Electronic Supplement

Appendix A. Implementation of the proposed pricing mechanism

To implement a dynamic real-time pricing mechanism, all consumers should be exposed to time-varying prices associated with ex ante estimates of generation costs that reflect system operating conditions (p. 81 of Borenstein et al. (2002)) so that they can adjust their demand in accordance to real-time prices as well as ex ante price estimates. The mechanism proposed in this paper is not an exception. The ex ante estimates of real-time prices can be developed by evaluating statistical relationships between historical real-time prices and various factors such as load forecast, weather predictions, and expected supply/demand balances (Borenstein et al., 2002).

We now provide a brief discussion of the details of a possible implementation of the proposed pricing mechanism:

Ex ante price estimates. Suppose that the exogenous state s_t is realized at the beginning of each stage t; for every possible realization of the trajectory (scenario) of future exogenous states $\{s_\tau\}_{\tau=t+1}^{t+T}$, consumers receive corresponding price estimates $\{\hat{p}_\tau\}_{\tau=t}^{t+T}$, $\{\hat{w}_\tau\}_{\tau=t}^{t+T}$, and $\{\hat{q}_\tau\}_{\tau=t}^{t+T}$, from utilities or an independent system operator. The consumers also know or receive the probabilities of the different trajectories. With the received price estimates (associated with possible trajectories of future exogenous states) and preset utility functions, each consumer's infrastructure solves a dynamic programming problem to maximize her expected payoff over the horizon from t to $t+T$. (The state at time τ in this dynamic program is comprised of $y_{i,\tau}$, and the history $(s_t, s_{t+1}, \ldots, s_\tau$.) The dimension of this state space grows with the time horizon T (because of the exponentially increasing number of histories). While this is unavoidable for models of this type, tractable approximations are possible, e.g. using a bounded length window, in the spirit of Weintraub et al. (2010).

Ex post prices. At each stage t, after the realization of the system demands A_{t-1} and A_t, consumers pay ex post prices (p_t, w_t, q_t) that are determined according to Eqs. (11) and (12).

Equilibrium. In a market with a large number of price-taking consumers, it is possible to make ex ante price estimates (contingent on the realized trajectories) that are close to ex post prices. If every consumer

> maximizes her own payoff in response to these pretty accurate price estimates, the resulting outcome should be close to that resulting from a Rational Expectations Equilibrium (REE). The results derived in this paper show that the expected social welfare will be approximately maximized, under the proposed mechanism.

We emphasize here that there remain several challenging implementation issues, e.g., the accuracy of future price estimates and the uncertainty of consumer response to ex ante price estimates. For example, Roozbehani et al. (2010) show that if consumers act myopically to highly inaccurate price estimates, real-time pricing may result in extreme price volatility. However, we note that these challenges are generic to almost all kinds of real-time pricing mechanisms.

Appendix B. Approximation of the supplier cost

In this appendix, we show via simulation that at least in some cases, the supplier cost (including the cost of ancillary service) can be captured by a simplified cost function of the form in (5). We consider a $(T+1)$-stage dynamic model with two energy resources, a primary energy resource and an ancillary energy resource. We assume that the forecast demand is met by the primary energy resource (e.g., coal-fired or nuclear power generators), and that at stage $t = 1, \ldots, T$, the deviations from the forecast demand, $\{w_t\}_{t=1}^{T}$, are independent random variables uniformly distributed on $[-\omega, \omega]$.[16] At the initial stage 0, we assume that the forecast error is zero, i.e., $w_0 = 0$.

We assume only two types of generating units, namely, that the primary energy resource is provided by base-load plants (e.g., coal and nuclear facilities), and that the ancillary energy resource is provided by intermediate peaking plants (e.g., oil/gas combustion turbines). We will assume that the primary energy resource has lower ramping rate and lower marginal cost than the ancillary energy resource.

Indeed, this simplified setting with two types of generating resources ignores many practical constraints in power systems. We note, however, that this simple model retains key aspects of power systems, and is therefore used

[16] This assumption may not hold in practical power systems, and is made to simplify our presentation and analysis. We note, however, that the performance of our supplier cost approximation is not sensitive to the distribution of the demand forecast error.

in previous works focusing on dynamic analysis of electricity markets (Murphy and Smeers, 2005; Baldick et al., 2006; Wu and Kapuscinski, 2013). We finally note that under more practical supplier models, the total supplier cost at stage t may depend not only on A_t and A_{t-1}, but on a finite window of the history, $A_t, \ldots, A_{t-\tau}$. In this case, through a similar approach we can show the asymptotic social optimality of a pricing mechanism that takes into account the ancillary cost associated with a consumer's action at **previous stages**.

At stage t, let b_t denote the difference between the actual output of the primary energy resource and the forecast demand, and let d_t denote the output of the ancillary energy resource. For simplicity, we will assume that the cost of a positive primary energy resource (respectively, ancillary energy resource) is b_t^2 (respectively, $10d_t^2$).

Let r_b be the ramping rate of the primary energy resource, and r_d be the ramping rate of the ancillary energy resource. At the initial stage 0, we assume that $b_0 = w_0 = 0$, and $d_0 = 0$. At stage $t \geq 1$, if $w_t < 0$, then $d_t = 0$, and we assume that $b_t = 0$, that is, the system operator maintains a high level of (potential) output in order to be able to deal with a possible unexpected demand surge in the future; if $w_t > 0$, we assume that $b_t = \min\{w_t, b_{t-1}+r_b\}$, where $b_{t-1}+r_b$ is the maximum possible output of the primary energy resource at stage t, and that $d_t = \min\{w_t - b_t, d_{t-1} + r_d\}$. The total supplier cost (excluding the cost to meet the forecast demand) is

$$C = \sum_{t=1}^{T} \left(b_t^2 + 10d_t^2\right). \tag{B.1}$$

For notational convenience, we let $(\cdot)^+ = \max\{\cdot, 0\}$. We use the following function to approximate the supplier cost:

$$\widetilde{C} = \sum_{t=1}^{T} \left(\widetilde{b}_t^2 + 10\widetilde{d}_t^2\right), \tag{B.2}$$

where $\widetilde{d}_t = \min\left\{r_d, \left(0, w_t - (w_{t-1})^+ - r_b\right)^+\right\}$, and $\widetilde{b}_t = (w_t - \widetilde{d}_t)^+$.

The function in (B.2) well approximates the supplier cost in (B.1), if for an unexpected demand surge at stage t, the system load at the previous stage, w_{t-1}, is met by the primary energy resource, and load shedding rarely occurs (so that $(w_t)^+$ typically equals $b_t + d_t$). Note that in (B.2), for each

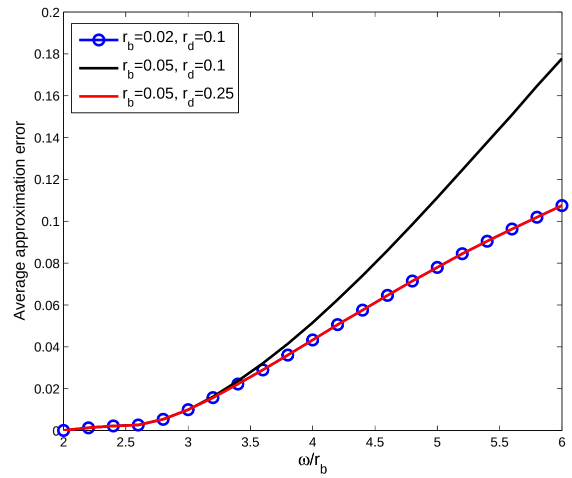

Figure B.2: A simulation experiment with $T = 24$, and $500,000$ trajectories for each ω/r_b on the horizontal axis. The approximation error is defined by $|C - \widetilde{C}|/C$. The average approximation error (vertical axis) is the mean value of the approximation errors of the $500,000$ trajectories.

stage t, the approximated cost depends only on w_{t-1} and w_t. Therefore, the approximated cost in (B.2) can be written as

$$\widetilde{C} = \sum_{t=1}^{T} \left(((w_t)^+)^2 + H(w_{t-1}, w_t) \right), \tag{B.3}$$

where $H(w_{t-1}, w_t) = \left(\widetilde{b}_t^2 + 10\widetilde{d}_t^2 - ((w_t)^+)^2 \right)^+$.

