On Global Games in Social Networks of Information Exchange *

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1. Introduction

It is a common perception that information plays an important role in crises: in particular, exchange of local information about economic or political fundamentals is crucial in determining the outcomes of crisis events. Coordination games of incomplete information have been used as stylized models of crisis phenomena such as currency attacks (e.g., Morris and Shin (1998)), debt crises (e.g., Morris and Shin (2004)), bank runs (e.g., Goldstein and Pauzner (2005)), and political regime change (e.g., Edmond (2005)). To the best of our knowledge, all existing applications of such games to crises assume a continuum of agents and a private (and possibly, in addition, a public) noisy signal about the fundamentals at each agent; there are no complex patterns of communication among the agents. In this work we provide a model of local information sharing through a social network (involving a finite number of discrete agents) and its effect on the outcomes. We seek to answer the question of how do the outcomes depend on the network topology.

We study a coordination game of incomplete information with a finite number of agents, in which each agent receives noisy signals concerning the strength of the status quo (i.e., the fundamentals) according to her position in a social network. The action space for each agent is binary: attack the

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status quo or not attack. Attacking can yield a positive, zero, or negative payoff and is thus the risky action; not attacking nets zero payoff, and is thus the safe action. Our payoff model exhibits strategic complementarities and state monotonicity: an agent's incentive to play the risky action is higher the more other agents coordinate on the risky action, and the weaker the fundamentals.

We model the exchange of idiosyncratic noisy signals about the fundamentals with a link in a graph that represents the social network. We identify the social network topology as a determining factor with respect to the dichotomy between multiplicity and uniqueness of equilibria, and pose the following question: what are necessary and sufficient conditions on the social network topology for uniqueness? The question of uniqueness versus multiplicity is intertwined with the question of predictability of outcomes, as well as the question of amenability of the model to policy implications through comparative statics analyses. We seek to quantify the connection between the topology of the social network and the predictability of individual behavior in large networks, as well as the connection between the topology of the social network and individual attitude towards risk.

Under our model, in a network consisting of finitely many disconnected agents, there is a unique strategy profile that survives iterated elimination of strictly dominated strategies, and therefore a unique Bayesian Nash equilibrium; such a uniqueness result is proven in Carlsson and van Damme (1993b), which assumes a model similar to ours. In sharp contrast, we show that introducing a single link between any two agents induces multiplicity. The network of disconnected agents and the network with a single link are examples of networks that are unions of disconnected cliques. We provide a characterization of strategies that survive iterated elimination of strictly dominated strategies (IESDS) for general networks that are unions of disconnected cliques. We prove that for each agent, all the information about the strength of the status quo can be summarized in a one-dimensional statistic, the average of the signals whose values she knows (her "observations"): in any strategy profile that survives IESDS, each agent chooses the risky action (attack) if the average of the observed signals is less than a certain threshold, and chooses the safe action (not attack) if the average of the observed signals is greater than some other threshold; in addition, any strategy profile that satisfies these two conditions survives IESDS. For the special case of cliques of equal size, we provide a characterization involving closed-form expressions for these thresholds. Our analysis proves multiplicity for unions of disconnected (non-trivial) cliques. We also study the case of asymptotically many agents, and we obtain sufficient conditions on the network topology for asymptotic uniqueness: if each clique grows sublinearly in the number of agents, then we have uniqueness. In contrast, linear growth induces multiplicity. This result can be interpreted as a tradeoff between predictability (uniqueness of equilibria) and degree of information sharing. In addition, the form of the thresholds indicates, at least for the graphs we have studied so far, that a society can become more amenable to risk-taking behavior by increasing the amount of information sharing.

As Angeletos and Werning (2006) put it, "it is a love-hate relationship: economists are at once fascinated and uncomfortable with multiple equilibria." In the economic literature, common knowledge of the fundamentals leads to the standard case of multiple equilibria due to the self-fulfilling nature of agents' beliefs. Morris and Shin (1998, 2000) and others propose that multiplicity vanishes once the economy/society is perturbed away from the complete-information benchmark. In this paper, we show that perturbation may or may not induce uniqueness in the context of a social network of discrete agents, depending on how the noisy signals are communicated, in other words depending on the topology of the social network.

Our game admits a variety of interpretations and applications, in all of which beliefs have the same self-fulfilling nature. Prominent examples are currency attacks (when a large speculative attack forces the central bank to abandon the peg), bank runs (when a large number of bank customers withdraw their deposits because they believe the bank is, or might become, insolvent), debt crises (when a country/company fails to coordinate its creditors to roll over its debt and is hence forced into bankruptcy), and political protests (when a large number of citizens decide whether or not to take actions to subvert a repressive dictator or some other political establishment).

The rest of this paper is organized as follows. Section 2 attempts a concise, and by no means exhaustive, review of the relevant literature. Section 3 introduces the model and defines the process of IESDS. Section 4 analyzes two specific examples of network topologies, the network without edges and the complete network, as well as the complete-information benchmark, for finitely many agents, as well as asymptotically. Section 5 characterizes the strategy profiles that survive IESDS for finite unions of cliques, showing multiplicity. Section 6 provides sufficient conditions on the network topology for asymptotic uniqueness.

2. Literature Review

Carlsson and van Damme (1993a) give the following definition for a global game: "a global game is an incomplete information game where the actual payoff structure is determined by a random draw from a given class of games and where each player makes a noisy observation of the selected game." Methodologically, global games between discrete agents have been studied as a tool for equilibrium selection; examples are Carlsson and van Damme (1993a) and Carlsson and van Damme (1993b). The former proves uniqueness of equilibrium in the limit as the noise about payoffs becomes small for the case of two-action games with two players receiving possibly correlated private signals, under mild technical assumptions; the latter proves uniqueness of equilibrium for n players receiving conditionally independent signals.

