Going public and the ownership structure of the firm

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Abstract

Going public is a complex process with distinct markets for dispersed shares and controlling blocks. It is important to design the sale of new shares with the final ownership structure in mind. An optimal strategy for going public starts with the IPO, which is particularly suited for the sale of dispersed holdings to small and passive investors. The marketing of potentially controlling blocks to active investors should occur subsequently. We develop a framework for evaluating alternative methods of sale and show that discriminating in favor of active investors can raise the market value of the firm for all shareholders. © 1998 Elsevier Science S.A. All rights reserved.

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1. Introduction

That ownership structure is an important determinant of firm value is now well established in the finance literature. Large, active investors often play a monitoring role that raises the value of all shares. The presence of large shareholders also improves the efficiency of the market for corporate control. How does the significance of ownership structure shape the process for issuing equity?
When a firm goes public, the large volume of new shares sold, as well as the large volume of existing shares transferred to new owners, lastingly shapes the firm’s ownership structure and thereby influences the firm’s value. To maximize the revenue raised from the shares sold in the public offering, it is important to design the sale of new shares with the final ownership structure in mind. Most investors will remain relatively small and passive holders of the firm’s shares, while others will seek a large block of shares and are prepared to actively shape the firm’s management, either as monitors of the current management or as proponents of an alternative strategy or management team (see Mikkelson and Ruback, 1985; Shleifer and Vishny, 1986; and Barclay and Holderness, 1989). The market for dispersed shareholdings is distinct from the market for potentially influential blocks; Hanley and Wilhelm (1995) provide evidence that the market for shares is segmented. Is it possible to ignore the heterogeneity among investors and design a sale of shares uniformly addressed to all buyers? If not, how should this heterogeneity shape the firm’s strategy for selling shares?

We establish that the optimal strategy involves a staged process of financing beginning with an IPO for small investors, then selling a controlling block (possibly at a discount), and concluding with a contingent sale of additional shares. Our results highlight the fact that going public is a complex and extended process. The IPO is particularly suited for the sale of dispersed shareholdings to small and passive investors but is not a good method for selling control. The marketing of potentially controlling blocks of shares to active investors should occur separately and, perhaps as important, after the IPO has taken place. If securities regulators prohibit price discrimination among different investors, then we show that the optimal strategy is to sell a controlling block subsequent to the IPO.

We show that favorable treatment for investors seeking potentially controlling blocks maximizes the revenue raised in the aggregate sale of shares. Because an active investor with a controlling block can benefit all shareholders, discriminating in favor of the active investor by offering the controlling block at a discount assures an efficient ownership structure and raises the market value of the firm. On the other hand, insofar as the active investor can use the controlling block to extract private benefits, the seller can raise the price at which a controlling block is offered. Consequently, whether the controlling block is offered at a discount or a premium depends upon the relative significance of the public and private benefits associated with the controlling block.

It is also necessary that active investors seeking controlling blocks be put into competition with small, passive investors seeking the same shares in dispersed allotments. This can be done, for example, by initially selling a portion of the shares to passive investors and then later putting a controlling block up for sale on terms determined in part by the price for the outstanding shares. This highlights the importance of treating the issuance of shares as a process incorporating transactions over time, instead of as a single event independent of the
firm’s plans for subsequent financing as has often been the case. The results of each sale affect the terms of later sales, and the terms of earlier sales are determined in part by the beneficial impact on ownership structure and the terms of later sales. One contribution of our paper is to explain why some privately held firms go public instead of selling control exclusively to another set of private investors, despite the apparent value loss associated with the free rider problem (Weston et al., 1990, p. 663).

A great deal of recent empirical evidence establishes that going public is an extended process and that, as its name implies, the IPO is but the first stage in this process. This literature shows that questions of ownership structure and control are at the center of this process and also seems to indicate that the IPO is not a good way of selling control. Rydqvist and Högholm (1994) present evidence that often the decision to go public cannot simply be explained by the growth experienced by the firm. According to Barry et al. (1990), venture capitalists have mechanisms to ensure that in many instances firms go public before any controlling blocks are sold. Indeed, in most of the cases analyzed, Barry et al. report that none of the venture capitalists sold shares during the IPO, although their ownership stakes gradually declined over time thereafter. Mikkelson et al. (1995) also find that initial owners rarely dispense controlling blocks at the IPO. Brennan and Franks (1995) provide evidence that firms manage the sale of shares with the purpose of discriminating between passive investors and applicants for large blocks and that the timing of the sale of large blocks is carefully chosen: most blocks remain intact during the IPO, but almost one-half of the offering company’s shares are sold subsequently. Indeed, the strategy of going public followed by a transfer of control seems to be a more frequent strategy in the sale of firms than it might appear at first. Evidence of control turnover after IPOs is documented by Holderness and Sheehan (1988) for the U.S., by Rydqvist and Högholm (1994) for Sweden and the U.K., and by Pagano et al. (1996) for Italy.

Although much of this literature deals with stand-alone firms, divisions of public corporations are sometimes also sold through a public offer of shares. The importance of seeing the sale as an extended process arises here as well. For example, Schipper and Smith (1986) and Klein et al. (1991) report that the initial public offering of shares in a wholly owned unit – an ‘equity carve-out’ – is frequently followed by the sale of the remaining interest by the parent corporation. Recent examples in which flotation has been proposed as the first stage of a complete ownership transfer are Agfa, currently owned by Bayer, Suburban Propane, a U.S. subsidiary of Hanson, the U.K. industrial conglomerate, and Thermo King, the transport refrigeration unit of Westinghouse (see the Financial Times, 11/8/95, 12/21/95; the New York Times, 11/14/96).

Other researchers have also begun to call attention to the important relationship between ownership structure and the process of going public. The papers most closely related to this one are Zingales (1995) and Stoughton and Zechn
Both of these papers analyze how the decision to go public is affected by considerations of corporate control and suggest that the sale of the company should proceed in stages. In a model with perfect information but in which the seller has better bargaining power against passive investors than against an investor seeking control, Zingales shows that first selling a portion of the firm’s shares to the passive investors and then selling a controlling block maximizes the seller’s revenue. Zingales also explores how the separation of control rights and cash flow rights can raise the seller’s revenue even more. Uncertainty about the value added by the active investor and uncertainty about the demand of small investors play a role in our setting that is not present in Zingales’. In our model the seller obtains valuable information from the sale of shares to small investors that is useful in negotiating the terms of a sale to an active investor. We interpret the IPO as a mechanism that provides the seller valuable information to set the conditions under which the controlling block will be sold.

Stoughton and Zechner (1998) emphasize as well the importance of favoring large investors. But because they abstract from the effects of asymmetric information among investors, their suggested optimal method is quite the opposite of ours—they suggest first selling shares to the large investor and then selling to small investors at the same price. This order of events does not seem to be supported by the empirical evidence. Stoughton and Zechner explain underpricing and rationing in an IPO as a second-best response to regulatory constraints on price discrimination which make implementation of the optimum ownership structure difficult, while we, on the other hand, argue that an IPO should be seen as the first step in a process of a staged financing used to implement an efficient ownership structure. To our knowledge this paper is the first that analyzes the discovery role of the IPO in models of the sale of control blocks when investors have different information. Indeed, in Zingales the IPO stage may not even be optimal when the value of the cash flow rights under the management of the new large investor is less than the value under the initial owner. In our case the IPO is always a good choice, because it helps to reveal information about the demand for dispersed shares and the market’s assessment of the value of the firm under the management of a potential new large investor. If this value is low, the current owner can then decide not to sell to the large investor, but that decision can only be made after the IPO.

The paper proceeds as follows. The next section describes the model and illustrates the relation between the allocation made in the public offering and the final ownership structure when investors are given the opportunity to trade in the firm’s shares. In Section 3 we develop a framework for analyzing strategies for going public, and in Section 4 we use this framework to evaluate alternative strategies, comparing the outcomes of methods of sale that have been most frequently used. In Section 5 we discuss some possible variations that will help obtain an optimal solution. The concluding section summarizes the major findings.
2. A model of the sale of shares with a secondary market

In this section we describe the basic framework of analysis and the fundamental assumptions. We model an initial sale together with a sequence of aftermarkets in which shares can be traded and out of which is established a final ownership structure. We then close the model with a concluding period in which all holdings are liquidated and any anticipated cash flows are actually realized. Finally, we present a numerical example that illustrates the relation between the allocation made in the public offering and the final ownership structure.