For different values of the parameters, r_b, r_d, and ω, we evaluate the performance of the approximation via simulation. Fig. 2 depicts some numerical results of a simulation experiment and we can make the following observations:

1. The main source of approximation error is from the following scenario: at stage $t-1$, the deviation in demand w_{t-1} is non-positive, $w_t > r_b$, and $w_{t+1} > 2r_b$. In this scenario, the output of the primary energy source at stage t is r_b, which is less than w_t. When $\omega/r_b \leq 2$, this scenario never occurs and we observe from Fig. 2 that the approximation error is close to zero, regardless of the value of r_d.

2. Comparing the black curve with the red curve in Fig. 2, we observe that when $\omega/r_b > 3$ (when $r_b = 0.05$ and $\omega > 0.15$), the approximation error for the case where $r_d = 0.1$ is larger than that for the case where $r_d = 0.25$. This is because for the case with $r_d = 0.1$, as ω/r_b increases from 3 to 6 (as ω increases from 0.15 to 0.3), the probability of load shedding increases, which deteriorates the performance of the approximation.
3. Finally, and perhaps most importantly, when the ramping rate of the ancillary energy resource is high enough to prevent any load shedding, the approximation error is an increasing function of the single parameter ω/r_b (e.g., the blue curve with circle markers for $r_b = 0.02, r_d = 0.1$ and the red curve for $r_b = 0.05, r_d = 0.25$ merge together in Fig. 2); in this case, we observe from Fig. 2 that the approximation error is less than 10% for a wide range of parameter values.

Appendix C. Proof of Theorem 1

We fix a DOE strategy ν and consider a sequence of n-consumer models where $n-1$ consumers (all except for consumer i) use the strategy ν. As the number of consumers increases to infinity, the randomness of consumer initial states averages out. Accordingly, in Step 1 we show that the aggregate demand (in an n-consumer model) at a history h_t is close to $n\widetilde{A}_{t|\nu,h_t}$ (defined in Eq. (16)), with high probability. As a consequence, we show in Step 2 that as $n \to \infty$, consumer i's expected payoff associated with any sequence of actions can be approximated by an oblivious value defined similarly to (21). Since the DOE strategy ν maximizes consumer i's oblivious value among all possible strategies, we argue in Step 3 that as $n \to \infty$, the maximum expected payoff consumer i can obtain is asymptotically no larger than the optimal oblivious value. In Step 4, we show that consumer i's optimal oblivious value can be approximately achieved if she uses the DOE strategy ν. We finally conclude with the AME property of the DOE strategy ν (the result stated in Theorem 1).

In what follows, we will be using the uniform metric over the set of probability distributions on the finite set $\mathcal{X}_0$. Specifically, if f and f' are two distributions on $\mathcal{X}_0$, we let

$$d(f, f') \triangleq \|f - f'\|_\infty = \max_{x \in \mathcal{X}_0} |f(x) - f'(x)| \,. \tag{C.1}$$

Step 1: *With high probability, the aggregate demand under a history h_t is close to $n\widetilde{A}_{t|\nu,h_t}$.*

Given an initial distribution $f^n_{-i,0}$, and if all consumers (excluding i) use a dynamic oblivious strategy ν, we write their aggregate demand at a history h_t as

$$A^n_{-i,t} = (n-1)\sum_{x\in\mathcal{X}_0} f^n_{-i,0}(x)\nu_t(x,h_t).$$

Recall that (cf. (16))

$$\widetilde{A}_{t|\nu,h_t} = \sum_{x\in\mathcal{X}_0} \eta_{s_0}(x)\cdot\nu_t(x,h_t).$$

Since $\nu_t(x,h_t)$ is always no more than B, we observe from the preceding two equalities that if $d(f^n_{-i,0},\eta_{s_0}) \le \delta/(XB)$, then at any history h_t, the following event,

$$\left| A^n_{-i,t} - (n-1)\widetilde{A}_{t|\nu,h_t} \right| \le \delta(n-1), \tag{C.2}$$

happens with probability at least $1-O(e^{-n})$. More precisely, since the consumers' initial states are independently drawn according to η_{s_0}, Hoeffding's inequality (Hoeffding (1963)) yields,

$$\begin{aligned}\mathbb{P}\left(d(F^{n-1}_{s_0},\eta_{s_0}) \ge \delta/(XB)\right) \le 2X\exp\left\{-2(n-1)\delta^2/(X^2B^2)\right\},\\ \forall s_0\in\mathcal{S},\ \ \forall\delta>0,\ \ \forall n\in\mathbb{N}^+,\end{aligned} \tag{C.3}$$

where X is the cardinality of the set $\mathcal{X}_0$ and $F^{n-1}_{s_0}$ is an X-dimensional random vector denoting the distribution of the initial states of the $n-1$ consumers (excluding i).

Step 2: *Under a given history h_T, consumer i's expected payoff can be approximated by a corresponding oblivious value, defined in (C.6) below.*

In an n-consumer model, suppose that all consumers other than i use the dynamic oblivious strategy ν. Given a complete history $h_T=(s_0,\dots,s_T)$, and consumer i's initial state $x_{i,0}$, her expected payoff under a history-dependent strategy κ^n is

$$V^n_{i,0}(x_{i,0},h_T \mid \kappa^n,\nu) = \mathbb{E}\left\{V^n_{i,0}(x_{i,0},h_T,f^n_{-i,0}\mid\kappa^n,\nu)\right\},$$

where the expectation is over the initial distribution, $f^n_{-i,0}$, and $V^n_{i,0}(x_{i,0},h_T,f^n_{-i,0}\mid \kappa^n,\nu)$ is consumer i's payoff under the given initial distribution $f^n_{-i,0}$,

$$V^n_{i,0}(x_{i,0},h_T,f^n_{-i,0}\mid\kappa^n,\nu) = \sum_{t=0}^{T}\pi^n_{i,t}(y_{i,t},h_t,f^n_{-i,t}\mid\kappa^n,\nu), \tag{C.4}$$

and where the stage payoff function, $\pi^n_{i,t}(\cdot)$, has been defined in (23). Note that given $f^n_{-i,0}$, and since all consumers other than i use a dynamic oblivious strategy, the distribution of their augmented states, $f^n_{-i,t}$, is completely determined by the history h_t. Therefore, given $f^n_{-i,0}$, consumer i's history-dependent strategy κ^n is equivalent to a dynamic oblivious strategy: the action it takes at stage t depends only on $x_{i,0}$ and h_t. We can therefore define an oblivious strategy $\widetilde{\nu}^n(\kappa^n, f^n_{-i,0})$ such that

$$\widetilde{\nu}_t(\kappa^n, f^n_{-i,0})(x_{i,0}, h_t) = \kappa^n_t(y_{i,t}, h_t, f^n_{-i,t}),$$

where $f^n_{-i,t}$ is the distribution of the $n-1$ consumers' augmented states under the history h_t, induced from the initial distribution $f^n_{-i,0}$ by the symmetric oblivious strategy profile $(\nu, \dots, \nu)$, and $y_{i,t}$ is consumer i's augmented state under the history h_t, induced from her initial state $x_{i,0}$ by the strategy κ^n.