Global games of regime change have been used extensively as stylized models of crisis phenomena. It is common in the relevant literature (e.g., Angeletos et al. (2007)) to model games of regime change so that payoffs incur a discrete change when the regime changes. In those models, the outcome of a collective attack against the regime is determined by the relative strength of the collective attack and the regime; once the outcome is determined, individual payoffs for attackers depend merely on the (binary) outcome, not on the relative strength of the collective attack and the regime. We consider a variation in which payoffs are not discrete: individual payoffs for attackers depend directly on the relative strength of the collective attack and the regime. This allows for a continuous modeling of the consequences of a collective attack as reflected on agents' utility.

Another strand of the literature has dealt with the interaction of private and public information, including the effect of public information on the unique rationalizable outcome (Morris and Shin (2002)) and sufficient conditions on the relative precision of private and public information for uniqueness (Hellwig (2002); Angeletos and Werning (2006)). Morris and Shin (2007) provides a characterization of rationalizable actions in a binary action coordination game in terms of beliefs and higher order beliefs, without making any reference to the relative precision of public and private signals.

Our work shares with Chwe (2000) the motivation that locality is represented by information and not necessarily by payoffs, thus building a model based on a local information game. Finally, for a concise review of theory and applications related to global games, we refer the reader to Morris and Shin (2003).

3. The Game

In this section, we introduce the model and define the process of IESDS. Our payoff function is similar to that in Carlsson and van Damme (1993b); it is our information structure that is crucially different from their setup, and this is what allows us to study the effect of local information exchange on the outcomes.

3.1 Agents and Payoffs

Consider a collection of agents $\mathcal{I} = \{1, 2, \ldots, n\}$ who face a status quo. Each agent has the option of attacking the status quo (the risky action R) or not attacking (the safe action S). The payoff to the attackers from taking the risky action depends on the number of agents who participate in the collective attack, as well as on the strength of the regime. The payoff to any given agent is increasing in the number of other agents who take the risky action (i.e., agents' actions are strategic complements), and decreasing in the underlying strength of the regime. The payoff of agents who decide to take the safe action of not attacking is zero, independent of the other agents' actions. More specifically, when the action profile $a = (a_1, \dots, a_n)$ is realized, the payoff of an agent i is given by

$$u_i(a) = \begin{cases} h(k/n) - \theta, & \text{if } a_i = R \text{ and } |\{a_i = R : i \in \mathcal{I}\}| = k, \\ 0, & \text{if } a_i = S, \end{cases}$$

where $\theta \in \mathbb{R}$ is the strength of the regime (i.e., the fundamentals, which we alternatively refer to as the state), and $h : [0,1] \longrightarrow [0,1]$ is a non-decreasing function such that $h(0) = 0$ and $h(1) = 1$. We assume that h is common knowledge among all agents.

3.2 Information Structure

The true strength of the regime is described by a random variable Θ; its realization θ is not known by any of the agents. The agents have a common (improper) prior: the uniform distribution over the entire real line. Each agent i receives an idiosyncratic noisy signal x_i about the state, which is a realization of a random variable X_i. Conditional on the state, agents' idiosyncratic signals are independent and identically distributed: $X_i = \Theta + \Xi_i$, where $\Xi_i \sim \mathcal{N}(0, \epsilon)$, are i.i.d. normal random variables, independent of Θ, with mean zero and variance $\epsilon > 0$.

In addition to her idiosyncratic signal, each agent i observes the signals of a subset $\mathcal{N}_i \subseteq \mathcal{I}$ of the other agents, called her *neighbors*. We specify this neighborhood relation by an undirected graph G, where each vertex corresponds to an agent, and where an edge (i, j) indicates that individuals i and j are neighbors. Throughout the report, we assume that G is common knowledge among all agents. We also use the convention that $i \in \mathcal{N}_i$ for all agents i. We use $\mathcal{V}(G)$ to denote the set of nodes of graph G.

3.3 Strategies and Iterated Elimination of Strictly Dominated Strategies

For any given agent, a strategy is a mapping from the information available to the set of actions. More specifically, a *pure strategy* of agent i is a mapping $s_i : \mathbb{R}^{|\mathcal{N}_i|} \to \{R, S\}$, where $|\mathcal{N}_i|$ denotes the size of agent i's neighborhood. Based on this mapping, each agent i chooses her action as a function of her idiosyncratic signal, as well as the signals observed by her neighbors. Similarly, one can define a *mixed strategy* for each agent as a measurable mapping from her set of observed signals to the set of all possible probability distributions on $\{R, S\}$. We denote the set of all possible mixed strategies of agent i by $\mathcal{S}_i^0$. For ease of readability, we define $y_i = (x_j)_{j\in\mathcal{N}_i}$ and use y_i instead of $(x_j)_{j\in\mathcal{N}_i}$ from now on. For a mixed strategy s_i, we will slightly abuse notation and write expressions such as $s_i(y_i) = 1$ to indicate that, upon signal y_i, the strategy assigns unit probability to the action $a_i = R$.

