$, and assume that the steady-state distribution of the underlying MC exists when the stationary policy $\pi$ is employed. Then, it follows

$$EWSAoI = T^2 \sum_{i=1}^{M} \alpha_i + \frac{T^2}{2} \sum_{i=1}^{M} \alpha_i \left( \frac{\text{Var}[I_i]}{\mathbb{E}[I_i]} + \mathbb{E}[I_i] \right)$$

where $I_i$ is the r.v. that represents the number of frames in the interval between two packet deliveries from client $i$, i.e. the interdelivery time.

Consider the AoI optimal policy $\pi^*$ and the associated throughput performance. Under the conditions of the Proposition, we know that from all policies with the same throughput, policy $\pi^*$ achieves the lowest value of $\text{Var}[I_i]$.

Outline / Contributions

• Age of Information (AoI) and Network Model

• Symmetric Network and the OPTIMALITY of the Greedy Policy

• General Network and the DESIGN of the Index Policy

• VALIDATION of the policies via Numerical Results

• COMPARISON of the Min AoI problem and the Max Throughput problem.
Supplementary Slides
Intuition of the proof: ideal channels

• Consider $M = 3$, $T = 1$ and $p = 1$ (ideal channels)
• Employ GREEDY policy. Deliveries are in green.

• One packet is scheduled and delivered in each frame ($T = 1$ slot).
• Greedy achieves the lowest $\sum_{i=1}^{M} h_{k,i}$ in every frame $k \rightarrow$ Greedy is optimal.
• Note that $h_{k,1} + h_{k,2} + h_{k,3} = 6, \forall k \geq 3$ (steady-state)
Intuition of the proof: coupling argument

• Consider $M = 3$, $T = 1$ and $p \in (0,1]$ (unreliable channels)

• Employ ARBITRARY policy. Deliveries are green. Failed transmissions are red.

• Fix any sample path for the state of the active channel:
  • channel is OFF: no room for improvement. All policies are equivalent.
  • channel is ON ≡ ideal channels: the best policy is Greedy. (example next)
Intuition of the proof: coupling argument

- Consider $M = 3$, $T = 1$ and $p \in (0, 1]$ (unreliable channels)
- Employ ARBITRARY policy. Deliveries are green. Failed transmissions are red.