In the corresponding continuum model, suppose that all consumers other than i use a dynamic oblivious strategy ν. For a given complete history h_T, we define consumer i's oblivious value under an initial distribution $f^n_{-i,0}$, her initial state $x_{i,0}$, and the history-dependent strategy κ^n:

$$\widetilde{V}_{i,0}(x_{i,0}, h_T, f^n_{-i,0} \mid \kappa^n, \nu) = \sum_{t=0}^{T} \widetilde{\pi}_{i,t}(y_{i,t}, h_t \mid \widetilde{\nu}(\kappa^n, f^n_{-i,0}), \nu), \tag{C.5}$$

where the oblivious stage value function $\widetilde{\pi}_{i,t}(\cdot)$ is given in (20). We define the expected oblivious value for consumer i under the history-dependent strategy κ^n, as[17]

$$\widetilde{V}_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu) = \mathbb{E}\left\{\widetilde{V}_{i,0}(x_{i,0}, h_T, f^n_{-i,0} \mid \kappa^n, \nu)\right\}, \tag{C.6}$$

where the expectation is over the initial distribution, $f^n_{-i,0}$. For any $\varepsilon > 0$, in this step we aim to show that there exists a positive integer N such that for any sequence of history-dependent strategies $\{\kappa^n\}$ and any $n \geq N$,

$$\left|\widetilde{V}_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu) - V^n_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu)\right| \leq \varepsilon, \quad \forall h_T \in \mathcal{H}_T, \quad \forall x_{i,0} \in \mathcal{X}_0. \tag{C.7}$$

[17] This is actually the oblivious value achieved by a mixed strategy under the complete history h_T. In the continuum model, under a history h_t, the mixed strategy takes an action $\widetilde{\nu}_t(\kappa^n, f^n_{-i,0})(x_{i,0}, h_t)$, if the distribution of the $n-1$ consumers' (excluding i's) initial states in the corresponding n-consumer model is realized as $f^n_{-i,0}$.

For a given s_0, let $\mathfrak{F}_{s_0}^{n-1}(\delta)$ be the set of $f_{-i,0}^n$ such that $d(f_{-i,0}^n, \eta_{s_0}) \leq \delta$. To verify (C.7), we first argue that for any $\varepsilon > 0$, there exists an positive integer N_1 and some $\delta > 0$ such that for any $f_{-i,0}^n \in \mathfrak{F}_{s_0}^{n-1}(\delta/(XB))$, any $n \geq N_1$, and any $h_T \in \mathcal{H}_T$,

$$\left| \widetilde{V}_{i,0}(x_{i,0}, h_T, f_{-i,0}^n \mid \kappa^n, \nu) - V_{i,0}^n(x_{i,0}, h_T, f_{-i,0}^n \mid \kappa^n, \nu) \right| \leq \varepsilon/2, \quad \forall x_{i,0} \in \mathcal{X}_0. \tag{C.8}$$

Under the uniform equicontinuity assumption for the derivatives of the cost functions (see Eqs. (25) and (26)), we know that a small deviation of the aggregate demand from $\widetilde{A}_{t|\nu,h_t}$ will result in prices that are only slightly different from the prices in the continuum model. We also note that consumer i cannot take an action larger than B, and her payoff is influenced by other consumers only through the prices. For any $\varepsilon > 0$, we can find some $\delta > 0$ and a positive integer N_1 such that for any given $(x_{i,0}, h_t)$, if $f_{-i,0}^n \in \mathfrak{F}_{s_0}^{n-1}(\delta/(XB))$, then the inequality in (C.2) holds for any history h_t, which implies that for any $n \geq N_1$,

$$\left| \widetilde{\pi}_{i,t}(y_{i,t}, h_t, \kappa_t^n(y_{i,t}, h_t, f_{-i,t}^n) \mid \nu) - \pi_{i,t}^n(y_{i,t}, h_t, f_{-i,t}^n \mid \kappa^n, \nu) \right| \leq \varepsilon/(2T+2), \tag{C.9}$$

i.e., consumer i's stage payoff (under the action $\kappa_t^n(y_{i,t}, h_t, f_{-i,t}^n)$) in the n-consumer model is close to her oblivious stage value (under the same action $\kappa_t^n(y_{i,t}, h_t, f_{-i,t}^n)$) in the continuum model, if the initial distribution in the n-consumer model, $f_{-i,0}^n$, is close to its expectation. The result in (C.8) follows from Eq. (C.9) and the definitions in (C.4) and (C.5). Note that $Q + 2BP$ is an upper bound on the stage payoff that consumer i could obtain, and $-2BP$ is a lower bound on consumer i's stage payoff, under Assumption 3. The desired result in (C.7) follows from (C.8), and the fact that the probability that $f_{-i,0}^n \notin \mathfrak{F}_{s_0}^{n-1}(\delta/(XB))$ decays exponentially with n (cf. Eq. (C.3)).

Step 3: *The maximum expected payoff consumer i can obtain is asymptotically no larger than the optimal oblivious value.*

In this step, we consider the case where all consumers in an n-consumer model except for i use a DOE strategy ν, and argue that for any sequence of history-dependent strategies $\{\kappa^n\}$,

$$\limsup_{n\to\infty} \left(V_{i,0}^n\left(x_{i,0}, s_0 \mid \kappa^n, \nu\right) - \widetilde{V}_{i,0}(x_{i,0}, s_0 \mid \nu, \nu) \right) \leq 0, \ \ \forall s_0 \in \mathcal{S}, \ \ \forall x_{i,0} \in \mathcal{X}_0, \tag{C.10}$$

where consumer i's expected payoff, $V^n_{i,0}(x, s \mid \kappa^n, \nu)$, is given in (24), and $\widetilde{V}_{i,0}(x, s \mid \nu, \nu)$ is the oblivious value function in (21). We first observe that

$$V^n_{i,0}(x_{i,0}, s_0 \mid \kappa^n, \nu) = \sum_{h_T \in \mathcal{H}_T(s_0)} \mathbb{P}(h_T \mid s_0) \cdot V^n_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu), \qquad \text{(C.11)}$$

where $\mathcal{H}_T(s_0)$ is the set of complete histories commencing at state s_0, and $\mathbb{P}(h_T \mid s_0)$ is the probability that the history h_T is realized, conditional on the initial global state being s_0. We define

$$\widetilde{V}_{i,0}(x_{i,0}, s_0 \mid \kappa^n, \nu) = \sum_{h_T \in \mathcal{H}_T(s_0)} \mathbb{P}(h_T \mid s_0) \cdot \widetilde{V}_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu). \qquad \text{(C.12)}$$

Note that if κ^n happens to be a dynamic oblivious strategy, this definition is consistent with the definition of an oblivious value function in (21).

For any $\varepsilon > 0$, let N be the integer defined in Eq. (C.7); for any sequence of history-dependent strategies $\{\kappa^n\}$, we argue that

$$\begin{aligned}\widetilde{V}_{i,0}(x_{i,0}, s_0 \mid \nu, \nu) &\geq \sum\nolimits_{h_T \in \mathcal{H}_T(s_0)} \mathbb{P}(h_T \mid s_0) \cdot \widetilde{V}_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu) \\ &\geq \sum\nolimits_{h_T \in \mathcal{H}_T(s_0)} \mathbb{P}(h_T \mid s_0) \cdot (V^n_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu) - \varepsilon) \\ &= V^n_{i,0}(x_{i,0}, s_0 \mid \kappa^n, \nu) - \varepsilon, \qquad \forall n \geq N, \qquad \forall x_{i,0} \in \mathcal{X}_0.\end{aligned} \qquad \text{(C.13)}$$

The DOE strategy ν, by definition, maximizes consumer i's oblivious value function among all possible dynamic oblivious strategies. The first inequality in (C.13) follows from the fact that $\widetilde{V}_{i,0}(x_{i,0}, s_0 \mid \kappa^n, \nu)$ is a weighted sum of the oblivious values achieved by a family of dynamic oblivious strategies[18]. The second inequality in (C.13) is due to (C.7), and the last equality in (C.13) follows from (C.11). The desired result, (C.10), follows.