To formally define iterated elimination of strictly dominated strategies (IESDS), we need a few auxiliary definitions. Let $V_i(s_{-i}|y_i)$ denote the expected payoff of agent i from taking the risky action, when she observes y_i, and the other agents play the strategy profile s_{-i}. We define the following sets recursively, for $m = 0, 1, \ldots$:

$$
\begin{aligned}
&A_i^0 = \emptyset, \qquad\qquad B_i^0 = \emptyset,\\
&\mathcal{S}_i^m = \left\{s_i \in \mathcal{S}_i^0 : s_i(y_i) = 1 \text{ if } y_i \in A_i^m, \text{ and } s_i(y_i) = 0 \text{ if } y_i \in B_i^m\right\},\\
&A_i^{m+1} = \left\{y_i : V_i\left(s_{-i}|y_i\right) > 0 \text{ for all } s_{-i} \in \mathcal{S}_{-i}^m\right\},\\
&B_i^{m+1} = \left\{y_i : V_i\left(s_{-i}|y_i\right) < 0 \text{ for all } s_{-i} \in \mathcal{S}_{-i}^m\right\}.
\end{aligned}
$$

In words, S_i^m is the set of mixed strategies of agent i that survive m rounds of iterated elimination of strictly dominated strategies. For $m = 0$, this is consistent with our earlier definition of $\mathcal{S}_i^0$, and $\mathcal{S}_i^{\epsilon,0} \neq \emptyset$. Note that we are using the notation $\mathcal{S}_{-i}^m$ to denote the Cartesian product of the sets $\mathcal{S}_j^m$, for $j \neq i$; this is the set of opponent strategy profiles s_{-i} such that $s_j \in \mathcal{S}_j^m$, for every $j \neq i$.

Note that as long as $x_j < h(1/n)$ for all $j \in \mathcal{N}_i$, taking the risky action is strictly dominant for agent i. Similarly, as long as $x_j > h(1)$ for all $j \in \mathcal{N}_i$, taking the safe action is strictly dominant for agent i. Therefore, there exist local signal profiles y_i such that, for all opponents' strategy profiles $s_{-i} \in \mathcal{S}_{-i}^0$, we have $V_i\left(s_{-i}|y_i\right) > 0$ (< 0). Since the set $\mathcal{S}_i^0$ is nonempty, it follows from the definitions that the sets A_i^1 and B_i^1 are disjoint. This implies that the set $\mathcal{S}_i^1$ is nonempty. Continuing inductively, we conclude that

$$S_i^m \neq \emptyset, \qquad \forall m > 0, \quad \forall i \in \mathcal{I},$$

and

$$A_i^m \cap B_i^m = \emptyset \qquad \forall m,\ \forall i.$$

Finally, a simple inductive argument shows that for all $i \in \mathcal{I}$ and all $m \geq 0$, the following relations hold:

$$
\begin{aligned}
A_i^m &\subseteq A_i^{m+1}, \\
B_i^m &\subseteq B_i^{m+1}, \\
S_i^{m+1} &\subseteq S_i^m.
\end{aligned}
$$

We now define the following sets, obtained in the limit of infinitely many rounds of iterated elimination of strictly dominated strategies:

$$
\begin{aligned}
A_i &= \bigcup_{m=0}^{\infty} A_i^m \\
B_i &= \bigcup_{m=0}^{\infty} B_i^m \\
S_i &= \bigcap_{m=0}^{\infty} S_i^m
\end{aligned}
$$

Definition 1. *We say that a mixed strategy profile* $\mathbf{s} = (s_1, \ldots, s_n)$ survives iterated elimination of strictly dominated strategies (IESDS) *if* $s_i \in S_i$ *for all* $i \in \mathcal{I}$.

4. Motivating Examples

In this section, we consider some examples that motivate our main research question, answered, to some extent, in Sections 5 and 6: how does the network topology determine uniqueness or multiplicity of equilibria? We study two networks that represent the two extremes of no communication between agents (a network with no edges) and full communication (the complete network). For these two networks, we characterize the strategy profiles that survive IESDS, for the case of a fixed n as well as for the case of an asymptotically large n. For the network without edges, there is an essentially unique strategy profile that survives IESDS, and therefore an essentially unique Bayesian Nash equilibrium.[1] For the complete network, infinitely many strategy profiles survive IESDS, and the game has infinitely many Bayesian Nash equilibria. We also showcase the case where agents observe the state perfectly (i.e., without noise), which yields infinitely many equilibria. What matters for uniqueness is the degree of strategic uncertainty: high strategic uncertainty, as in the case of the network without edges, causes uniqueness; no strategic uncertainty, as in the cases of the complete network and complete information, makes coordination on each of a vast space of different strategies

[1] Nonuniqueness is also present because ties can be broken arbitrarily. However, ties occur only on a set of measure zero. "Essential uniqueness" means that any two strategies that survive IESDS take the same actions with probability one. In the sequel, we will use the simpler term "unique" to mean "essentially unique."

possible.

4.1 Finitely Many Agents

In this subsection, we study the network without edges, the complete network, as well as the case of complete information, all for fixed n.

Example 1 (The Network without Edges)**.** *In this case, $\mathcal{N}_i = \{i\}$, for all $i \in \mathcal{I}$. A variation of this model, and a derivation of the associated uniqueness result, was first presented in Carlsson and van Damme (1993b).*

Proposition 1. *A strategy profile survives IESDS if and only if it satisfies the following: each agent i chooses R if she observes $x_i < \frac{1}{n}\sum_{k=1}^{n} h\left(\frac{k}{n}\right) \equiv t_R$, and S if she observes $x_i > \frac{1}{n}\sum_{k=1}^{n} h\left(\frac{k}{n}\right) \equiv t_S$. In particular, the game has a unique Bayesian Nash equilibrium.*

Proof. For ease of exposition, we only present the proof for the case when $h(x) = x$.

