Special Seminar in Operations Research:
“Computational Methods for Infinite Dimensional Optimization”

15.099 Spring 2019
Course Syllabus

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Lectures  Fridays, 10:00am – 12:00pm, E62-550
Course page  https://canvas.mit.edu/courses/1446

Course Objectives

In keeping with tradition, 15.099 will focus on an advanced operations research topic related of interest to a portion of the MIT community. This Spring the seminar will study papers on the area of infinite dimensional optimization with a strong emphasis on computational methods that can (approximately) solve these problems in practice. Students will get in touch with the frontier of current research and they will be exposed to several open problems in this field and in the related application areas. This will allow students

- To be exposed to theory and application areas that might not otherwise be covered in their program.
- To deal with advanced material, typically based on original literature from the forefront of current research.
- To develop and strengthen their skill of reading and understanding original scientific articles.
- To practice and enhance their presentation skills and to receive feedback as well.

Subject Description

A wide range of problems in engineering and statistics can be formulated as optimization problems in infinite-dimensional functional spaces. Such problems include optimal control problems and differential equations, which are routinely solved via exact and numerical methods. However, when viewed from a generic optimization perspective they can reveal various theoretical challenges that can make their practical solution seem like a daunting task. In this seminar we will explore several recent computational approaches that allow the solution of fairly general infinite-dimensional optimization problems in practice.

Course Organization

The seminar will meet most Fridays from 10:00am to 12:00pm in room E62-550. The seminar aims to provide a friendly setting for developing presentation skills and practicing teaching. Each week one or two students will be responsible for making a presentation. Students are expected to send in their presentation several days in advance so that I can provide them feedback and help them improve their presentation skills. The seminar is intended to be an informal forum for discussion and for interaction among the student and faculty participants, and works best when everyone contributes. Since we are collectively exploring an advanced topic, the discussion leader(s) need not feel that he or she must have a complete grasp of everything being presented – often coming to class with questions leads to the most lively discussion and the greatest learning.
Preliminary List of Topics/Papers

Possible papers and topics will be discussed during the first lecture on Friday, February 8th. Topics will focus on both computational methods and applications, so we encourage students to propose their own research or applications of interest as possible presentations. All references below should be accessible within the MIT network. Presentations may focus on a single paper or a group of papers.

1. **Conic certificates for non-negativity of functions**: Sum-of-squares or semi-definite programming certificates for the non-negativity of polynomials have been extensively used for optimization problems in polynomial spaces [BPT12]. Similar approaches exist for other functional spaces including non-negativity certificates for polynomials using the power cone [GM12] and non-negativity certificates for signomial functions using the exponential cone [CS16]. In addition, the dual approach concerning moments of measures can sometimes provide significant computational improvements in this context [HT18]. These approaches have been (or could be) used to solve a wide range of problems in engineering and physics [TMTR10, PKT17, HT18, ZMV17, DPS02].

2. **Simplex algorithm for countably-infinite linear programming (CILP)**: CILP problems are linear optimization problems with a countably-infinite number of variables AND constraints [RS98, RTV16]. Under some assumptions the Simplex algorithm for traditional linear optimization can be extended to CILPs and can sometimes lead to a practical computational method [LERS17, RSE18].

3. **Multi-stage adaptive or dynamic integer programming**: A combination of MIP and dynamic programming techniques can be effective for dynamic decision making with integer variables [BD16, BDFK16, PS17, AG17].

4. **Dynamically-generated time-expanded networks**: A traditional approach for network problems over time is to approximate using a fixed discretization of time. Newer methods can dynamically generate a time-expanded network using a sequence of manageable problems [BHVS17, BHMS17].

5. **Mixed-integer optimal control**: Optimization problems whose constraints include ordinary differential equations (ODE) or partial differential equations (PDE) are extremely challenging, but recent methods have led to some practical success [SBD12, SCM15, Han17, Han16, Sag13, BKM18, DT15].

6. **Modeling and automatic discretization for optimization with differential and algebraic equations**: One way to solve optimization and control problems with (partial) differential and algebraic equations is to discretize them and solve this discretization with an appropriate finite-dimensional optimization software. This can be a complicated and error-prone process that can be significantly improved with appropriate modeling tools [NSW+18].

7. **Model predictive control (MPC) and multi-parametric optimization**: MPC [AZ12] solves optimal control problems through a time-discretized look-ahead optimization problem. Practical MPC requires real-time solution of instances from a family of optimization problems parametrized by the systems state. The sub-field of explicit MPC [Pis09, AB09] combines off-line evaluation of these problems with techniques from multi-parametric optimization [ODN+16, PGD07b, PGD07a] to store the associated control functions.

8. **Approximate dynamic programming techniques**: Building dynamically refined approximations of cost-to-go value functions can lead to practical algorithms for stochastic dynamic programming problems with infinite horizons and/or infinite/continuous state- and action-spaces [LS19, Dow18, WBL17].

9. **Computational Optimal Transport** The classical optimal transport problem can be seen as an infinite dimensional generalization of the assignment problem and has been a rich area of research since the beginnings of linear optimization. Recent improvements in the scalability of computational methods for this problem have re-energized many related areas [PC18].

10. **Gaussian Processes for Global Optimization**: [OGR09]
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<td>First lecture: course overview and discussion</td>
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<td>15th</td>
<td>Guest Lecture by Rahul Mazumder: <em>Non-parametric smooth function estimation with shape constraints</em></td>
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<td>22nd</td>
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**References**