Step 4: *Consumer i's optimal oblivious value can be asymptotically achieved at an n-consumer game under a DOE strategy.*

In this step, we consider the case where all consumers in an n-consumer model use a DOE strategy ν, and show that

$$\lim_{n \to \infty} \left(\widetilde{V}_{i,0}(x_{i,0}, s_0 \mid \nu, \nu) - V^n_{i,0}(x_{i,0}, s_0, \mid \nu, \nu) \right) = 0, \quad \forall s_0 \in \mathcal{S}, \quad \forall x_{i,0} \in \mathcal{X}_0. \qquad \text{(C.14)}$$

[18]Note that for a given $f^n_{-i,0}$, the action taken by κ^n depends only on $x_{i,0}$ and h_t, and that $\widetilde{V}_{i,0}(x_{i,0}, h_T \mid \kappa^n, \nu)$ is the oblivious value achieved by a mixed strategy; cf. the footnote associated with (C.6).

According to (C.7), with $\kappa^n = \nu$, for any $\varepsilon > 0$, we can find some N such that for any $n \geq N$

$$\left| \widetilde{V}_{i,0}(x_{i,0}, h_T \mid \nu, \nu) - V^n_{i,0}(x_{i,0}, h_T \mid \nu, \nu) \right| \leq \varepsilon, \quad \forall h_T \in \mathcal{H}_T, \quad \forall x_{i,0} \in \mathcal{X}_0.$$

The desired result in (C.14) follows from (C.11) and (C.12). Theorem 1 follows from (C.10) and (C.14).

Appendix D. Proof of Theorem 2

Appendix D.1. Proof of Part (a)

We will show that in a continuum model, a DOE strategy maximizes the expected social welfare among all possible symmetric dynamic oblivious strategy profiles (part (a) of the theorem), i.e., that if ν is DOE, then

$$\widetilde{\mathcal{W}}_0(s_0 \mid \nu) = \sup_{\vartheta \in \mathfrak{V}} \widetilde{\mathcal{W}}_0(s_0 \mid \vartheta), \qquad \forall s_0 \in \mathcal{S}. \tag{D.1}$$

Let S and X be the cardinality of $\mathcal{S}$ and $\mathcal{X}_0$, respectively. Given the initial global state s_0, the number of possible histories of length $t+1$ is S^t. Hence, the number of all possible histories commencing at state s_0 is $\sum_{t=0}^{T} S^t$. Given an initial global state s_0, the expected social welfare defined in (32) is a deterministic function of the following $(X \sum_{t=0}^{T} S^t)$-dimensional action vector:

$$\left\{\nu_t\left(x, h_t\right)\right\}_{x \in \mathcal{X}_0,\ h_t \in \mathcal{H}(s_0)}, \tag{D.2}$$

where $\mathcal{H}(s_0)$ is the set of positive probability histories commencing at state s_0. Under Assumption 4, the expected social welfare defined in (32) is a concave function of the vector in (D.2). Therefore, the following conditions are necessary and sufficient for the action vector (in the form of (D.2)) associated with the DOE strategy ν to maximize the expected social welfare, among all possible dynamic oblivious strategies[19]:

$$\begin{cases} \dfrac{\partial_+ U_t(l_{\nu,h_t}(x), s_t, \nu_t(x,h_t))}{\partial_+ \nu_t(x,h_t)} \le \widetilde{p}_{t|\nu,h_t} + \widetilde{w}_{t|\nu,h_t} + 1_{\tau<T} \cdot g^+_{t|\nu,h_t}(x), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \ \nu_t(x,h_t) < B, \\ \dfrac{\partial_- U_t(l_{\nu,h_t}(x), s_t, \nu_t(x,h_t))}{\partial_- \nu_t(x,h_t)} \ge \widetilde{p}_{t|\nu,h_t} + \widetilde{w}_{t|\nu,h_t} + 1_{\tau<T} \cdot g^-_{t|\nu,h_t}(x), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } \ \nu_t(x,h_t) > 0, \end{cases} \tag{D.3}$$

where $l_{\nu,h_t}(x)$ is consumer i's state, $x_{i,t}$, under the initial condition $x_{i,0} = x$, a (positive probability) history h_t and the strategy ν (cf. p.15), the prices, $\widetilde{p}_{t|\nu,h_t}$ and $\widetilde{w}_{t|\nu,h_t}$ are given in (17) and (18), and where, if $k_{h_\tau}(\cdot)$ (cf. the definition in (35)) is nondecreasing in $a_{i,t}$ for any $t < \tau \le T$, then $g^+_{t|\nu,h_t}(x)$ is given by[20]

$$g^+_{t|\nu,h_t}(x) = \mathbb{E}\left\{ \widetilde{q}_{t+1|\nu,h_{t+1}} - \sum_{\tau=t+1}^{T} \frac{\partial_+ U_\tau(x, z_{i,\tau}, s_\tau, a_{i,\tau})}{\partial_+ z_{i,\tau}} \cdot \frac{\partial_+ z_{i,\tau}}{\partial_+ a_{i,t}} \right\}, \tag{D.4}$$

where the price, $\widetilde{q}_{t+1|\nu,h_{t+1}}$, is defined in (18), the expectation is over the future global states, $\{s_\tau\}_{t+1}^{T}$, $z_{i,\tau} = k_{h_\tau}(x, a_{i,0}, \ldots, a_{i,\tau-1})$ for $\tau > t$, and $a_{i,\tau} = \nu_\tau(x_{i,\tau}, h_\tau)$ for $\tau \ge t$. The expression (D.4) is the part of the right derivative of the expected social welfare (32) with respect to the action $a_{i,t}$, which reflects the influence of consumer i's action at stage t on the ancillary cost $\widetilde{H}(\widetilde{A}_t, \widetilde{A}_{t+1}, \overline{s}_{t+1})$ at the next stage, and on her future utility (due to the influence of the action $a_{i,t}$ on the future state $z_{i,\tau}$, through the functions $k_{h_\tau}(\cdot)$). In (D.3), $g^-_{t|\nu,h_t}(x)$ can be defined by replacing the right (left) partial derivatives in (D.4) with left (respectively, right) partial derivatives.

Given an initial global state s_0, and the initial state of consumer i, x, her oblivious value, defined in (21), is a deterministic, concave function of the vector

$$\{\nu_t(x, h_t)\}_{h_t \in \mathcal{H}(s_0)} \tag{D.5}$$

of actions that she would take at any given stage and for any given history. Since the DOE strategy ν maximizes consumer i's oblivious value, it is easily

[19]We use the notations $\partial_+ f$ and $\partial_- f$ to denote the right and left, respectively, derivatives of a function f.

[20]If for some $\tau > t$, $k_{h_\tau}(\cdot)$ is decreasing in $a_{i,t}$, then the right partial derivative of $U_\tau(x_{i,\tau}, s_\tau, a_{i,\tau})$ with respect to $z_{i,\tau}$ in (D.4) should be replaced by its left partial derivative.

checked that the vector in (D.5) must satisfy the conditions (D.3). Since this is true for any $x \in \mathcal{X}_0$, we conclude that the action vector (D.2) (which is comprised of the vectors in (D.5), for different types x in the set $\mathcal{X}_0$) satisfies the conditions (D.3). Thus, the DOE ν satisfies the sufficient condition for optimality and the result (D.1) follows.