By symmetry, $A_1^m = A_2^m = \cdots = A_n^m$, for all m, and $A_1 = A_2 = \cdots = A_n$; $B_1^m = B_2^m = \cdots = B_n^m$, for all m, and $B_1 = B_2 = \cdots = B_n$. Let us focus on agent i, for some fixed i.

Let $s_{-i}^{A_{-i}^m}$ be the strategy profile in which every agent $j \neq i$ plays R if $x_j \in A_j^m$, and plays S otherwise. Let $s_{-i}^{A_{-i}}$ be the strategy profile in which every agent $j \neq i$ plays R if $x_j \in A_j$, and plays S otherwise.

Let $s_{-i}^{B_{-i}^m}$ be the strategy profile in which every agent $j \neq i$ plays S if $x_j \in B_j^m$, and plays R otherwise. Let $s_{-i}^{B_{-i}}$ be the strategy profile in which every agent $j \neq i$ plays S if $x_j \in B_j$, and plays R otherwise.

We first provide an outline of the argument. For every $m \geq 1$, we define the function $g_m : \mathbb{R} \longrightarrow \mathbb{R}$ by

$$g_m(x_i) = V_i\left(s_{-i}^{A_{-i}^{m-1}} | x_i\right), \forall x_i \in \mathbb{R}.$$

We will show that, for every m, g_m is continuously differentiable; and that there exist positive constants K_1, K_2 such that $dg_m(x_i)/dx_i < 0$ and $K_1 \leq |dg_m(x_i)/dx_i| \leq K_2$, for all x_i and m.

We define the function $g : \mathbb{R} \longrightarrow \mathbb{R}$ by

$$g(x_i) = V_i\left(s_{-i}^{A_{-i}} | x_i\right), \forall x_i \in \mathbb{R}.$$

It then follows that Lipschitz continuity (with the same Lipschitz constants) holds for the (pointwise) limit of the sequence of functions $\{g_m : m \geq 1\}$, which is $g(\cdot) = V_i\left(s_{-i}^{A_{-i}} | \cdot\right)$. A similar argument establishes the Lipschitz continuity of $V_i\left(s_{-i}^{B_{-i}} | \cdot\right)$. Uniqueness then follows easily.

We now continue with the formal proof. Clearly, $A_i^0 = \emptyset$ and $A_i^1 = \left(-\infty, \frac{1}{n}\right)$. Also, $g_1(x_i) \equiv V_i\left(s_{-i}^{A_{-i}^0} | x_i\right) = 1/n - x_i$ is continuously differentiable, and $dg_1/dx_i = -1$.

Assume, for the purpose of induction, that, for some $m > 1$, g_m is continuously differentiable, and there exist positive constants K_1, K_2 such that $dg_m(x_i)/dx_i < 0$, and $K_1 \leq |dg_m(x_i)/dx_i| \leq K_2$, for all x_i, with $K_1 = 1$ and $K_2 = \frac{n-1}{n}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\epsilon}} + 1$. Then there exists a (unique) real number t_m such that $g_m(x) = 0$ if and only if $x = t_m$ and $g_m(x) > 0$ if and only if $x < t_m$.

We have[2]

$$\begin{aligned} g_{m+1}(x_i) &= V_1\left(s_{-i}^{A_{-i}^m} | x_i\right) \\ &= \frac{1}{n}\sum_{j\neq i} \mathbb{P}\left(X_j \in A_j^m \mid x_i\right) + \frac{1}{n} - x_i \\ &= \frac{1}{n}\sum_{j\neq i} \mathbb{P}\left(g_m(X_j) > 0 \mid x_i\right) + \frac{1}{n} - x_i \\ &= \frac{1}{n}\sum_{j\neq i} \mathbb{P}\left(X_j < t_m \mid x_i\right) + \frac{1}{n} - x_i \\ &= \frac{n-1}{n}\Phi\left(\frac{t_m - x_i}{\sqrt{2\epsilon}}\right) + \frac{1}{n} - x_i, \end{aligned} \tag{1}$$

where we use Φ to denote the cumulative distribution function of the standard normal $\mathcal{N}(0,1)$. (By standard Gaussian updating, the distribution of Θ conditional on $\{X_i = x_i\}$ is $\mathcal{N}(x_i, \epsilon)$. Therefore the posterior on $X_j, j \neq i$ is $\mathcal{N}(x_i, 2\epsilon)$.)

We see that g_{m+1} is continuously differentiable. In addition,

$$\begin{aligned} \left|\frac{dg_{m+1}(x_i)}{dx_i}\right| &= \left|\frac{n-1}{n}\phi\left(\frac{t_m - x_i}{\sqrt{2\epsilon}}\right)\frac{1}{\sqrt{2\epsilon}}(-1) - 1\right| \\ &\leq \frac{n-1}{n}\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{2\epsilon}} + 1 \\ &= K_2, \end{aligned}$$

where we use ϕ to denote the probability density function of the standard normal $\mathcal{N}(0,1)$. Therefore $dg_m(x_i)/dx_i < 0$ and $K_1 \leq \left|\frac{dg_m}{dx_i}\right| \leq K_2$, for all x_i. The induction is complete.

It follows that the pointwise limit g of the functions $g_m, m \geq 1$, is Lipschitz continuous in x_i, with the same constants as the sequence $\{g_m, m \geq 1\}$. Therefore there exists a (unique) real number t_R such that $g(x) = 0$ if and only if $x = t_R$ and $g(x) > 0$ if and only if $x < t_R$.

[2]Abusing notation, we use $\mathbb{P}(\cdot \mid x_i)$ to denote $\mathbb{P}(\cdot \mid X_i = x_i)$, here and elsewhere in the paper.