2.1. The population of investors

Investors are assumed to be risk neutral and to differ in their demands for the shares of the firm as well as in their degree of influence on the firm once they become shareholders. There is a population of small investors for whom the value of a share in the firm is the sum of several components. One component, \( y \), captures the value calculated under a common metric and about which there is no uncertainty. Added to this common component is an idiosyncratic component, \( w \), which is private information to each investor. For example, the small investor’s tax status or liquidity preference might affect his or her valuation of the firm’s expected cash flows. Diversity in the population of small investors is described by the distribution \( G(w|x) \) on the range \([w_{\text{min}}(x), w_{\text{max}}(x)] \subseteq R^+ \) with \( \int_{w_{\text{min}}}^{w_{\text{max}}} dG(w|x) = 1 \). The parameter \( x \) is a random variable in \( R \) that captures variation in the aggregate valuation of small investors, and has the distribution \( H(x) \). We assume that \( \partial G/\partial x > 0 \) everywhere that \( H(x) \in (0, 1) \). Each small investor knows its own type, \( w \), but does not directly observe the population parameter \( x \) and is therefore uncertain about aggregate demand. We normalize each small investor’s demand for a share with the condition that aggregate demand at any fixed price, \( p \), is given by \( \int p \to y \, dG(w|x) = 1 - G(p - y|x) \).

In addition, there is a large, active investor denoted by ‘a’ who seeks a controlling block in order to actively influence future management’s decisions. This investor might be interested in acquiring control because of information about a strategy for using the assets of the firm that could increase the value of the firm’s cash flows. Control is reached with 50% of the shares, although a lower proportion can be used to illustrate the problem. Upon achieving control of the firm, the large shareholder can implement changes in the operations of the firm that will increase the value of the firm’s expected future cash flows by an amount \( z \). Initially, the value of \( z \) is private information to the active investor. The seller and the small investors view the active investor’s control premium as a random variable drawn from a distribution \( F(z) \). The large investor’s idiosyncratic component of the value is \( w_{a} \). The model can be extended to include multiple active investors without changing the essential dynamics of the problem, but it is...
necessary that the active investors not be atomless – size matters. According to Wruck (1989, p. 10), the most common case is indeed that of one large buyer per sale.

Since the efficiency of the allocation of shares plays a central role in this paper, it is useful to characterize the efficient allocation. Consider first the allocation among the class of small shareholders. For an arbitrary quantity of shares for the class as a whole, the optimal allocation is to assign the shares first to those with the highest private valuation and then to shareholders with lower and lower private valuations until the quantity available is exhausted. The last small investor to receive a share is the marginal small investor, given the quantity to be allocated. Consider next the allocation of shares either to the active investor or to the pool of small investors. It will either be efficient to allocate a controlling block or not. Efficiency requires that we allocate the controlling block to the active investor whenever the per unit value of the shares in the active investor’s hands, \( w_a + z \), is greater than the average per unit value of the shares in the hands of the small investors displaced, \( \int_{\hat{w}_{1/2}(z)}^{w_{1/2}} w \ dG(w|z) \), where \( \hat{w}_{1/2}(z) \) denotes the marginal small investor when the active investor has received exactly a controlling stake and \( \hat{w}_0(z) \) denotes the marginal small investor when the active investor has received zero shares. Note that the efficient decision of whether to allocate the controlling block to the active investor is contingent upon the parameter \( a \), which measures the aggregate demand of small investors. We can summarize these results in formal notation as follows. Denote by \( q^*_a(z, z, w) \) the efficient allocation to the active investor contingent on all realizations of the two parameters \( a \) and \( z \), for a given value of \( w_a \). Then \( q^*_a = \frac{1}{2} \) whenever \( w_a + z \geq \bar{z}(z) = \int_{\hat{w}_{1/2}(z)}^{w_{1/2}} w \ dG(w|z) \). Denote by \( q^*_a(w, z, z) \) the efficient allocation among small investors. If \( q^*_a = \frac{1}{2} \), then \( q^*_a(w, z, z) = \frac{1}{2} \) for \( w \geq \hat{w}_{1/2}(z) \) and zero otherwise, and if \( q^*_a = 0 \), then \( q^*_a(w, z, z) = 1 \) for all \( w \geq \hat{w}_0(z) \).

2.2. The sequence of markets

Consider a firm that is for sale by its owner(s). There are many reasons that could explain this exit decision, including the benefits of diversification, liquidity preferences, the realization of gains from selling to better-positioned parties, exploiting favorable market conditions, gains from focus, etc. In this paper we take the decision to sell as given and concentrate on the issues surrounding the implementation of the sale.

The full sequence of events is broken down into six periods, \( \tau = 0, \ldots, 5 \), as shown in Fig. 1. At \( \tau = 0 \), the risk neutral seller makes public the choice of method used in the initial sale. The sale is open to all interested investors. At \( \tau = 1 \), the initial sale of shares takes place according to the rules established by the seller, the resulting allocation, \( q_1^*(w_a, z, z) \), is made public, and all investors update their beliefs about the unknown parameters \( z \) and \( z \) based upon this information.
At $\tau = 2$, small investors trade shares in a secondary market at a competitive, rational expectations equilibrium price, $p^2$. This price can be informative about the unknown parameters. The active investor cannot trade anonymously in the secondary market at $\tau = 2$ and must instead make all trades in a tender offer market at $\tau = 3$. The active investor can use the tender offer market to buy any quantity of shares or to sell some or all of any holdings accumulated in the initial offering. The active investor's decision to make a tender offer at the price $p^3$ can be also informative about the unknown parameters. The results of the tender offer are public information. At $\tau = 4$, competitive trading among the small investors resumes at a rational expectations equilibrium price $p^4$. At $\tau = 5$, the firm is liquidated and shareholders receive their prorated share of the firm's cash flows, valued as described earlier.

Variations on this sequencing are also possible. For example, the initial sale can take place in stages within period $\tau = 1$, and in Section 4 we consider a sale in which some of the shares are distributed at $\tau = 1$ and a second portion is distributed after a secondary market price is established.

### 2.3. Trade and information in the secondary market

Investors can buy shares in the primary market as well as later when the secondary market opens for trading. One question that immediately arises is whether the extended trading opportunities effectively provide for an optimal ownership structure in the firm. For example, is it always possible for the active investor to accumulate a block in the secondary market when the optimal outcome includes block ownership by an active investor? If so, then naturally that will reshape the equilibrium bidding in the initial sale. Also, why should the seller worry about designing a method of sale if investors can revise their allocations in the secondary market? If a passive investor can buy shares in the
secondary market at a competitive price, then it would appear impossible to design the initial sale to raise any more than the expected value of a share in the secondary market. Faced with the secondary market as a constraint on the price paid by the passive investor, it would seem that rules of sale that strategically invite the participation of specific groups of investors are relatively innocuous and ultimately implemented at the expense of the seller.

We show that the markets for shares cannot ensure an optimal ownership structure in the firm all the time, and therefore the choice of the method of sale is an important consideration. Suppose that in equilibrium the initial sale always establishes the efficient ownership of the firm. At the start of period \( \tau = 2 \), passive investors know whether the active investor has been allocated a controlling block and therefore whether the ultimate value of a share to them incorporates a control premium. Moreover, small investors infer from the active investor’s allocation in the initial sale whether \( z \geq \bar{z}(x) \). When \( q_a^1 = 0 \), a small investor demands a share if and only if \( p^2 < y + w \). The market-clearing price is therefore \( p^2 = y + \bar{w}_0(z) \). When \( q_a^1 = \frac{1}{2} \), a small investor must make an estimate of the size of the control premium, \( z \). The small investor’s own private valuation provides information about the likely values for \( x \) and therefore for \( z \). The active investor’s allocation tells the small investor that \( z \geq \bar{z}(x) \). Finally, the equilibrium price can itself provide additional information about the likely values for \( x \) and \( z \). The small investor demands a share if and only if \( p^2 < y + w + \bar{z}(z|w_a, z \geq \bar{z}(x), p^2) \). The rational expectations equilibrium price function \( p^2 = y + \bar{w}_{1/2}(z) + \bar{z}(z|w_a, x, z \geq \bar{z}(x)) \) fully reveals \( x \), making each small investor’s own private valuation entirely superfluous in conditioning the posterior distribution on \( z \), and clears the market.

We can summarize the results of the competitive market with the following characterization of the market price as a function of the underlying parameters:

\[
p^*(x, z) = \begin{cases} 
  y + \bar{w}_0(z), & z < \bar{z}(x), \\
  y + \bar{w}_{1/2}(z) + \bar{z}(z|w_a, x, z \geq \bar{z}(x)), & z \geq \bar{z}(x).
\end{cases}
\]

This market price acts as an important constraint on the original terms of sale that can be imposed on small investors. Knowing that it is always possible to obtain a share in the competitive market on these terms, each small investor puts a limit on the price he or she is willing to pay in the initial sale. We discuss these issues in detail in the next section, but first it is necessary to complete our discussion of the secondary market.