Appendix D.2. Proof of Part (b)

For a given initial global state s_0, let us fix some initial distribution f_0^n with $d(f_0^n, \eta_{s_0}) \leq \delta$, where δ is small. In Step 1, we compare the social welfare achieved by various strategy profiles and show that

$$\frac{1}{n}\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n) \leq \frac{1}{n}\mathcal{W}_0^n(s_0 \mid \boldsymbol{\vartheta}^{n,f_0^n}) \approx \widetilde{\mathcal{W}}_0(s_0 \mid \vartheta^{n,f_0^n}).$$

Here, κ^n is a general history-dependent strategy profile for the n-consumer model (cf. (22)). The symmetric strategy profile $\boldsymbol{\vartheta}^{n,f_0^n} = (\vartheta^{n,f_0^n}, \ldots, \vartheta^{n,f_0^n})$ is one that maximizes expected social welfare given the initial population state f_0^n. In Step 1, we will argue that ϑ^{n,f_0^n} can be identified with a dynamic oblivious strategy. In the approximate equality we are comparing the expected (over future global states, $\{s_t\}_{t=1}^T$) social welfare under the same oblivious strategy ϑ^{n,f_0^n} (hence the same sequence of actions for each consumer type $x \in \mathcal{X}_0$) under two different initial population states (initial distributions of consumer types), f_0^n and η_{s_0}.

Since ν is a DOE, part (a) of the theorem implies that

$$\widetilde{\mathcal{W}}_0(s_0 \mid \vartheta^{n,f_0^n}) \leq \widetilde{\mathcal{W}}_0(s_0 \mid \nu).$$

Note that as the number of consumer grows large, with high probability the initial population state f_0^n is close to its expectation, η_{s_0}. In Step 2, we complete the proof of part (b) by showing that

$$\widetilde{\mathcal{W}}_0(s_0 \mid \nu) \approx \frac{1}{n}\mathbb{E}\{\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\nu}^n)\},$$

where the expectation is over the initial population state f_0^n.

Step 1: *If the initial population state is close to its expectation, the optimal social welfare in an n-consumer model can be approximated by the social welfare achieved by a dynamic oblivious strategy in the corresponding continuum model.*

In this step, we aim to show that in an n-consumer model, for any given initial global state s_0 and any $\varepsilon > 0$, there exists some $\delta > 0$ such that for any initial distribution f_0^n with $d(f_0^n, \eta_{s_0}) \le \delta$, we can find a dynamic oblivious strategy ϑ^{n,f_0^n} that satisfies

$$\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n) \le n\widetilde{\mathcal{W}}_0(s_0 \mid \vartheta^{n,f_0^n}) + \varepsilon n, \tag{D.6}$$

for all symmetric history-dependent strategy profiles, $\boldsymbol{\kappa}^n = (\kappa^n, \ldots, \kappa^n)$. Given an initial global state s_0 and an initial population state f_0^n, we observe that the social welfare, $\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n)$, is a deterministic, concave function of the following vector of consumers' actions under different histories,

$$\left\{\kappa_t^n\left(m_{i,\kappa^n,h_t}(x_{i,0}), h_t, f_{-i,t}^n\right)\right\}_{h_t \in \mathcal{H}(s_0),\ x_{i,0} \in \mathcal{X}_0,\ i=1,\ldots,n}, \tag{D.7}$$

where $m_{i,\kappa^n,h_t} : \mathcal{X}_0 \to \mathcal{Y}_t$ maps consumer i's initial state into her augmented state at the history h_t, under the strategy profile $\boldsymbol{\kappa}^n$, and $f_{-i,t}^n$ is the distribution of other consumers' augmented states at the history h_t, under the strategy profile $\boldsymbol{\kappa}^n$. Note that given the initial population state f_0^n, the strategy profile $\boldsymbol{\kappa}^n$, and a history h_t, the augmented state of consumer i at stage t depends only on her initial state $x_{i,0}$.

Since the social welfare $\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n)$ is concave in the action vector in (D.7), there exists a symmetric solution, $\vartheta^{n,f_0^n} = \{\vartheta_t^{n,f_0^n}\}_{t=0}^T$, such that if at any history $h_t \in \mathcal{H}(s_0)$, all consumers with the same initial state take the same action according to

$$a_{i,t} = \vartheta_t^{n,f_0^n}(x_{i,0}, h_t), \quad i = 1, \ldots, n, \tag{D.8}$$

then the expected social welfare, $\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n)$, is maximized among all possible symmetric history-dependent strategy profiles[21]. In (D.8) we have defined a dynamic oblivious strategy ϑ^{n,f_0^n} that maximizes the expected social welfare in the n-consumer model, conditional on the initial global state being s_0, and the initial population state being f_0^n. That is, in an n-consumer model, for any given s_0 and f_0^n, there exists a dynamic oblivious strategy ϑ^{n,f_0^n} such that

$$\sup_{\kappa^n} \mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n) = \mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\vartheta}^{n,f_0^n}), \tag{D.9}$$

[21] The fact that the supremum is attained is a consequence of our continuity assumption and the fact that the various variables of interest can be restricted to be in a compact set.

where $\boldsymbol{\vartheta}^{n,f_0^n} = (\vartheta^{n,f_0^n}, \dots, \vartheta^{n,f_0^n})$ is the corresponding symmetric dynamic oblivious strategy profile. To verify (D.6), it suffices to show that for any $\varepsilon > 0$, there exists some $\delta > 0$ such that for any f_0^n with $d(f_0^n, \eta_{s_0}) \leq \delta$,

$$\left| \mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\vartheta}^{n,f_0^n}) - n\widetilde{\mathcal{W}}_0(s_0 \mid \vartheta^{n,f_0^n}) \right| \leq \varepsilon n, \tag{D.10}$$

i.e., if all consumers use the strategy ϑ^{n,f_0^n}, the difference between the optimal social welfare achieved in an n-consumer model and the social welfare achieved in the corresponding continuum model can be made arbitrarily small, if the initial population state is close enough to its expectation, η_{s_0}. We next argue that the result in (D.10) holds for any dynamic oblivious strategy ϑ.

To prove (D.10), we first upper bound the difference between the supplier cost in an n-consumer model and that in the corresponding continuum model. Since all cost functions are Lipschitz continuous (see Eqs. (27) and (28)), for any $\varepsilon > 0$, there exists some $\delta_1 > 0$ such that if

$$\left| A_t^n - n\widetilde{A}_{t|\vartheta,h_t} \right| \leq X\delta_1 Bn, \qquad t = 0, \dots, T, \qquad \forall h_t \in \mathcal{H}(s_0), \tag{D.11}$$

then

$$\left| C^n(A_t^n, s_t) - C^n(n\widetilde{A}_{t|\vartheta,h_t}, s_t) \right| \leq n\varepsilon/(3T+3), \quad t = 0, \dots, T, \quad \forall h_t \in \mathcal{H}_t(s_0), \tag{D.12}$$

$$\left| H_0^n(A_0^n, s_0) - H_0^n(n\widetilde{A}_{0|\vartheta,h_0}, s_0) \right| \leq n\varepsilon/(3T+3), \tag{D.13}$$

and for $t = 1, \dots, T$ and any $h_t \in \mathcal{H}_t(s_0)$,

$$\left| H^n(A_{t-1}^n, A_t^n, \bar{s}_t) - H^n(n\widetilde{A}_{t-1|\vartheta,h_{t-1}}, n\widetilde{A}_{t|\vartheta,h_t}, \bar{s}_t) \right| \leq n\varepsilon/(3T+3), \tag{D.14}$$

where $\mathcal{H}_t(s_0)$ is the set of all histories of length $t+1$ commencing at state s_0. Given an initial population state f_0^n, if all consumers use the strategy ϑ, the aggregate demand under a history h_t is

$$A_t^n = n \sum_{x \in \mathcal{X}_0} f_0^n(x) \vartheta_t(x, h_t).$$

From (16) we observe that if $d(f_0^n, \eta_{s_0}) \leq \delta_1$, the condition in (D.11) holds, and then Eqs. (D.12)-(D.14) are verified.