Similarly, the pointwise limit f of the functions $f_m, m \geq 1$, defined by

$$f_m(x_i) = V_i\left(s_{-i}^{B_{-i}^{m-1}}|x_i\right), \forall x_i \in \mathbb{R},$$

is Lipschitz continuous in x_i, with the same constants as the sequence $\{f_m, m \geq 1\}$. Therefore, there exists a unique real number t_S such that $f(x) = 0$ if and only if $x = t_S$ and $f(x) < 0$ if and only if $x > t_S$.

We have

$$\begin{aligned} V_i\left(s_{-i}^{A_{-i}}|x_i\right) &= \frac{1}{n}\sum_{j\neq i}\mathbb{P}\left(X_j \in A_j|x_i\right) + \frac{1}{n} - x_i \\ &= \frac{1}{n}\sum_{j\neq i}\mathbb{P}\left(g(X_j) > 0|x_i\right) + \frac{1}{n} - x_i \\ &= \frac{1}{n}\sum_{j\neq i}\mathbb{P}\left(X_j < t_R|x_i\right) + \frac{1}{n} - x_i \\ &= \frac{n-1}{n}\Phi\left(\frac{t_R - x_i}{\sqrt{2\epsilon}}\right) + \frac{1}{n} - x_i, \end{aligned} \tag{2}$$

where the second equality follows from Lebesgue's dominated convergence theorem. Because $V_i\left(s_{-i}^{A_{-i}}|t_R\right) = 0$ by the definition of t_R, and $\Phi(0) = 1/2$, it follows that $t_R = \dfrac{n+1}{2n}\left(= \dfrac{1}{n}\sum_{k=1}^{n}\dfrac{k}{n}\right)$.

We also have

$$\begin{aligned} V_i\left(s_{-i}^{B_{-i}}|x_i\right) &= \frac{1}{n}\sum_{j\neq i}\mathbb{P}\left(X_j \notin B_j|x_i\right) + \frac{1}{n} - x_i \\ &= \frac{1}{n}\sum_{j\neq i}\mathbb{P}\left(f(X_j) \geq 0|x_i\right) + \frac{1}{n} - x_i \\ &= \frac{1}{n}\sum_{j\neq i}\mathbb{P}\left(X_j \leq t_S|x_i\right) + \frac{1}{n} - x_i \\ &= \frac{n-1}{n}\Phi\left(\frac{t_S - x_i}{\sqrt{2\epsilon}}\right) + \frac{1}{n} - x_i, \end{aligned} \tag{3}$$

where the second equality follows again from Lebesgue's dominated convergence theorem.

Because $V_i\left(s_{-i}^{B_{-i}}|t_S\right) = 0$ by the definition of t_S, it follows similarly that $t_S = t_R = \dfrac{n+1}{2n}\left(= \dfrac{1}{n}\sum_{k=1}^{n}\dfrac{k}{n}\right)$. We have shown that $A_i = (-\infty, t_R)$ and $B_i = (t_S, +\infty)$, with $t_R = t_S$, which implies uniqueness.

□

Example 2 (The Complete Network). *In this case, $\mathcal{N}_i = \mathcal{I}$, for all $i \in \mathcal{I}$.*

Proposition 2. *A strategy profile survives IESDS if and only if it satisfies the following: each agent i chooses R if $\frac{\sum_{i=1}^n x_i}{n} < h\left(\frac{1}{n}\right) \equiv t_R$, and S if $\frac{\sum_{i=1}^n x_i}{n} > h(1) \equiv t_S$. The game has infinitely many Bayesian Nash equilibria. In particular, fix some t with $h(1/n) \leq t \leq h(1)$ and consider a strategy profile whereby each agent chooses R is she observes $\frac{\sum_{i=1}^n x_i}{n} < t$, and S if she observes $\frac{\sum_{i=1}^n x_i}{n} > t$. Such a strategy profile is a Bayesian Nash equilibrium.*

Proof. By symmetry, $A_1^m = A_2^m = \cdots = A_n^m$, for all m, and $A_1 = A_2 = \cdots = A_n$; $B_1^m = B_2^m = \cdots = B_n^m$, for all m, and $B_1 = B_2 = \cdots = B_n$. Fix agent i. Let $s_{-i}^{A_{-i}^m}$ be the strategy profile in which every agent $j \neq i$ plays R if $(x_1, \ldots, x_n) \in A_j^m$, and plays S otherwise. Let $s_{-i}^{B_{-i}^m}$ be the strategy profile in which every agent $j \neq i$ plays S if $(x_1, \ldots, x_n) \in B_j^m$, and plays R otherwise.

Is is easy to see that

$$A_i^1 = \left\{(x_1, \ldots, x_n) : \frac{\sum_{j=1}^n x_j}{n} < h\left(\frac{1}{n}\right)\right\}$$

and

$$B_i^1 = \left\{(x_1, \ldots, x_n) : \frac{\sum_{j=1}^n x_j}{n} > h(1)\right\}.$$

Then,

$$\begin{aligned} V_i\left(s_{-i}^{A_{-i}^1} \middle| x_1, \ldots, x_n\right) &= h(1) \cdot \mathbb{P}\left(\frac{\sum_{j=1}^n x_j}{n} < h\left(\frac{1}{n}\right) \middle| x_1, \ldots, x_n\right) \\ &\quad + h\left(\frac{1}{n}\right) \cdot \mathbb{P}\left(\frac{\sum_{j=1}^n x_j}{n} \geq h\left(\frac{1}{n}\right) \middle| x_1, \ldots, x_n\right) - \frac{\sum_{j=1}^n x_j}{n} \\ &= \begin{cases} h(1) - \frac{\sum_{j=1}^n x_j}{n}, & \text{if } \frac{\sum_{j=1}^n x_j}{n} < h\left(\frac{1}{n}\right), \\ h\left(\frac{1}{n}\right) - \frac{\sum_{j=1}^n x_j}{n}, & \text{if } \frac{\sum_{j=1}^n x_j}{n} \geq h\left(\frac{1}{n}\right). \end{cases} \end{aligned}$$