In the analysis above we assume that the initial sale of shares always establishes an efficient ownership structure for the firm. However, the derivations of the equilibrium price function actually only rely upon the assumption that the initial sale establishes a controlling block whenever that is efficient. Consequently, the same equilibrium price function will obtain regardless of the allocation of shares among small investors. Small investors with private...
valuations greater than the marginal valuation impounded into the price, \( w \geq \hat{w}(z) \), will purchase a share if they do not own one, and small investors with a private valuation below the marginal valuation, \( w < \hat{w}(z) \), will sell a share if they have one. The secondary market can be relied upon to establish the efficient allocation of shares among small investors, even when the initial sale does not.

However, the tender offer market cannot be relied upon to the same degree. An active investor seeking to obtain a controlling stake makes a single tender offer at a take-it-or-leave-it price \( p^3 \), for the specified number of shares, \( 1/2 - q_\alpha^1 \). If the tender is successful, the active investor’s net profits are \( 1/2(y + w_a + z) - (1/2 - q_\alpha^1)p^3 \). Alternatively, the active investor can sell his or her stake, which must be done at the price \( y + w_0(z) \), netting profits of \( q_\alpha^1(y + w_0(z)) \). A tender offer to buy at the price \( p^3 \) is optimal for the active investor only if \( 1/2(y + w_a + z) - (1/2 - q_\alpha^1)p^3 \geq q_\alpha^1(y + w_0(z)) \). For convenience we restate this condition as \( z \geq d \) where \( d = (1 - 2q_\alpha^1)(p^3 - y) - 2(1/2w_a - q_\alpha^1w_0(z)) \). In other words, if the active investor does not receive a controlling block in the secondary market, even when that would be efficient, it is not always possible to acquire one in the tender offer market.

**Lemma 1.** The tender offer market does not always allow a controlling stake to be accumulated.

**Proof.** For a tender offer to succeed, a sufficient number of the small investors currently owning a share must be willing to tender, i.e., all \( w \leq \hat{w}_{1/2}(z) \), or

\[
\frac{1}{2}(y + w_a + z) - q_\alpha^1(y + w_0(z)) \geq p^3 \geq y + \hat{w}_{1/2}(z) + \varepsilon(z|z > d).
\]

The marginal small investor accepts the tender if the offered price is greater than his or her valuation of a share incorporating the information that \( z \geq d \): \( p^3 \geq y + \hat{w}_{1/2} + \varepsilon(z|z \geq d) \). Whenever \( q_\alpha^1 = 0 \) we have \( d = p^3 - (y + w_a) \). This condition implies the impossible result that \( p^3 - (y + w_a) > \varepsilon(z|z \geq p^3 - (y + w_a)) \), and consequently a successful tender offer is not possible whenever \( w_a < \hat{w}_{1/2} \), regardless of the value of \( z \). This is due to the free rider problem first demonstrated by Grossman and Hart (1980). When \( q_\alpha^1 > 0 \), as Shleifer and Vishny (1986) and Hirshleifer and Titman (1990) point out, a successful tender offer will be possible for sufficiently large \( z \). However, a tender offer might not succeed for all values of \( z \) for which a controlling allocation is efficient. Indeed, it is always possible to choose reasonable parameter values for which the final allocation is everywhere efficient only if the previous allocation that results from the original sale is everywhere efficient. To see this, let \( \hat{w}_0 \approx \hat{w}_{1/2} \), in which case \( q_\alpha^*(z, z) = 1/2 \) for all \( w_a + z > \frac{1}{2}w + \varepsilon \), with \( \varepsilon \) arbitrarily close to zero. Then, for an active investor with \( z = \frac{1}{2}w - w_a + \varepsilon + \delta \), with \( \delta \) arbitrarily close to zero, the
feasibility condition for a successful tender offer is certainly violated, and unless the sale efficiently allocates one-half of the shares, the active investor, being unable to buy enough shares from the passive investors who demand too a high price, chooses instead to sell the original allocation: \( \{q_3^1 < \frac{1}{2}\} \Rightarrow \{q_a^3 = 0\} \)

2.4. A numerical example

The following example illustrates the relation between the allocation made in the original sale and the result of the tender offer market. It also illustrates the relation between the price prevailing in the secondary market at \( \tau = 2 \) and the tender offer price at \( \tau = 3 \).

Let \( H(z) \) be the uniform distribution over \([0, 1]\), let \( G(w|z) \) be the uniform distribution over \([z, z+1]\), and let \( F(z) \) be the uniform distribution over \([0, 1]\). For simplicity, assume \( w_a = 0 \). Then, efficiency requires that \( q^*_a(x, z) = \frac{1}{2} \) whenever \( z > 1/2z + 1/8 \), and zero otherwise. Fig. 2 shows the parameter space, \([z, z]\). In region I it is efficient for all of the shares to go to the small investors and for no controlling block to be allocated. In all other regions it is efficient to allocate the active investor a controlling block. Since our focus is on whether the secondary and tender offer markets assure efficiency in the final allocation, we assume an inefficient but otherwise arbitrary result from the original sale and explore what happens in the secondary and tender offer markets that follow.

![Fig. 2.](image-url)
Suppose, for example, that the equilibrium allocation from the original sale yields

\[ q_1^*(x, z) = \begin{cases} 
0, & \text{if } z < \frac{1}{2}x + \frac{3}{8}, \\
 z - (\frac{1}{2}x + \frac{3}{8}), & \text{if } \frac{1}{2}x + \frac{3}{8} < z \leq \frac{7}{8}, \\
\frac{1}{2}, & \text{if } \frac{7}{8} < z.
\end{cases} \]

This allocation is inefficient whenever \( \frac{1}{2}x + \frac{7}{8} > z \geq \frac{1}{2}x + \frac{1}{8} \) since the active investor should have obtained control but in fact only obtains \( q_1^*(x, z) < 1/2 \). The competitive market-clearing price at \( \tau = 2 \) reveals \( x \) and, given the allocation at \( \tau = 1 \), also reveals \( z \). A successful tender offer at \( \tau = 3 \) is feasible whenever

\[ z \geq \frac{x + \frac{1}{2}}{2q_1^*(x, z)} - \frac{1}{2}, \]

which, for the relevant values of \( q_1^*(x, z) \), is equivalent to

\[ z \geq -\frac{1}{2}(x - \frac{1}{4}) + \frac{1}{8}x^2 + \frac{3}{2}x + \frac{7}{16}. \]

Otherwise, the tender offer will not succeed. So the original sale fails to generate an efficient final allocation whenever

\[ \frac{1}{2}x + \frac{1}{8} < z < -\frac{1}{2}(x - \frac{1}{4}) + \frac{1}{8}x^2 + \frac{3}{2}x + \frac{7}{16}. \]

Fig. 2 displays the five regions yielding different histories of allocations to the active investor in this example. In region I the efficient allocation is \( q_1^*(x, z) = 0 \), and this also matches the original allocation \( q_1^*(x, z) = 0 \). In region II the efficient allocation is \( q_1^*(x, z) = \frac{1}{2} \), while the original allocation is \( q_1^*(x, z) = 0 \). The active investor fails to make a successful tender offer so that \( q_1^* = 0 \). In region III the efficient allocation is \( q_1^*(x, z) = \frac{1}{2} \), while the original allocation is \( q_1^*(x, z) \in (0, \frac{1}{2}) \). The active investor cannot make a successful tender offer to obtain control and instead sells shares at the tender price \( p^3 = y + w_0(x) \) so that again \( q_1^* = 0 \). In region IV the efficient allocation is \( q_1^*(x, z) = \frac{1}{2} \), while the original allocation is \( q_1^*(x, z) \in (0, \frac{1}{2}) \). The active investor has a large enough stake to make a successful tender offer, buying shares at \( p^3 = y + w_{1/2}(x) + z \) so that \( q_1^* = \frac{1}{2} \). In region V the efficient allocation is \( q_1^*(x, z) = \frac{1}{2} \), which the active investor succeeds in obtaining in the original sale, \( q_1^*(x, z) = \frac{1}{2} \), so that a tender offer is unnecessary. Regions I and V are those in which the initial sale establishes an efficient allocation. In regions II–IV, the initial allocation is not efficient and an efficient tender market is needed, although only in region IV is one successful. The outcome in region II is like that described in Grossman and Hart (1980): it is not possible for an investor without an initial stake in the firm to make a successful tender offer because of the free rider problem. The outcome in region IV is like that described by Shleifer and Vishny (1986): the investor who begins with
a large enough stake is able to make a credible tender offer. Region III contains those allocations in which an investor has some initial stake but it is not large enough to make a successful tender offer credible.