We now show that if the initial population state is close to its expectation, the total utility obtained by all consumers is close to its counterpart in the corresponding continuum model. Given an initial population state f_0^n, we write the total utility obtained by all consumers under a history h_t as

$$\sum_{i=1}^{n} U_t(x_{i,t}, s_t, a_{i,t}) = n \sum_{x\in\mathcal{X}_0} f_0^n(x) U_t\left(l_{\vartheta,h_t}(x), \vartheta_t(x,h_t), s_t\right).$$

On the other hand, the utility achieved in the corresponding continuum model is given by

$$\widetilde{U}_{t|\vartheta,h_t} \triangleq \sum_{x\in\mathcal{X}_0} \eta_{s_0}(x) U_t\left(l_{\vartheta,h_t}(x), \vartheta_t(x,h_t), s_t\right).$$

We have that if $d(f_0^n, \eta_{s_0}) \le \varepsilon/(3XQ(T+1))$, then for any $n \in \mathbb{N}^+$,

$$\left|\sum\nolimits_{i=1}^{n} U_t(x_{i,t}, s_t, a_{i,t}) - n\widetilde{U}_{t|\vartheta,h_t}\right| \le n\varepsilon/(3T+3), \quad t = 0,\ldots,T, \quad \forall h_t \in \mathcal{H}_t(s_0), \tag{D.15}$$

Let $\delta = \min\{\delta_1, \varepsilon/(3XQ(T+1))\}$. If $d(f_0^n, \eta_{s_0}) \le \delta$, from (D.12)-(D.15) we have

$$\left|\widetilde{W}_t(h_t \mid \vartheta) - W_t^n(f_t^n, h_t \mid \boldsymbol{\vartheta}^n)\right| \le n\varepsilon/(T+1), \quad t = 0,\ldots,T, \quad \forall h_t \in \mathcal{H}_t(s_0).$$

Eq. (D.10) follows from the definition of expected social welfare in an n-consumer model (31), and in a continuum model (32). The desired result in (D.6) follows.

Step 2: *Asymptotic social optimality of a DOE.*

In this step, we complete the proof of part (b) of the theorem, using the fact that as the number of consumers grows large, with high probability the initial population state is close to its expectation. Note that the action space is $[0, B]$, so that $|A_t| \le nB$. Using Assumption 3, $C^n(\cdot)/n$ is therefore bounded. A similar argument holds for $H_0^n(\cdot)/n$ and $H^n(\cdot)/n$. Furthermore, the total utility per consumer is also bounded. Thus, there exists some constant D that upper bounds $|\mathcal{W}_0^n/n|$. We define $\mathfrak{F}_{s_0}^n(\delta)$ as the set of initial population states such that $d(f_0^n, \eta_{s_0}) \le \delta$. By the law of large numbers, for any pair of positive real numbers, ε and δ, we can find an integer N such that for any $n \ge N$

$$\sum_{f_0^n \notin \mathfrak{F}_{s_0}^n(\delta)} \mathbb{P}\Big(F_{s_0}^n = f_0^n\Big) \cdot \sup_{\kappa^n} |\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n)| \le D\mathbb{P}(d(f_0^n, \eta_{s_0}) > \delta) \le \varepsilon n. \tag{D.16}$$

For any $\varepsilon > 0$, let δ be the positive real number defined in (D.6), and let N be the positive integer given in (D.16); for any $n \geq N$ and any symmetric history-dependent strategy profile $\boldsymbol{\kappa}^n$, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n)\right\} \\
&\leq \sum\nolimits_{f_0^n \in \mathfrak{F}_{s_0}^n(\delta)} \mathbb{P}\big(F_{s_0}^n = f_0^n\big) \cdot \mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\kappa}^n) + \varepsilon n \\
&\leq \sum\nolimits_{f_0^n \in \mathfrak{F}_{s_0}^n(\delta)} \mathbb{P}\big(F_{s_0}^n = f_0^n\big) \cdot \Big(n\widetilde{\mathcal{W}}_0(s_0 \mid \vartheta^{n, f_0^n}) + \varepsilon n\Big) + \varepsilon n \\
&\leq \sum\nolimits_{f_0^n \in \mathfrak{F}_{s_0}^n(\delta)} \mathbb{P}\big(F_{s_0}^n = f_0^n\big) \cdot \Big(n\widetilde{\mathcal{W}}_0(s_0 \mid \nu) + \varepsilon n\Big) + \varepsilon n \\
&\leq \sum\nolimits_{f_0^n \in \mathfrak{F}_{s_0}^n(\delta)} \mathbb{P}\big(F_{s_0}^n = f_0^n\big) \cdot (\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\nu}^n) + 2\varepsilon n) + \varepsilon n \\
&\leq \mathbb{E}\left\{\mathcal{W}_0^n(f_0^n, s_0 \mid \boldsymbol{\nu}^n)\right\} + 4\varepsilon n,
\end{aligned}
$$

where the first inequality follows from (D.16), the second inequality is due to (D.6), the third inequality follows from the optimality property of the DOE ν (part (a) of the theorem), the fourth inequality follows similar to (D.10) (the proof of Eq. (D.10) remains valid for any dynamic oblivious strategy), and the last inequality follows from (D.16).

Appendix E. Numerical Results

In this section we give a numerical example to compare the proposed pricing mechanism with marginal cost pricing. The comparison is carried out in terms of DOEs and the resulting social welfare under the corresponding continuum model. Towards this purpose, we first define the DOE for a continuum model under the marginal cost pricing mechanism, in Appendix E.1. In Appendix E.2, we consider a two-stage dynamic model in which the consumers' marginal utility and demand increase at the second stage. We calculate the equilibria resulting from the two pricing mechanisms, and compare the potential of the two pricing mechanisms to improve social welfare and reduce peak load.

Appendix E.1. Equilibrium under Marginal Cost Pricing

In an n-consumer model, at stage $t \geq 1$, the supplier's marginal cost is

$$(C^n)'(A_t^n, s_t) + \frac{\partial H^n(A_{t-1}^n, A_t^n, \overline{s}_t)}{\partial A_t^n} = p_t^n + w_t^n, \qquad t = 1, \ldots, T. \tag{E.1}$$

At stage 0, the supplier's marginal cost is

$$(C^n)'(A_0^n, s_0) + (H_0^n)'(A_0^n, s_0) = p_0^n + w_0^n. \tag{E.2}$$

Under marginal cost pricing, each consumer's stage payoff is

$$\pi(y_{i,t}, \overline{s}_t, a_{i,t}, f_{-i,t}^n, u_{-i,t}^n) = U(x_{i,t}, s_t, a_{i,t}) - (p_t^n + w_t^n) \cdot a_{i,t}, \tag{E.3}$$

where the stage marginal cost, $p_t^n + w_t^n$, is given in (E.1) and (E.2), and $y_{i,t} = (x_{i,t}, a_{i,t-1})$.

For marginal cost pricing, we now define the non-atomic equilibrium concept in the corresponding continuum model. Suppose that all consumers other than i use a dynamic oblivious strategy ν. Consumer i's oblivious stage value under marginal cost pricing is given by

$$\widetilde{\pi}_{i,t}(y_{i,t}, \overline{s}_t, f_{t|\nu,h_t}, a_{i,t} \mid \nu) = U_t(x_{i,t}, s_t, a_{i,t}) - (\widetilde{p}_{t|\nu,h_t} + \widetilde{w}_{t|\nu,h_t}) \cdot a_{i,t}, \tag{E.4}$$

where $\widetilde{p}_{t|\nu,h_t}$ and $\widetilde{w}_{t|\nu,h_t}$ are defined in (17) and (18). Replacing the oblivious stage value function in (19) with that given in (E.4), we can define an equilibrium concept for the marginal cost pricing mechanism in a similar way as for the DOE in Section 4.

Appendix E.2. Numerical Example

In current wholesale electricity markets, the highest daily wholesale price usually occurs when the system load increases quickly (cf. Fig. 1 in Section 1). Inspired by the above observation, we construct a two-stage dynamic model, in which the aggregate demand increases quickly at the second stage, to compare the performance of the proposed mechanism with marginal cost pricing. For simplicity, we assume that there is a continuum of identical consumers indexed by $i \in [0, 1]$. Each consumer would like to consume $1 + x$ and $1.2 - x$ at the two stages, where $x \in [0, E]$. Here, $E \in [0, 0.1]$ (a given constant) is the amount of electricity demand that can be shifted from the second stage to the first stage. The value of E will be called demand substitutability[22].