Because $A_i^2 = \left\{(x_1, \ldots, x_n) : V_i\left(s_{-i}^{A_{-i}^1} \middle| x_1, \ldots, x_n\right) > 0\right\}$, it follows that

$$A_i^2 = A_i^1 = \left\{(x_1, \ldots, x_n) : \frac{\sum_{j=1}^n x_j}{n} < h\left(\frac{1}{n}\right)\right\}.$$

Furthermore,

$$\begin{aligned} V_i\left(s_{-i}^{B_{-i}^1}|x_1,\ldots,x_n\right) &= h(1)\cdot\mathbb{P}\left(\frac{\sum_{j=1}^n x_j}{n}\leq h(1)\big|x_1,\ldots,x_n\right)\\ &\quad + h\left(\frac{1}{n}\right)\cdot\mathbb{P}\left(\frac{\sum_{j=1}^n x_j}{n}> h(1)\big|x_1,\ldots,x_n\right)-\frac{\sum_{j=1}^n x_j}{n}\\ &= \begin{cases} h(1)-\frac{\sum_{j=1}^n x_j}{n}, & \text{if } \frac{\sum_{j=1}^n x_j}{n}\leq h(1),\\ h\left(\frac{1}{n}\right)-\frac{\sum_{j=1}^n x_j}{n}, & \text{if } \frac{\sum_{j=1}^n x_j}{n}> h(1). \end{cases} \end{aligned}$$

Because $B_i^2=\left\{(x_1,\ldots,x_n): V_i\left(s_{-i}^{B_{-i}^1}\big|x_1,\ldots,x_n\right)<0\right\}$, it follows that

$$B_i^2=B_i^1=\left\{(x_1,\ldots,x_n):\frac{\sum_{j=1}^n x_j}{n}>h(1)\right\}.$$

Similarly, $A_i^m=\left\{(x_1,\ldots,x_n):\frac{\sum_{j=1}^n x_j}{n}<h\left(\frac{1}{n}\right)\right\}$ and $B_i^m=\left\{(x_1,\ldots,x_n):\frac{\sum_{j=1}^n x_j}{n}>h(1)\right\}$, for all rounds of elimination $m\geq 1$. It follows that $A_i=\left\{(x_1,\ldots,x_n):\frac{\sum_{j=1}^n x_j}{n}<h\left(\frac{1}{n}\right)\right\}$ and $B_i=\left\{(x_1,\ldots,x_n):\frac{\sum_{j=1}^n x_j}{n}>h(1)\right\}$.

That the strategy profiles of the form described in the second part of the proposition are indeed Bayesian Nash equilibria can be verified by inspection. □

Example 3 (Complete Information)**.** *In this case, we assume that $x_i=\theta$, for all $i\in\mathcal{I}$. Informally, this can be viewed as a version of our earlier model, with zero observation noise, i.e., with $\epsilon=0$. With this model, common knowledge of the fundamentals gives rise to the standard case of multiple equilibria due to the self-fulfilling nature of agents' beliefs. Clearly, in this case, the structure of the network is immaterial.*

Proposition 3. *A strategy profile survives IESDS if and only if it satisfies the following: each agent i chooses R if she observes that $\theta<h\left(\frac{1}{n}\right)\equiv t_R$, and S if she observes that $\theta>h(1)\equiv t_S$. The game has infinitely many Bayesian Nash equilibibria. In particular, fix some t with $h\left(\frac{1}{n}\right)\leq t\leq h(1)$ and consider a strategy profile whereby each agent chooses R is she observes that $\theta<t$, and S if she observes that $\theta>t$. Such a strategy profile is a Bayesian Nash equilibrium.*

Proof. As before, we use the definition $y_i=(x_j)_{j\in\mathcal{N}_i}$ for ease of readability.

By symmetry, $A_1^m=A_2^m=\cdots=A_n^m$, for all m, and $A_1=A_2=\cdots=A_n$; $B_1^m=B_2^m=\cdots=B_n^m$, for all m, and $B_1=B_2=\ldots=B_n$. Fix agent i. Let $s_{-i}^{A_{-i}^m}$ be the strategy profile in which every agent

$j \neq i$ plays R if $y_j \in A_j^m$, and plays S otherwise. Let $s_{-i}^{B_{-i}^m}$ be the strategy profile in which every agent $j \neq i$ plays S if $y_j \in B_j^m$, and plays R otherwise.