It is interesting to take note of the equilibrium price for our example in the different regions. In regions I–III, the competitive secondary market price is 
\[ p^2 = p^4 = y + w_0(z). \]
In region III the active investor sells shares in a tender offer at this price, 
\[ p^3 = p^2. \]
In regions IV and V the competitive market price is 
\[ p^2 = p^4 = y + \bar{w}_{1/2}(z) + \gamma. \]
In region IV the active investor purchases the shares necessary for a controlling stake in a tender offer with the price 
\[ p^3 = p^2. \]

Of course, for higher values of \( w_a \) an efficient allocation of the controlling block will easily result. Indeed, large buyers with 
\[ w_a > w_{\text{max}}(z) \]
will always be able to obtain control independent of the number of shares sold in the public offering. Therefore, the problem is only interesting when 
\[ w_a < w_{\text{max}}(z), \]
and the large shareholder’s idiosyncratic component of value is not absolutely bigger than that of the population of small investors.

Having shown that the tender offer market does not always allow a controlling stake to be accumulated, we now turn to the question of how the tender offer market functions in allowing an active investor to sell excess shares received in the initial sale.

Lemma 2. The active investor does not always use the tender offer market to sell excess shares.

Proof. When \( q^1_a > \frac{1}{2} \) the large investor wishes to sell \( q^1_a - \frac{1}{2} \). A tender offer to sell is optimal for the active investor at 
\[ p^3 \geq y + w_a + \gamma. \]
Faced with a tender offer to sell, the marginal passive investor is only willing to buy if 
\[ y + \bar{w}_{1/2} + \mathcal{E}(z|w_a, z \leq p^3 - (y + w_a)) \geq p^3, \]
which can be rewritten as 
\[ \bar{w}_{1/2} + \mathcal{E}(z|w_a, z \leq p^3 - (y + w_a)) \geq p^3 - y. \]
If \( \bar{w}_{1/2} \) is close to zero, then this is impossible. That is, if the passive investors are uncertain about the value of \( z \), then 
\( z - \mathcal{E}(z|w_a, z < p^3 - (y + w_a)) \) could be large enough to violate the necessary conditions for a successful sale of the block to occur. The active investor is frustrated by the familiar adverse selection or lemons problem.\(^1\)

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\(^1\) In our model, the active investor’s private valuation, \( w_a \), is proportional to his or her ownership stake. That may not be the case. The private benefits can flow from control and be independent of the number of shares in excess of the minimum required for control. The proof given above need be only modestly revised. A tender offer to sell excess shares is optimal for the active investor at 
\[ p^3 \geq y + \bar{w}_{1/2} + \mathcal{E}(z|w_a, z < p^3 - y) \geq p^3 \]
which can be rewritten as 
\[ \bar{w}_{1/2} + \mathcal{E}(z|w_a, z < p^3 - y) \geq p^3 - y. \]
If \( \bar{w}_{1/2} \) is close to zero, then this is impossible. That is, if the passive investors are uncertain about the value of \( z \), then 
\( z - \mathcal{E}(z|w_a, z < p^3 - y) \) could be large enough to violate the necessary conditions for a successful sale of the block to occur.
From the above lemmas the tender offer market is doubly disadvantaged in assuring an efficient ownership structure: it neither assures the accumulation of a controlling block where useful, nor assures the liquidation of an inefficiently large block.

**Lemma 3.** Whenever the original allocation to the active investor is efficient, the final allocation of shares is everywhere efficient.

**Proof.** As noted above, the competitive market always assures an efficient allocation of that portion of shares allocated to the passive investors. Here we note that in equilibrium the active investor does not use the tender offer market to trade away from the efficient allocation, regardless of the expectations and information going into the tender offer market. The assumption that the active investor cannot trade anonymously is important for this and previous results. Of course, the more the large shareholder is able to internalize the gains by trading shares prior to being identified, the more closely will the final equilibrium allocation match the efficient allocation regardless of the allocation made in the initial sale. As an example, it can arise that \( q^*_1 = 1/2 \) and that \( p^2 = y + w(x^2) + \epsilon^2(z) < y + w_a + z \), so that the active investor would like to offer to buy additional shares at the prevailing competitive market price. However, to try and do so would alter the expectations of the passive investors so that the necessary tender offer price would be higher than the active investor would be willing to pay. Consequently no trade would take place (see Milgrom and Stokey, 1982).

So although the competitive secondary market assures an efficient allocation among passive investors, the tender offer market does not always assure an efficient allocation of shares to the active investor. Relying upon the secondary market to determine whether a transfer of control will take place, which in turn affects the proceeds from the sale and how the surplus is divided between the seller and the

3. **Optimal design for the sale of shares with asymmetric investors**

The results in the previous section show that the choice of method of sale influences whether a transfer of control will take place, which in turn affects the proceeds from the sale and how the surplus is divided between the seller and the
various interested buyers. In this section we evaluate a wide variety of alternative strategies for going public. One research option is to analyze each alternative individually, solving for the equilibrium secondary market price and for equilibrium investor strategies and then calculating the revenue raised by the firm. Unfortunately, the enormous range of alternatives available for the sale of shares makes this option impractical. The majority of papers on IPOs and staged equity financing resolve this predicament by restricting consideration to a pair of alternative financing strategies and establishing the relative benefits of one over the other. As a result, only a restricted number of financing strategies are considered and the optimal choice in the class is derived. We overcome this limitation by utilizing the methods employed in the mechanism design literature, in particular, the revelation principle. The advantage of using this approach is that it allows us to characterize the maximum revenue that can be raised from any alternative financing strategy, given the informational assumptions of our model. After characterizing the maximum revenue raised from any financing strategy, we can compare and evaluate specific alternatives and identify the optimal strategy. In this section we provide the general characterization, while the next section provides an analysis of specific alternatives.

In brief, the mechanism design approach analyzes the subset of all possible rules for sale that are direct mechanisms, i.e., those in which the strategy space is a simple reporting of each investor’s type, \( w \) (which yields an \( \hat{a} \) and \( \hat{z} \)), and the outcome is an allocation of shares, \([q^1(w, \hat{a}, \hat{z}), q^1_a(w, \hat{a}, \hat{z})]\), and payments to the seller, \([x^1_s(w, \hat{a}, \hat{z}), x^1_a(w, \hat{a}, \hat{z})]\). Without loss of generality, we can restrict attention further to the set of incentive-compatible mechanisms in which it is optimal for each investor to truthfully report his or her type, \( \hat{w} = w \) (and therefore \( \hat{a} = a \)) and \( \hat{z} = z \). This limited focus is admissible since the revelation principle establishes that each strategy for sale from the complete set of available strategies is equivalent in equilibrium to some incentive-compatible direct mechanism. Consequently, the maximum revenue attainable in the set of incentive compatible direct mechanisms is also the maximum revenue attainable in the full set of available strategies for the sale of shares.

Lemmas 4 and 5 describe the maximum revenue that can be raised from the passive and active investors, respectively. Proposition 1 then establishes that the revenue-maximizing strategy for the sale of shares involves a discount to the active investor that is contingent on the amount by which the investor’s control raises the firm’s value. All proofs are contained in the Appendix.

Lemma 4. For any efficient allocation \( q^1 = q^* \), the maximum average revenue received from a small investor, \( X^*_s(w) \), satisfies the following condition:

\[
X^*_s(w) = \int_{\hat{z}} \int_{\hat{a}} p^*(\hat{z}, \hat{a})q^*_s(w, \hat{a}, \hat{z}) \, dH(\hat{a}/w) \, dF(\hat{z}).
\]
The lemma says that the maximum price paid by any small investor is the secondary market price in a fully revealing rational expectations equilibrium with the efficient allocation, \( p^*(z, z) \). It is not possible to price discriminate among small investors with different information: all small investors buying a share pay the same fixed price. Small investors are unable to capture any economic rents on the private information they originally have about the value of the firm. The information possessed by any individual small investor and reflected in his or her demand has zero marginal value, since the demands of the other small investors are sufficient to fully reveal the unknown parameter, determining aggregate demand. In the language of Milgrom and Weber (1982), the information of an individual small investor is completely substitutable. It is important to note that both the inability of the small investors to capture information rents and the inability of the seller to discriminate among small investors are consequences of the future existence of a competitive secondary market in the shares of the firm. The price paid by any small investor is always the value of the share to the marginal small investor.

**Lemma 5.** For any efficient allocation \( q^1 = q^* \), the maximum average revenue received from an active investor, \( X_a^*(z, w_a) \), satisfies the following condition:

\[
X_a^*(z, w_a) = \int_0^z (y + w_a + z) q_a^*(w_a, x, z) \, dH(x)
- \int_0^z \int_{z_{\min}}^z q_a^*(w_a, z, s) \, ds \, dH(z).
\]

The lemma says that the maximum price charged to the active investor can be divided into two parts. The first part is the value of the shares received in the efficient allocation. This would be the price if the seller had full information on the active investor’s valuation and so could extract all of the value of the shares. However, lacking full information on the active investor’s valuation, the seller must set a price that is discounted from the full valuation. The active investor is thereby able to capture a portion of the surplus. The second part of the price is this discount from the full valuation. The discount depends upon the seller’s uncertainty about the active investor’s type, \( z \), relative to the demand by small investors, \( H(x) \), and upon the contingent probability of obtaining the controlling block at various types, \( q_a^*(w_a, x, s) \). This is the usual basis for the division of surplus when the seller has incomplete information about the buyer’s valuation (see Myerson, 1981). If there were multiple active investors competing for the controlling block, then the informational advantage would be partially dissipated and the seller would be able to capture a greater portion of the surplus: i.e., the significance of the second part of the equation would decline.