[22] There are two types of elasticity of consumers' demand: (i) consumers may curtail their demand at a high price, and (ii) they may shift their demand to a less expensive time. The first type of demand response is a price elasticity, and the second type is an elasticity of substitution across time. The first type of elasticity is incorporated in our model through the utility functions, and the second type of elasticity is incorporated through E.

Formally, consumer i's state at each stage denotes the maximum amount of electricity she could use at the stage[23]. For a given consumer i, we have $x_{i,0} = 1 + E$, and her state at stage 1 is determined as follows:

1. if $a_{i,0} \leq 1$, the maximum amount of electricity she could use at stage 1 is $1.2 - E + E$, i.e., $x_{i,1} = 1.2$;
2. if $1 < a_{i,0} \leq x_{i,0}$, the maximum amount of electricity she could use at stage 1 is $x_{i,1}$ =$1.2 - E + (x_{i,0} - a_{i,0}) = 2.2 - a_{i,0}$;
3. if $x_{i,0} < a_{i,0}$, the maximum amount of electricity she could use at stage 1 is $x_{i,1}$ =$1.2 - E$.

To summarize, we have

$$x_{i,1} = 1.2 - E + \max\{0, x_{i,0} - \max\{a_{i,0}, 1\}\}.$$

For each stage t, the utility functions are given by

$$U_t(x_{i,t}, s_t, a_{i,t}) = \begin{cases} d_t a_{i,t}, & \text{if} \quad 0 \leq a_{i,t} \leq x_{i,t}, \\ d_t x_{i,t}, & \text{if} \quad a_{i,t} > x_{i,t}, \end{cases}$$

where the slopes are $d_0 = 10$ and $d_1 = 12$. Here, we assumed that the consumers place a larger value on electricity during peak hours, and that shifting peak load to off-peak hours hurts consumer utility. For example, rescheduling kitchen and laundry activities may cause inconvenience for residential consumers; similarly, industrial consumers may face higher labor cost premiums for off-peak production.

The primary cost function (cf. Section 2) is $\widetilde{C}(A, s) = A^2$, for any s. We assume that the capacity available at each stage is proportional to the system load, i.e.,

$$G_t = b_t A_t, \quad t = 0, 1,$$

and that the ancillary cost depends only on the difference between the capacity available at two consecutive stages. At the second stage (peak hour), we assume that the system operator maintains a reserve margin of 10%, i.e., $b_1 = 1.1$. We will consider two different system operator policies: (i) the system operator does not forecast the load jump at the second stage, and

[23]Since all consumers are of the same type, the consumer state space in this example is a subset of $[0, \infty)$.

uses a conservative policy under which $b_0 = 1.12$, and (ii) the system operator predicts the load jump at the second stage, and ramps up the system capacity in advance, by letting $b_0 = 1.2$.

For simplicity, we use a quadratic function to approximate the ancillary cost associated with load fluctuations:

$$\widetilde{H}_0(A_0, s_0) = 10(\max\{b_0 A_0 - 1.12, 0\})^2,$$
$$\widetilde{H}(A_0, A_1, \overline{s}_1) = 20(\max\{b_1 A_1 - b_0 A_0, 0\})^2,$$

where 1.12 represents the capacity available at the stage before the initial stage[24]. We assumed a higher coefficient, 20, for the ancillary cost at the second stage, due to the increase of the system load.

For different levels of demand substitutability E, and two different system operator policies (b_0 equal to 1.12 or $b_0 = 1.2$), we compare the social welfare (in Section Appendix E.2.1) and the peak load (in Section Appendix E.2.2) resulting from the equilibria of the two pricing mechanisms.

Appendix E.2.1. Social welfare gain.

For various levels of demand substitutability ($E \in [0, 0.1]$), and the two different system operator policies, we calculate the equilibria resulting from the two pricing mechanisms. Fig. 3 compares the social welfare achieved by the proposed mechanism and the marginal cost pricing mechanism. We observe from Fig. 3 the following.

1. **System operator's policy:** When the consumers have a low level of demand substitutability, the policy with $b_0 = 1.2$ achieves a much higher social welfare than the conservative policy ($b_0 = 1.12$), under both the proposed and the marginal cost pricing mechanisms. (This is to be expected, because when $b_0 = 1.12$, and with the demand at stage 1 more or less fixed, the difference $b_1 A_1 - b_0 A_0$ is necessarily large.) For consumers with a high level of demand substitutability, the policy with $b_0 = 1.2$ achieves a slightly smaller social welfare than the conservative policy ($b_0 = 1.12$), because the policy with $b_0 = 1.2$ results in a lower price at the second stage than the conservative one, and therefore does not provide enough encouragement to the consumers to shift their peak load (cf. the discussion in Section Appendix E.2.2).

[24] Suppose that the load at stage "-1" is 1, and that the capacity available at stage -1 is 1.12, under an average reserve margin of 12%.

2. **Social welfare gain at a low level of demand substitutability:** At a low level of demand substitutability, e.g., when $E \leq 0.02$, and under the system operator's conservative policy ($b_0 = 1.12$), we observe that the proposed pricing mechanism achieves significantly more social welfare gain (the social welfare achieved by flat rate pricing[25] is used a reference) than marginal cost pricing; if the system operator ramps up the capacity in advance ($b_0 = 1.2$), both pricing mechanisms achieve approximately the same social welfare as flat rate pricing.
3. **Social welfare gain at a high level of demand substitutability:** If the consumers have a high demand substitutability, e.g., when $E \geq 0.08$, the proposed pricing mechanism achieves approximately 5% more social welfare gain than marginal cost pricing under the system operator's conservative policy ($b_0 = 1.12$); if the system operator ramps up the capacity in advance ($b_0 = 1.2$), the proposed pricing mechanism achieves approximately 50% more social welfare gain than marginal cost pricing.

Let us now derive some insights by considering the special case of zero demand substitutability ($E = 0$) and $b_0 = 1.12$. The one-stage aggregate demand and the social welfare resulting from the three pricing mechanisms are given in Table E.1. The prices faced by consumers are given in Table E.2, where the average retail price is the ratio of the total money a consumer pays at an equilibrium to her total demand during the two stages.[26] Note that under the proposed pricing mechanism, a consumer pays

$$(p_0 + w_0 + q_1)a_{i,0} + (p_1 + w_1)a_{i,1},$$

while she would pay $(p_0 + w_0)a_{i,0} + (p_1 + w_1)a_{i,1}$ under marginal cost pricing. In general, the price q_1 will be negative and will be even smaller if we were to increase the aggregate demand at the second stage. That is, a higher peak

[25] Under flat rate pricing, consumers pay a fixed (time-invariant) retail price for the electricity they consume. Since the average retail price is less than the consumers' marginal utility (see Tables E.2 and E.4), the payoff-maximizing consumer demand at the two stages is 1 and 1.2, respectively. Since all consumers are identical, the aggregate demand at the two stages is 1 and 1.2.

[26] Note that only consumers under flat rate pricing pay this price. We list the average prices for the two real-time pricing mechanisms to compare the consumers' expense under different pricing mechanisms.