It is easy to see that

$$\begin{aligned} A_i^1 &= \left\{ y_i : x_i < h\left(\frac{1}{n}\right) \right\}, \\ B_i^1 &= \left\{ y_i : x_i > h(1) \right\}. \end{aligned}$$

Then

$$\begin{aligned} V_i\left(s_{-i}^{A_{-i}^1} | y_i\right) &= h(1) \cdot \mathbb{P}\left(x_i < h\left(\frac{1}{n}\right) | y_i\right) \\ &\quad + h\left(\frac{1}{n}\right) \cdot \mathbb{P}\left(x_i \geq h\left(\frac{1}{n}\right) | y_i\right) - x_i \\ &= \begin{cases} h(1) - x_i, & \text{if } x_i < h\left(\frac{1}{n}\right), \\ h\left(\frac{1}{n}\right) - x_i, & \text{if } x_i \geq h\left(\frac{1}{n}\right). \end{cases} \end{aligned}$$

Because $A_i^2 = \left\{ y_i : V_i\left(s_{-i}^{A_{-i}^1} | y_i\right) > 0 \right\}$, it follows that

$$A_i^2 = A_i^1 = \left\{ y_i : x_i < h\left(\frac{1}{n}\right) \right\}.$$

Furthermore,

$$\begin{aligned} V_i\left(s_{-i}^{B_{-i}^1} | y_i\right) &= h(1) \cdot \mathbb{P}\left(x_i \leq h(1) | y_i\right) \\ &\quad + h\left(\frac{1}{n}\right) \cdot \mathbb{P}\left(x_i > h(1) | y_i\right) - x_i \\ &= \begin{cases} h(1) - x_i, & \text{if } x_i \leq h(1), \\ h\left(\frac{1}{n}\right) - x_i, & \text{if } x_i > h(1). \end{cases} \end{aligned}$$

Because $B_i^2 = \left\{ y_i : V_i\left(s_{-i}^{B_{-i}^1} | y_i\right) < 0 \right\}$, it follows that

$$B_i^2 = B_i^1 = \left\{ y_i : x_i > h(1) \right\}.$$

Similarly, $A_i^m = \left\{ y_i : x_i < h\left(\frac{1}{n}\right) \right\}$ and $B_i^m = \left\{ y_i : x_i > h(1) \right\}$, for all rounds of elimination $m \geq 1$. It follows that $A_i = \left\{ y_i : x_i < h\left(\frac{1}{n}\right) \right\}$ and $B_i = \left\{ y_i : x_i > h(1) \right\}$.

That the strategy profiles of the form described in the second part of the proposition are indeed Bayesian Nash equilibria can be verified by inspection. □

4.2 Asymptotically Many Agents

In all of the preceding examples, the set of strategies that survive IESDS are characterized by certain thresholds. We now consider the limit as the number of agents n increases, and the behavior of the associated thresholds.

For the network without edges, we have $\lim_{n\to\infty} t_R = \lim_{n\to\infty} t_S = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} h\left(\frac{k}{n}\right) = \int_0^1 h(x)dx$. (The Riemann-integrability of h follows from its monotonicity.) For the complete network, as well as for the case of complete information, we have $\lim_{n\to\infty} t_R = \lim_{n\to\infty} h\left(\frac{1}{n}\right) = h(0) = 0$, assuming continuity of h at 0.

Tables 1 and 2 summarize the results for the three examples, for fixed number of agents as well as for an asymptotically large number of agents, and for positive noise as well as for zero noise. As in the statements of the Propositions, we use t_R (t_S) to denote the threshold pertaining to playing R (S).

Table 1: Thresholds for strategies that survive IESDS for the network without edges and for the complete information case.

Network without Edges	Fixed n	$n \to \infty$
$\epsilon > 0$	$t_R = \frac{1}{n}\sum_{k=1}^{n} h(k/n)$	$t_R = \int_0^1 h(x)dx$
$\epsilon > 0$	$t_S = \frac{1}{n}\sum_{k=1}^{n} h(k/n)$	$t_S = \int_0^1 h(x)dx$
$\epsilon = 0$	$t_R = h(1/n)$	$t_R = h(0)$
$\epsilon = 0$	$t_S = h(1)$	$t_S = h(1)$

Table 2: Thresholds for strategies that survive IESDS for the complete network and for the complete information case.

Complete Network	Fixed n	$n \to \infty$
$\epsilon > 0$	$t_R = h(1/n)$	$t_R = h(0)$
$\epsilon > 0$	$t_S = h(1)$	$t_S = h(1)$
$\epsilon = 0$	$t_R = h(1/n)$	$t_R = h(0)$
$\epsilon = 0$	$t_S = h(1)$	$t_S = h(1)$

5. Characterization of Strategy Profiles that Survive IESDS for Finite Unions of Cliques

In this section, we characterize the set of strategy profiles that survive IESDS for finite networks that are unions of disconnected cliques. We first provide a generic solution, which we then use to come up with a closed-form solution for the special case of cliques of equal size. Our results establish multiplicity of equilibria for the case of finitely many agents. This multiplicity arises because agents in the same clique can use their shared information to coordinate their actions in multiple ways.

5.1 Generic Characterization

Assume that the network consists of M disconnected cliques; clique $i \in \{1, \ldots, M\}$ has n_i nodes, and $\sum_{i=1}^{M} n_i = n$. We have the following result:

Proposition 4 (Characterization for finite unions of cliques)**.** *There exist thresholds $\{t_R^c, t_S^c\}_{c=1}^M$ such that a strategy profile survives IESDS if and only if it satisfies the following: each agent i in clique c chooses R if $\frac{\sum_{j\in c} x_j}{n_c} < t_R^c$ and S if $\frac{\sum_{j\in c} x_j}{n_c} > t_S^c$. Furthermore, the thresholds $\{t_R^c, t_S^c\}_{c=1}^M$ solve the following system of equations (here a choice of l corresponds to selecting r out of the $M-1$ cliques):*