Both the price and the discount, measured as a percent of the full valuation, increase with the parameter \( z \), the active investor’s contribution to value.
through control. Note that the price also increases with the active investor’s private benefits, \(w_a\). The price charged to the active investor is discounted from the full valuation inclusive of the private benefits. Note also that the price paid by the active investor is contingent on the parameter of aggregate demand, \(\alpha\). Although unable to extract everything from the large buyer’s valuation, the seller is able to increase his or her share of the surplus by making the allocation to the active investor a function of \(\alpha\). The commitment to sell a fraction of the shares to small investors is an effective way of putting pressure on active investors to compete more aggressively.

The optimal method of sale must ensure that the efficient allocation of shares maximizes the expected value of the proceeds from the sale. Lemmas 1–3 imply that \(q^1 = q^*\) and Lemmas 4 and 5 yield the conditions on the payment rule. This establishes the following proposition:

**Proposition 1.** A method of sale is optimal if and only if \(q^1 = q^*\) and \(X^1\) satisfies the conditions of Lemmas 4 and 5.

Lemma 4 does not directly compare the price paid by the active investor with the price paid by the small, passive investors. This is done in the following proposition.

**Proposition 2.** Maximizing revenue from a sale that establishes an efficient ownership structure requires giving the active shareholder a discount whenever the expected external benefits derived from large block ownership are larger than the private benefits from control.

Among the optimal pricing rules satisfying Lemma 4 is the rule in which \(x^*_a(w, \alpha, z) = p(\alpha, z)q^*_a(w, \alpha, z)\). Using Lemma 5, an optimal pricing rule for the active investor is given by \(x^*_a(\alpha, z, w_a) = (p_a(\alpha, z) - \pi(\alpha, z, w_a))q^*_a(w_a, \alpha, z)\), where

\[
\pi(\alpha, z, w_a) = (\tilde{w}_{1/2}(\alpha) - w_a) + \int_{z_{\min}}^{z} q^*_a(w_a, \alpha, s) \, ds.
\]

Whenever \(w_a < \tilde{w}_{1/2}(\alpha) + \int_{z_{\min}}^{z} q^*_a(w_a, \alpha, s) \, ds\), the price paid by the active investor is discounted, and when the inequality is reversed, the price paid contains a premium. The discount or premium, \(\pi\), has a straightforward interpretation in terms of its two components. The first component, \((\tilde{w}_{1/2}(\alpha) - w_a)\), is the typical discount given in a monopolist’s price discrimination problem when the exante expected valuation of two classes of buyers differs. The current owner behaves as a monopolist who discriminates by setting a higher price for that class of buyers with the higher expected valuation. If the active investor’s private benefits from control are significant, then this component is negative and
potentially transforms the discount into a premium. The second component, the one central to this paper, is the discount given specifically due to the public benefit of the active investor’s ownership in the firm. Although the large shareholder receives a discount relative to the price paid by small shareholders, giving this relative discount benefits the seller by raising the expected value of the firm and therefore the average price of the shares.

Proposition 2 also helps us understand the optimal method of sale when there exist private benefits from control (see Barclay and Holderness, 1989). When the private benefits to the large shareholder are expected to be large, i.e., \( w_a > \tilde{w}_{1/2}(z) + \int_{z_a}^{z} q^*(w_a, x, s) \, ds \), then the first component is negative and dominates the second component, so that the large shareholder would actually pay a premium. This seems to be the case analyzed in Brennan and Franks (1995) when they conjecture that discriminatory pricing is strategically used to screen applicants for shares and increase the price for acquiring a block.

In Lemmas 4 and 5 and Propositions 1 and 2 we restrict ourselves to mechanisms that yield efficient allocations and characterize the revenue-maximizing payment rules for this class. It is a well known and general result that in a sale to buyers with private information, introducing the right kind of inefficiency in the allocation can allow a seller to raise the revenue extracted. In a few cases when it is efficient to allocate shares to an active investor who has a low valuation, the seller may be better off by denying the active investor the controlling block. This is because reducing the allocation to active investors with low valuations raises the price the seller can charge to active investors with high valuations. Bebchuk and Zingales (1995) illustrate this possibility in another model of the sale of shares. In a model with private information and a secondary and tender offer market, the problem is more complicated because the seller’s allocation in the initial sale is not the final allocation. The seller must determine the right amount of inefficiency to induce in the final allocation as well as the allocation in the initial sale that induces the right allocation in the final sale. While these considerations can modify the allocation that is optimal from the seller’s point of view, the tradeoffs in pricing the shares sold to different classes of investors documented in the propositions above remain.

4. An evaluation of alternative strategies for going public

The previous section develops a framework for analyzing methods for going public. In this section we apply this framework to evaluate and compare the methods that have been most used in countries with developed capital markets.

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2 There exist optimal payment rules which, for a particular realization of the parameters \( z \) and \( z \), have the active investor paying more per share.
The best strategy is to design a method of sale that satisfies a number of conditions: (i) investors differ in their size and role in the firm; (ii) value is contributed by active investors; (iii) all investors are in competition with one another; and (iv) trading opportunities are provided by the opening of a secondary market in the shares of the firm. The third condition arises out of the desire of the seller to extract surplus from active investors potentially interested in obtaining control. This condition is of particular importance, for example, in setting the price of the initial public offering. Indeed, the inability of the seller to discriminate among small investors makes an IPO consisting of just a portion of the shares of the firm and sold at a fixed price an optimal choice. This establishes a market in the firm’s shares and a public price that reflects the information and demands of the small investors. Subsequently, the firm markets a controlling block to an interested active investor. The firm negotiates a price for the block that is based upon the market price, but with a discount that reflects the perceived contribution to value by the large investor. If the active investor does not purchase the block, then the firm organizes a seasoned public offer to sell the shares to the passive shareholders at a new market-clearing price. This result is formalized as follows:

**Corollary 1.** A sequential sale beginning with an initial public offering of dispersed shares, followed first by a negotiated sale of a controlling block and then by a seasoned offering, is optimal:

(i) with a partial sale of $q = \frac{1}{2}$ unit at a rational expectations equilibrium or clearing price equal to

$$p^1 = y + \tilde{w}_0(z) F(\tilde{z}(x)) + (\tilde{w}_{1/2}(x) + \delta' z|x, a, z \geq \tilde{z}(x))(1 - F(\tilde{z}(x)))$$

(ii) with the seller making a take-it-or-leave-it offer to the active investor at $\tau = 3$ contingent on the revelation of $z$ from this initial public offer and the secondary market price at $\tau = 2$ and at the price

$$p^3 = y + w_a + \tilde{z}(x),$$

(iii) and, should the active investor reject the offer, with the seller then making a seasoned public offering at $\tau = 4$ and at the secondary market price

$$p^4 = y + \tilde{w}_0(x).$$

This view that the initial public offering of shares is not an isolated step but part of a more elaborate process for selling shares seems to be confirmed by the existing empirical evidence. Barry et al. (1990) report that IPOs represent the most frequently used method of selling a fraction of the shares by venture capital firms and that during this first transaction the equity holdings of venture capital investors do not change much. Later on these investors sell a significant portion of their stakes, either to another investor or to another company or through a follow-on offering. Control turnover subsequent to the IPO is also found by
Holderness and Sheehan (1988) for the U.S., Rydqvist and Högholm (1994) for Sweden and the U.K., and Pagano et al. (1996) for Italy. This also seems to be the case in many equity carve-outs, according to Schipper and Smith (1986) and Klein et al. (1991). The findings on the transfer of control raise certain doubts about Barry et al.’s interpretation of the decision not to sell during the IPO as reflecting the venture capitalists’ willingness to bind themselves to the value of new issues by maintaining their holdings beyond the IPO. Our view is that an immediate exit strategy may not be optimal, so that temporarily retaining a stake can tell small investors participating in the IPO that a block is reserved for a possible future sale. Also important, by selling the block afterwards, the owner can make the price of the block dependent on the conditions prevailing in the market for dispersed shares.

