REAL ANALYSIS 18.100C: WHAT’S ON THE FINAL?

Here is a list of topics which I think are important and you should definitely be familiar with.

Chapter 4: Continuous functions, relation between continuous functions and open/closed/compact/connected sets, uniform continuity.

Chapter 5: Definition of differentiation, Mean Value theorems: 5.8, 5.9, 5.10, 5.11, Taylor’s theorem.


Here are some practice questions:

Determine whether the following statements are true or false.

1. If \( f \in \mathcal{R}(\alpha) \) on \([a,b]\), then \( f \in \mathcal{R}(\alpha) \) on any \([c,d] \subseteq [a,b]\).
2. Suppose that \( f,\alpha : [a,b] \rightarrow (0,75) \) are discontinuous at the same point \( x \in [a,b] \), then \( f \notin \mathcal{R}(\alpha) \).
3. The uniform closure \( \mathcal{F} \) of an equicontinuous set \( \mathcal{F} \subseteq \mathcal{C}([0,1]) \) is equicontinuous.
4. A finite set of continuous functions is equicontinuous.
5. The set \( \{x^n\} \subseteq \mathcal{C}^0([0,1]) \) is equicontinuous.
6. If \( f^2 \) is integrable, then \( f \) is integrable.
7. If \( f^3 \) is integrable, then \( f \) is integrable.
8. If \( f \in \mathcal{R} \) with \( \int f = 0 \) and \( f \geq 0 \), then \( f = 0 \).
9. In a complete metric space, closed and bounded sets are compact.
10. The pointwise limit of a sequence of continuous functions on a compact set is continuous.
11. The preimage of a compact set by a continuous function is compact.
12. Suppose that \( f_n \rightarrow f \) uniformly and that \( f_n \) is differentiable. Then, \( f'_n \rightarrow f' \) pointwise.
13. The set \( \{f_n\} \) is equicontinuous if \( f_n \rightarrow f \) uniformly.
14. The pointwise limit of a sequence of polynomials, \( P_n : [0,1] \rightarrow \mathbb{R} \), can be discontinuous.
15. The uniform limit of a sequence of polynomials, \( P_n : [0,1] \rightarrow \mathbb{R} \), can be discontinuous.
16. The uniform limit of a sequence of polynomials, \( P_n : [0,1] \rightarrow \mathbb{R} \), can be non-differentiable everywhere.