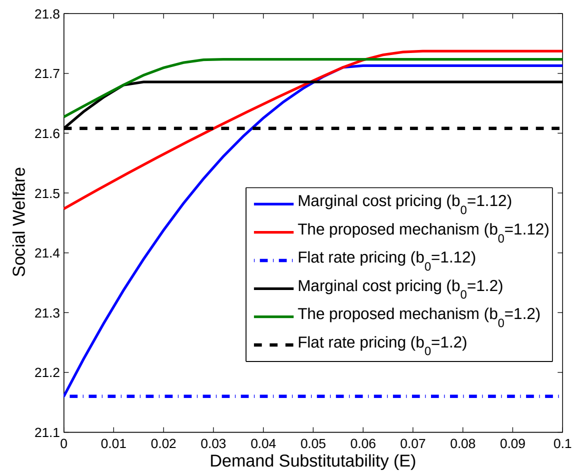

Figure E.3: The social welfare achieved by the proposed pricing mechanism, the marginal cost pricing mechanism and the flat rate pricing mechanism, as a function of the demand substitutability, E.

Table E.1: Demand and social welfare (per consumer) at $E = 0$ and $b_0 = 1.12$

	a_0	a_1	Social welfare
Flat rate	1	1.2	21.16
Marginal cost	1	1.2	21.16
Proposed	1.0901	1.2	21.4735

load results in a lower price at the first stage, which encourages consumers to increase their demand at the off-peak hour, even if they do not derive any additional utility from such an increase. In fact, from Table E.2 we observe that at the DOE, the proposed pricing mechanism offers each consumer a zero total price on a_0. This may appear illogical at first sight. The reason is that due to the conservative reserve policy, with $b_0 = 1.12$, a demand of $a_0 = 1$ results in a large increase from $b_0 a_0$ to $b_1 a_1$ and hence a large ancillary cost. The increase of the demand a_0 beyond 1 does not provide any utility to the consumer, but reduces the ancillary cost. Thus, the counterintuitive choice of $a_0 = 1.0901$ serves to mitigate a conservative and somewhat deficient reserve policy. This suggests that further research is needed that will include an intertemporal optimization of the reserve policy as well.

Table E.2: Price fluctuation at $E = 0$ and $b_0 = 1.12$. The price $p_t + w_t$ equals the marginal cost at stage t.

	$p_0 + w_0$	$p_1 + w_1$	q_1	Average (retail) price
Flat rate	2	11.2	-	7.0182
Marginal cost	2	11.2	-	7.0182
Proposed	4.4401	6.7609	-4.4401	5.6562

Table E.3: Demand and social welfare (per consumer) at $E = 0.08$ and $b_0 = 1.2$

	a_0	a_1	Social welfare
Flat rate	1	1.2	21.608
Marginal cost	1.0131	1.1869	21.6857
Proposed	1.0308	1.1692	21.7237

For the case where the system operator ramps up the capacity in advance ($b_0 = 1.2$), and consumers have a high level of demand substitutability ($E = 0.08$), the one-stage aggregate demand and the social welfare resulting from the three pricing mechanisms are given in Table E.3. The prices faced by consumers are given in Table E.4. From Table E.3 we observe that under the proposed pricing mechanism, consumers would like to shift 0.031 peak load to off-peak hours, while under the marginal cost pricing mechanism, consumers are willing to shift less than 0.014 peak load to off-peak hours. Compared to marginal cost pricing, the more flattened load curve resulting from the proposed pricing mechanism leads to 50% more social welfare gain.

For a given load curve, the proposed pricing mechanism results in a larger price difference between stage 1 and stage 0 than marginal cost pricing, because of the negative price q_1. The negative price q_1 creates an additional incentive for consumers to shift their load from stage 1 to stage 0. In this way, the proposed pricing mechanism results in a more flattened load curve

Table E.4: Price fluctuation at $E = 0.08$ and $b_0 = 1.2$. The price $p_t + w_t$ equals the marginal cost at stage t.

	$p_0 + w_0$	$p_1 + w_1$	q_1	Average (retail) price
Flat rate	3.92	7.68	-	5.971
Marginal cost	4.324	6.324	-	5.403
Proposed	4.868	4.505	-2.363	3.568

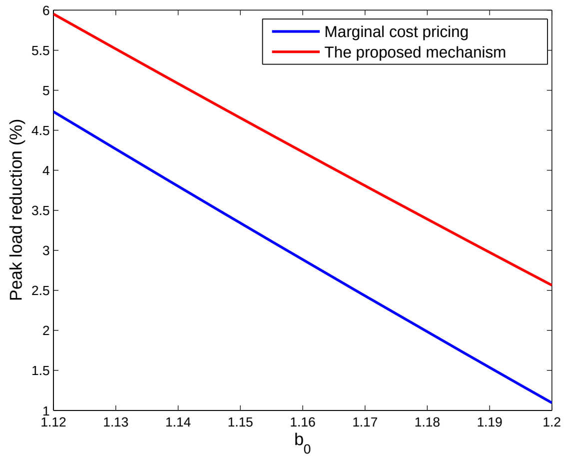

Figure E.4: Comparison of the percentage of peak load reduction (the peak load under flat rate pricing, 1.2, is used a reference) resulting from the proposed pricing mechanism and the marginal cost pricing mechanism, as a function b_0.

and a higher social welfare than marginal cost pricing (cf. Table E.3).

Appendix E.2.2. Peak load reduction.

Under flat rate pricing, the peak load (the aggregate demand at the second stage) is 1.2, because consumers do not have an incentive to shift their load to off-peak hours. Given a pricing mechanism and a system operator's policy (b_0), consumers are willing to substitute across time only up to a certain level. Even with a high level of demand substitutability, consumers prefer not to shift much of their peak load, to avoid the utility loss caused by peak load shifting. For example, with $b_0 = 1.2$ and $E = 0.08$, consumers under marginal cost pricing choose to shift at most 0.013 peak load (cf. Table E.3). In Fig. 4, for different values of b_0, we compare the maximum amount of peak load consumers choose to shift under the proposed pricing mechanism and the marginal cost pricing mechanism.

We observe from Fig. 4 that the amount of peak load consumers will shift decreases with b_0. This is because a larger reserve at the first stage lowers the price at the second stage, which in turn discourages consumers from shifting their peak load. The proposed pricing mechanism results in a peak load which is approximately 1.5 percent lower than that resulting from marginal cost pricing, regardless of the value of b_0. If the system operator ramps up

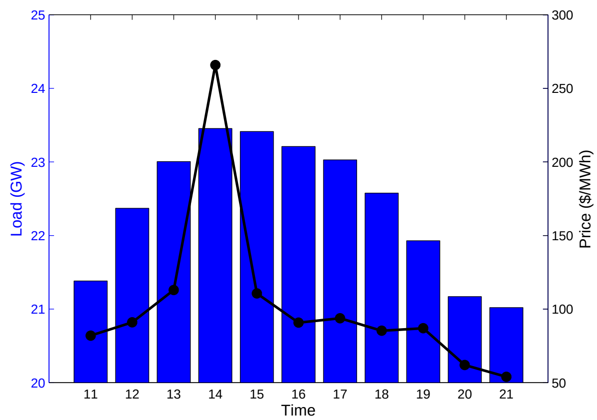

Figure E.5: Real-time prices and actual system loads on August 01, 2011, ISO New England Inc. Blue bars represent the real-time system loads and the dots connected by a black line represent the hourly prices.

the system capacity in advance ($b_0 = 1.2$), marginal cost pricing reduces the system peak load resulting from flat rate pricing by approximately one percent. Compared to marginal cost pricing, the negative price q_1 in the proposed mechanism encourages consumers to make a larger shift of their peak load (cf. the discussion at the end of Section Appendix E.2.1).

Fig. 5 plots the real-time system loads and prices on August 1, 2011, a typical hot summer day in New England[27]. If consumers are able to shift some of their load to the morning (possibly at the expense of losing some utility), the proposed pricing mechanism encourages consumers to shift more of their peak load than marginal cost pricing. Since the highest peak load determines the generation capacity necessary for system reliability, the proposed pricing mechanism has a greater potential to reduce the long-term capacity investment.

[27] www.ferc.gov/market-oversight/mkt-electric/new-england/2011/08-2011-elec-isone-dly.pdf