Partial sales have been advocated by other authors studying IPOs, for a different reason. This branch of the literature highlights the opportunity to signal the quality of the seller, in contrast to the signal on the demand described here.\(^3\) We believe that both supply and demand aspects are important to a better understanding of the pricing and method for selling equity stakes, especially in IPOs.\(^4\)

The sequential sales suggested above can be contrasted, for example, to a public offering of all shares at a single fixed price. In the offer for sale by tender, employed both in the U.K. and in France, investors place bids for the shares indicating both quantity and price. After the bids have been received, a single sale price is set and all buyers pay that price. It is possible to show, however, that issuing all shares at a single price is not an optimal strategy.

**Corollary 2. A public offering of all shares at a uniform price is not optimal.**

This result is a direct implication of Proposition 2. The active investor must receive a discount that reflects the expected contribution of block ownership to the firm’s value. The discount gives the large investor an incentive to bid


\(^4\) There may be some additional advantages to another sort of partial sale in which the size of the controlling block is contingent on the bidding of the active investor or investors due to the additional surplus that can be extracted when the investor’s valuation is ex post observable through the realized income of the firm. For example, in a privatization it may be possible for the seller to extract a larger amount of surplus by reserving for the state a portion of the non-controlling shares and having the size of this reserve determined by the active investor bidding for control. Similarly, in acquisitions payment can be in cash or various securities, and an auction in terms of shares can possibly extract greater surplus from bidders than an auction in cash payments (see Riley, 1988; Hansen, 1985).
competitively for the controlling block. Without this the resulting proceeds from the sale are not maximized.

Consider now a sale in which investors tender bids and those bidding the highest pay the price bid, i.e., the traditional sealed-bid, discriminatory auction. Discrimination can be accomplished in a number of ways. Even in the U.S., for example, where shares in an IPO must be offered at a uniform price, there is evidence that discrimination among buyers occurs through the allocation of oversubscribed issues and the expectation that buyers will sometimes accept allotments of undersubscribed issues (see Hanley and Wilhelm, 1995). Brennan and Franks (1995) also present evidence of discrimination in the U.K. market. Although it is not possible to evaluate specific discriminatory practices without more detailed information on the allocation rules, we find that the standard discriminatory sale is not optimal.

**Corollary 3.** A discriminatory auction in which each bidder pays the amount bid is not optimal.

Consider next a sequential sale in which a controlling block is first sold to the active investor, after which there follows a public offering to small investors. At times this method has been used by some private companies, as well as by governments in privatizations. In mixed offers, as they are also called, a tender offer is first made to large shareholders and the resulting tender price is then used to set the fixed price for the sale to small investors of the remaining shares. Besides violating the optimality condition that prescribes some form of contingent discriminatory rule, the mixed offer does not follow the right sequence, as shown next.

**Corollary 4.** Selling the controlling block before the IPO is, in general, not optimal.

Selling the controlling block first avoids the free rider problem (see Grossman and Hart, 1980), and it can also help reduce the winner’s curse problem faced by small investors (see Rock, 1986). However, by selling the controlling block first, the active investor’s allocation cannot be contingent on the parameter of aggregate demand by small investors, \( z \). Furthermore, if the active investor is assured a controlling block, then it is impossible to extract the maximum revenue from most types of active investors. Since large investors with low valuations are assured shares, there is no leverage with which to force a higher price from those with higher valuations. As a result, conducting the sale sequentially assures them an even greater discount than the optimal method. However, in the special case in which \( w_a \geq w_{\max}(z_{\max}) \), it is obvious that the block should go to the active investor and negotiating this sale first is not disadvantageous.
Corollary 4 shows that it is not optimal to sell shares using a mixed offer in which a tender offer is made first to large investors and the tender price is then used to set the fixed price in the offer for sale to small investors. This conclusion is not shared by Stoughton and Zechner (1998), who advocate a mixed offer as their main prescription. In their model there is no uncertainty about the demand of small investors, and so there is no loss from selling to the active investor prior to learning about aggregate demand and estimates of value. This result highlights that simply selling a fraction of the company in an IPO is not always an optimal mechanism, but with the addition of some clauses and by following the right sequence of offerings the seller can adjust the ownership structure to obtain a higher sale price.

Selling some amount of non-controlling shares allows the seller to obtain information about the aggregate demand of small investors on which the price of the controlling block can be made contingent. Although the sale occurs in stages and the active investor never directly bids against the small investors, competition between them arises through the conditions imposed on the sale to the large investor based on the results of the first sale to small investors. If the model included more than one potential large shareholder, then it would be necessary to run an auction or contest among them. However, the advantage of selling some shares to the public ahead of time remains. We should note here that the expression for $X^*$ derived in Lemma 2 would be slightly more involved with multiple active investors, but in form it would remain very much the same.

The same result can potentially be achieved in some single-period offerings by making the allocations to different classes of investors contingent on the orders received.

**Corollary 5.** There is an optimal single-period discriminatory pricing rule. The large investor gets control only with a bid that is higher than the average bid of the marginal small investors competing for the shares of the controlling block. Upon the successful sale of the block, small investors pay a uniform price equal to the marginal small investor’s bid for one-half of the shares plus a control premium. The large investor receives a reduction from the price paid by small investors based upon the public benefits of control and pays a premium over the price paid by small investors based upon the private benefits of control. If the large investor does not obtain control, all shares are sold to the small investors at the clearing bid.

Perhaps because of the elaborate procedures it involves, a simultaneous sale of shares with a discriminatory allocation of a potentially controlling block is not very common. However, Brennan and Franks (1995) refer to a method that has recently been experimented with in the U.K. and Australia and has features similar to those just described. This method combines a private placement, targeting large investors, with a simultaneous public offering. It could be an optimal method depending on the allocation and pricing rules used for investors.
of different types. It is worth noting that whenever it has been used, this method usually includes a clawback provision. According to our model, perhaps this is done intentionally, because the clawback provision makes the allocation to large investors dependent on the demand by small investors, therefore creating competition among investors of different types.

Although we have focused only on comparing various methods of sale, there are other mechanisms that could be used to help reduce the uncertainty surrounding both the demand and the valuation of different investors. For example, if it were possible to open a when-issued market in the new stock, then the price in this market would provide crucial information to the seller that would make it possible to extract the maximum surplus given the need to assure a successful sale. Another alternative would be to ask investors to submit indications of interest, just as they do under ‘book-building’, knowing that some bidders would be eliminated from the subsequent sale. Although the NASD Rules of Fair Practice require that all investors pay a uniform price in the offering, the seller can always discriminate in the allocation of shares. Alternatively, and recognizing that a sale of control should not be carried out through the IPO, the seller can use a simultaneous public and private offer, as in Corollary 5, or follow the sequence in Corollary 1 and choose an allocation in the IPO that is influenced by the prospect of a future sale of the controlling block. In both cases, however, the book-building effort will provide the seller with information about the aggregate demand of passive investors on which the price of the controlling block can be made contingent.

5. Other determinants of the ownership structure

This paper indicates that the design of the sale increases the value of the firm. It emphasizes that the choice of method of sale is important because the capital market does not establish an optimal ownership structure for the firm. The idea that the process of going public cannot be left to the capital market to achieve an efficient outcome goes back to Berle and Means (1932) and has also been recently analyzed, among others, by Kahan (1993), Bebchuk (1994), and Bebchuk and Zingales (1995). Bebchuk discusses how voting arrangements and freeze-out schemes improve the transfer of ownership, while Bebchuk and Zingales discuss deviations from one vote per share that allow the seller to obtain greater revenue. Given that control considerations are also the focus of these papers, it is important to see how the ideas proposed in our model relate to the legal arrangements suggested in this literature.

Bebchuk evaluates two rules governing sale of control transactions: the market rule, followed in the U.S., and the equal opportunity rule, followed in many European countries and to be adopted by the European Union. In the market rule (MR), a control block can be transferred without the participation
of minority shareholders. Under the equal opportunity rule (EOR), minority shareholders are entitled to participate in the transaction on the same terms as the control seller. Bebchuk shows that when the buyers’ private benefits are significant, the MR might not avoid inefficient transfers, while the EOR prevents all inefficient transfers. However, the EOR fares worse in facilitating efficient transfers. As Bebchuk points out, neither of the rules dominates the other in all instances. But how about a mixture of the two rules? From the previous section, it is easy to see that a suitable combination of the two rules is what is implicit in the optimal method of sale proposed in this paper. First, all investors are entitled to participate in the primary market, so in that sense the first stage of the ownership transfer is equivalent to the EOR. Recognizing, however, that the EOR might discourage efficient transfers, the terms of the sale are set in a way that favors large shareholders who contribute to the firm. The discriminating clause, which depends on the characteristics of the large shareholder, appropriately modifies the rules governing the transfer of the controlling block and, in so doing, retains the good features of the MR.

This conclusion is also clear from Bebchuk’s discussion of legal arrangements that improve the outcomes of the MR and the EOR, such as voting arrangements and freeze-out clauses. In order to facilitate efficient transfers, Bebchuk claims that voting arrangements would have to be strengthened to enable small shareholders to approve a payment to the large investor. In the same vein, he contends that a freeze-out prior to the sale of the block would move the transfer of control close to the first best, although this would deny small shareholders the gain from selling their shares at the market price. Again, the legal arrangements suggested by Bebchuk are simply alternative forms of benefiting the large shareholder, the outcome of our Proposition 2. Although they have merit in theory, it is questionable whether in practice a sale of shares organized without recognizing that investors are different, combined with strengthened voting and freeze-out provisions, if allowed, would be easier to implement. Most probably, asymmetric information and problems of moral hazard would make it difficult to determine the right compensation involved. The advantage of making the allocation and the price of the controlling block contingent on demands by small investors is that it helps to determine the compensation to be attributed to the large shareholder with greater transparency and without costly haggling between shareholders.

So far we have assumed that the firm issues equity with corresponding voting power. However, given that the market for control cannot always establish an efficient ownership structure, it is important to consider the possibility of issuing dual class shares, or equity with differential voting rights. More specifically, the seller could allocate a disproportional voting power to the active shareholder, while selling shares that give the right to future cash flows to passive investors. This would, in principle, seem a revenue-maximizing solution, since it would apparently minimize the number of shares that would have to be offered at
a discount to a potential controller. In the context of our model, however, all the value-enhancing activities performed by the large shareholder are reflected in higher cash flows, not in private benefits of control. Thus, in the limiting case in which all cash flow rights would be sold to small shareholders, the large shareholder would gain nothing from contributing to the firm’s value and there would thus be no reason to bid for control. Without the ability to attract a control buyer, the seller would not expect to capture any surplus from selling cash flow rights to passive shareholders, who would not value the firm as highly as if it were under the control of a large shareholder. Of course, with a disproportionately large fraction of the votes the controller could always try later on to dilute the rights of small shareholders. But if this type of action were admissible, rational small shareholders would discount the negative effects in the price of the shares. Thus, there seems to be no obvious way for the seller to profit from deviating from the rule of one vote per share.

The discussion above seems to point out that the discount per share offered to the control buyer presumably depends on the size of the allocated block, $q^1_a$. A larger block requires a smaller discount per share and a smaller block requires a larger discount per share, so that the total discount, $pq^1_a$, is always approximately the same amount.

It is interesting to contrast these results with those in Bebchuk and Zingales (1995), who advocate a deviation from the rule of one vote per share as a way to increase the expected revenue of the seller. Their suggestion, however, relies on the complete separation of cash flow rights and private rights to control. Even if the seller disperses cash flow rights so as to extract more surplus from the large investor, the large investor will still be interested in obtaining control to realize his or her private benefits. Interestingly, in their model the private benefits of control are assumed to increase with the fraction of the cash flow rights sold to small shareholders, and therefore include an implicit form of dilution that makes these benefits even more attractive to a controller. Whether the valuation of the company to a large shareholder essentially comes from the private benefits of control or from improved future expected cash flows is an empirical question that is still to be resolved. What our results, on the one hand, and Bebchuk and Zingales’s, on the other hand, seem to imply is that when private benefits of control are significant and voting rights can be isolated from cash flow rights it may be best for the seller to deviate from one vote per share, but not otherwise.

6. Conclusions

It is clear that ownership structure matters for the value of a corporation and its future performance. We address how different methods for the sale of shares fare in establishing the appropriate ownership and maximizing revenue. Our results are an important contribution to the ongoing debate over the
importance of treating controlling blocks distinctively in selling a firm. Does it matter that the sale disperses the shares when interested parties can trade in a secondary market that includes a tender offer market? Or is it better to pass on a block to someone who wants a controlling stake? And if this alternative is advantageous, then how should the firm design a sale of shares to maximize expected revenue?

Because large shareholders can provide the public good associated with monitoring activity, it is always better to make sure that they participate in the sale, and if this depends on their chances of getting control, the sale should be designed to benefit them. The seller can always recover part of the added value of having a large investor through the higher bids posted by small investors who profit from the monitoring activities of the large investor. But guaranteeing the large shareholder a controlling stake would eliminate the competitive pressure to bid aggressively. Therefore, it is crucial that the method of sale promote the participation of potential large shareholders and at the same time make their allocations and payments contingent on the demands of the small investors. This is necessary because in many instances the large shareholder will be unable to assemble a controlling block later in the secondary market, due to the free rider problem. But an active secondary market also prevents the seller from extracting higher payments that would make investors turn to this market, instead of buying the shares in the original sale.

We provide an analysis of the problem and show that commonly used methods of sale are in general not optimal. We characterize various optimal selling strategies, which have features of some existing methods. We are able to explain why some privately held firms go public despite the apparent value loss associated with the free rider problem. Viewing the IPO as a step in a more complete process of selling the firm is the result of considering the inherent asymmetry of investors together with the strategic behavior on the part of the seller. By taking into account the fact that firms manage the sale of shares with the purpose of discriminating between small investors and applicants for large blocks, it is possible to improve our understanding of the pricing and method of selling companies.

Appendix A. Proofs

Proof of Lemma 4: The utility of a small investor of type \( w \) and who reports that type to be \( \hat{w} \), given a mechanism with allocation rule \( q^* \) and payment rule \( x \), is given by

\[
U_d(w, \hat{w}|q^*, x) = \int_{x} \int_{z} v(w, z, q^*_d(\hat{w}, z, zd) - x_d(\hat{w}, z, zd) \ dF(z) \ dH(z|w).
\]
Notice that the investor's valuation, $v$, is a function of the investor's actual valuation and not of the reported valuation, while the investor's allocation and payment, $q_s^*$ and $x_s$, are a function of the investor's reported valuation and not the actual valuation. Notice also that the seller is always able to infer correctly $z$, regardless of this investor's report. The population of small investors is atomless and the distribution of reports by the other investors is sufficient to identify $z$. The report of a single small investor is immaterial to this inference. This fact is central to the results of the model – in the competitive market rational expectations equilibrium as well as here in the incentive compatibility design – since it effectively determines that the small investor’s information has no market value and does not earn the small investor any return.

The set of truthful direct revelation mechanisms with allocation $q^*$ is defined by three constraints on the payment $x_s$ extracted from the small investors. First, the mechanism must be incentive compatible, i.e., the small investor’s utility is maximized with a truthful report. Second, the mechanism must be individually rational, i.e., the small investor’s utility must be at least as great as if he or she simply withdrew and did not participate in the sale of shares. Third, the mechanism must be dynamically rational, i.e., the small investor’s utility must be at least as great as if the shares were purchased instead in the secondary market. We show that setting the payment equal to the maximum allowed under the third constraint yields the expected revenue shown in the statement of the lemma, and also satisfies the other constraints.

The third constraint is written

$$\forall w \quad U_s(w, w|q, x) \geq \int \int (v(w, z, q_s^*) - p^*(z, z))q_s^*(w, z, z) \, dF(z) \, dH(x|w),$$

which can be rearranged to yield

$$\int \int p^*(z, z)q_s^*(w, z, z) \, dF(z) \, dH(x|w) \geq \int \int x_s(\hat{w}, z, z) \, dF(z) \, dH(x|w).$$

The left-hand side is the upper bound which appears in the statement of the lemma, $X_s^*(w)$. Clearly, setting $x_s(\hat{w}, z, z) = p^*(\hat{w}, z, z)q_s^*(\hat{w}, z, z)$ satisfies the constraint with equality. It also clearly satisfies the individual rationality constraint, $U_s(w, w|q^*, x) \geq 0$. It remains to be shown that it satisfies the incentive-compatibility constraint:

$$\forall w \text{ and } \forall \hat{w} \neq w \quad U_s(w, w|q^*, x) \geq U_s(w, \hat{w} \mid q^*, x).$$
The two sides of the inequality can be expanded as follows:

\[
\int \int_{a \leq z} v(w, z, q_a^*) q_a^*(w, \alpha, z) - x_\delta(w, \alpha, z) \, dF(z) \, dH(z|w) \\
\geq \int \int_{a \leq z} v(w, z, q_a^*) q_a^*(\hat{w}, \alpha, z) - x_\delta(\hat{w}, \alpha, z) \, dF(z) \, dH(z|w).
\]

which, upon rewriting \(x_\delta(\hat{w}, \alpha, z) = p^*(\alpha, z) q_a^*(\hat{w}, \alpha, z)\), become

\[
\int \int_{a \leq z} v(w, z, q_a^*) q_a^*(w, \alpha, z) - p^*(\alpha, z) q_a^*(w, \alpha, z) \, dF(z) \, dH(z|w) \\
\geq \int \int_{a \leq z} v(w, z, q_a^*) q_a^*(\hat{w}, \alpha, z) - p^*(\alpha, z) q_a^*(\hat{w}, \alpha, z) \, dF(z) \, dH(z|w).
\]

Rearranging, we have

\[
\int \int_{a \leq z} (v(w, z, q_a^*) - p^*(\alpha, z))(q_a^*(w, \alpha, z) - q_a^*(\hat{w}, \alpha, z)) \, dF(z) \, dH(z|w) \geq 0.
\]

Clearly, it is sufficient to show that \(\forall \alpha\)

\[
\int_{a \leq z} (v(w, z, q_a^*) - p^*(\alpha, z))(q_a^*(w, \alpha, z) - q_a^*(\hat{w}, \alpha, z)) \, dF(z) \geq 0.
\]

Conducting the integration over two discrete regions, the condition is again rewritten as follows:

\[
\int_{z < z(\alpha)} (v(w, z, q_a^*) - p^*(\alpha, z))(q_a^*(w, \alpha, z) - q_a^*(\hat{w}, \alpha, z)) \, dF(z) \\
+ \int_{z \geq z(\alpha)} (v(w, z, q_a^*) - p^*(\alpha, z))(q_a^*(w, \alpha, z) - q_a^*(\hat{w}, \alpha, z)) \, dF(z) \geq 0.
\]
Solving for \( v - p^* \) within each region of integration yields

\[

to z(x) = \int_{z < z(x)} (w - \bar{w}_0(z)) \, dF(z) (q^*_a(w, x, z) - q^*_a(\hat{w}, x, z)) \, z < z(x)
\]

\[
+ \int_{z \geq z(x)} (w - \bar{w}_{1/2}(z) + z - \delta(z|z \geq z(x)) \, dF(z) (q^*_a(w, x, z) - q^*_a(\hat{w}, x, z)) \, z \geq z(x) \geq 0.
\]

And finally,

\[
\int_{z < z(x)} (w - \bar{w}_0(z)) \, dF(z) (q^*_a(w, x, z) - q^*_a(\hat{w}, x, z)) \, z < z(x)
\]

\[
+ \int_{z \geq z(x)} (w - \bar{w}_{1/2}(z)) \, dF(z) (q^*_a(w, x, z) - q^*_a(\hat{w}, x, z)) \, z \geq z(x) \geq 0.
\]

To see that this condition is satisfied, consider for example \( w < (\bar{w}_0(z), \bar{w}_{1/2}(z)) \). The allocation given a truthful report is \( q^*_a(w, x, z) = 1 \) when \( z < \bar{z}(x) \) and \( q^*_a(w, x, z) = \frac{1}{2} \) when \( z \geq \bar{z}(x) \). A report of \( \hat{w} > \bar{w}_{1/2}(z) \) would increase the allocation whenever \( z \geq \bar{z}(x) \), but this would only lower utility since in those events \( w < \bar{w}_{1/2}(z) \). □

**Proof of Lemma 5.** The incentive compatibility of a mechanism \([q^*, x]\) for the active investor requires that \( \forall z \) and \( \forall \bar{z} \),

\[
U_a(z, z|q^*, x) \geq U_a(z, \bar{z}|q^*, x).
\]

Having a control component of an investor’s allocation is, in the setting of Myerson (1981), like having a revision function where the \( n \)th player is the active investor. Unlike in Myerson, this component of the valuation is allocation contingent. By the same steps found in Myerson’s Lemma 2, then, incentive compatibility requires

\[
U_a(z, z|q^*, x) = U_a(z_{min}, z_{min}|q^*, x) + \int_{z_{min}}^z q^*_a(w, x, s) \, dH(x) \, ds.
\]

Expanding the left-hand side and rearranging yields

\[
\int_{z} x_d(x, z) \, dH(x) = \int_{z} \left( (y + w_d + z)q_a(x, z) - \int_{z_{min}}^z q^*_a(x, s) \, ds \right) \, dH(x)
\]

\[- U_a(z_{min}, z_{min}|q^*, x) .
\]
The last equality says that given an allocation rule, \( q^*_s \), the average payment for any type other than the lowest type, \( x_a(z + w_a \neq z_{\min} + w_a) \), is completely determined by the utility afforded to the lowest type and the incentive-compatibility constraints. Since the average payment for any type of active investor increases with the payment of the lowest type, the revenue-maximizing rule sets this at the largest possible amount as determined by the individual rationality constraint for the lowest-type active investor, i.e., so that \( U_a(z_{\min}, z_{\min}q^*, x) = 0 \). Then, the average payment made by any type of active investor is as given in the statement of the lemma. □

**Proof of Proposition 2.** As mentioned in the discussion of Lemma 4, the revenue-maximizing payment function \( X^*_s(w) \) is equivalent to charging the single price \( p^*(z, z) \) to all small investors. The revenue-maximizing payment function for the active investor \( X^*_a(z, w_a) \) can be decomposed into the price charged the small investors, \( p^*(z, z) \), and a discount \( \bar{\pi}(z, z, w_a) \), where \( \bar{\pi}(z, z, w_a) = (\bar{w}_1(z) - w_a) + \int_{z_{\min}}^{z} q^*_a(w_a, z, s) \, ds \). The price paid by the active investor is discounted if \( \bar{\pi}(z, z, w_a) > 0 \). By Lemmas 4 and 5, any other pricing rule that maximizes revenue is equivalent in expected revenue to this pricing rule and so must yield in expectation an equivalent contingent discount to the active investor. □

**Proof of Corollary 3.** For the equilibrium bidding functions of the active investor, \( b_a(z, w_a) \), and of the passive investors, \( b_s(w) \), to yield \( X^*_a(z, w_a) \) and \( X^*_s(w) \) as each investor’s average payment, the bidding functions would have to satisfy \( b_a(z) = X^*_a(z, w_a)/Q^*_a(z) \) and \( b_s(z) = X^*_s(w)/Q^*_s(w) \), where \( Q^*_a(z) = \int_{z_{\min}}^{z} q^*_a(z, z) \, dH(z) \) and \( Q^*_s(w) = \int_{s}^{w} q^*_s(z, z) \, dF(z) \, dH(z|w) \). And for the equilibrium bidding functions to guarantee the efficient allocation, it would have to be the case that for every \( z \), \( b_a(\bar{z}(z), w_a) \geq b_s(\bar{w}(z)) \). Together these conditions require that for every \( z \) it must be the case that

\[
\frac{X^*_a(\bar{z}(z), w_a)}{Q^*_a(\bar{z}(z))} \geq \frac{X^*_s(\bar{w}(z))}{Q^*_s(\bar{w}(z))}.
\]

This cannot always hold: for example, for \( z = z_{\min} \) and setting \( w_a < w_{\min}(z_{\min}) \) and \( z_{\min} = 0 \) we have

\[
\frac{X^*_a(\bar{z}(z), w_a)}{Q^*_a(\bar{z}(z))} = y + w_a \leq y + w_{\min}(z_{\min}) < \frac{X^*_s(\bar{w}(z))}{Q^*_s(\bar{w}(z))}
\]

which shows that a standard discriminatory auction is generally not optimal. □

**Proof of Corollary 5.** Denote the active investor’s bid for the controlling block by \( b_a(z, w_a) \) and a small investor’s bid for a share by \( b_s = b(w) \). Also, denote by
\( B = y + \int_{b_0}^{b_1} 2b \, dG(b) \) the average bid of the marginal small investors competing for the controlling block, i.e., the average of the lower half of the bids, with \( b_0 \) as the marginal bid when small investors are allocated all of the shares, \( b_0 = b(w_0) \), and \( b_{1/2} \) as the marginal bid when small investors are allocated half of the shares, \( b_{1/2} = b(\overline{w}_{1/2}) \). Then, the conditions restated are the following:

(i) bids are placed simultaneously;

(ii) allocate one-half unit to the large investor and one-half unit to small investors whenever \( b_s \geq B \); charge the small investors \( \overline{b}_s + b_a \), and charge the large investor \( \overline{b}_s + b_a - \pi \), where \( \pi = \overline{b}_s - B + b_a - w_a \);

(iii) allocate the full unit to the small investors whenever \( b_s < B \) and charge \( b_0 \).

To confirm that this method is optimal, note that in equilibrium each investor bids his or her type, \( b_s(z, w_a) = z + w_a + y \) and \( b_a(w) = w + y \), and that the allocation and pricing rules therefore yield \( q^* \) and \( x^* \). In this sale all small investors pay a uniform price equal to the marginal small investor’s bid plus a control premium when a block is successfully sold, while the large investor pays a discount, \( \pi \) (always positive), from that same price. Note that the marginal small investor pays a price higher than originally bid, though ex post the higher price is acceptable because it is also the equilibrium price in the secondary market. \( \square \)

References