$$\begin{aligned}
t_R^c &= h\left(\frac{n-n_c+1}{n}\right)\mathbb{P}\left(\forall d \neq c, \frac{\sum_{j\in d} X_j}{n_d} < t_R^d \,\Big|\, \frac{\sum_{j\in c} X_j}{n_c} = t_R^c\right) \\
&\quad + \sum_{r=1}^{M-2}\sum_{l=1}^{\binom{M-1}{r}} h\left(\frac{n-\sum_{d\neq c, d \text{ selected by } l} n_d - n_c + 1}{n}\right) p_R^{r,l} \\
&\quad + h\left(\frac{1}{n}\right)\mathbb{P}\left(\forall d \neq c, \frac{\sum_{j\in d} X_j}{n_d} \geq t_R^d \,\Big|\, \frac{\sum_{j\in c} X_j}{n_c} = t_R^c\right), \qquad \forall c \in \{1, \ldots, M\}, \\
t_S^c &= h(1)\mathbb{P}\left(\forall d \neq c, \frac{\sum_{j\in d} X_j}{n_d} \leq t_S^d \mid \frac{\sum_{j\in c} X_j}{n_c} = t_S^c\right) \\
&\quad + \sum_{r=1}^{M-2}\sum_{l=1}^{\binom{M-1}{r}} h\left(\frac{n-\sum_{d\neq c, d \text{ selected by } l} n_d}{n}\right) p_S^{r,l} \\
&\quad + h\left(\frac{n_c}{n}\right)\mathbb{P}\left(\forall d \neq c, \frac{\sum_{j\in d} X_j}{n_d} > t_S^d \mid \frac{\sum_{j\in c} X_j}{n_c} = t_S^c\right), \qquad \forall c \in \{1, \ldots, M\},
\end{aligned}$$

and where

$$p_R^{r,l} = \mathbb{P}\left(\text{only for the } r \text{ cliques selected by } l \ \frac{\sum_{j\in d} X_j}{n_d} \geq t_R^d \Big| \ \frac{\sum_{j\in c} X_j}{n_c} = t_R^c\right)$$

and

$$p_S^{r,l} = \mathbb{P}\left(\text{only for the } r \text{ cliques selected by } l \ \frac{\sum_{j\in d} X_j}{n_d} > t_S^d \mid \frac{\sum_{j\in c} X_j}{n_c} = t_S^c\right).$$

Notice that due to our normality assumptions, $\frac{\sum_{j\in c} x_j}{n_c}$ is a sufficient statistic for $\{x_j\}_{j\in c}$ with respect to θ, and hence with respect to the signals of other cliques, for all cliques c.

Example 4. *We showcase the characterization for the simple network of Figure 1. The relevant thresholds,*

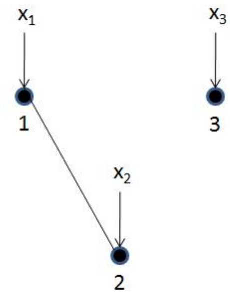

Figure 1: A simple network.

$\{t_R^{\{1,2\}}, t_S^{\{1,2\}}, t_R^{\{3\}}, t_S^{\{3\}}\}$, *satisfy the following system of equations:*

$$\begin{aligned}
t_R^{\{1,2\}} &= h\left(\frac{2}{3}\right)\mathbb{P}\left(X_3 < t_R^{\{3\}} \,\middle|\, \frac{X_1+X_2}{2} = t_R^{\{1,2\}}\right) + h\left(\frac{1}{3}\right)\mathbb{P}\left(X_3 \geq t_R^{\{3\}} \,\middle|\, \frac{X_1+X_2}{2} = t_R^{\{1,2\}}\right)\\
t_S^{\{1,2\}} &= h(1)\mathbb{P}\left(X_3 \leq t_S^{\{3\}} \,\middle|\, \frac{X_1+X_2}{2} = t_S^{\{1,2\}}\right) + h\left(\frac{2}{3}\right)\mathbb{P}\left(X_3 > t_S^{\{3\}} \,\middle|\, \frac{X_1+X_2}{2} = t_S^{\{1,2\}}\right)\\
t_R^{\{3\}} &= h(1)\mathbb{P}\left(\frac{X_1+X_2}{2} < t_R^{\{1,2\}} \,\middle|\, X_3 = t_R^{\{3\}}\right) + h\left(\frac{1}{3}\right)\mathbb{P}\left(\frac{X_1+X_2}{2} \geq t_R^{\{1,2\}} \,\middle|\, X_3 = t_R^{\{3\}}\right)\\
t_S^{\{3\}} &= h(1)\mathbb{P}\left(\frac{X_1+X_2}{2} \leq t_S^{\{1,2\}} \,\middle|\, X_3 = t_S^{\{3\}}\right) + h\left(\frac{1}{3}\right)\mathbb{P}\left(\frac{X_1+X_2}{2} \geq t_S^{\{1,2\}} \,\middle|\, X_3 = t_S^{\{3\}}\right).
\end{aligned}$$

The system can be solved numerically. The solutions for different values of ϵ, *for the case when* $h(x) = x$, *are shown in Table 3. For small values of* ϵ, *the thresholds for the clique of two agents and the single agent are*

Table 3: Thresholds for strategies that survive IESDS for the network in Figure 1, for $h(x) = x$, and different values of ϵ.

	$\epsilon = 0.0001$	$\epsilon = 1$	$\epsilon = 10000$
Agents 1, 2	$t_R = 0.5474, t_S = 0.7859$	$t_R = 0.5132, t_S = 0.8202$	$t_R = 0.5002, t_S = 0.8332$
Agent 3	$t_R = 0.5518, t_S = 0.7815$	$t_R = 0.6345, t_S = 0.6989$	$t_R = 0.6662, t_S = 0.6671$

close. As the noise increases, the extent of multiplicity (more formally, the difference $t_S - t_R$*) grows larger for the two agents in the clique* $\{1,2\}$*, but smaller for the single agent* 3*. For the clique of two agents, less precise signals lead to multiplicity of greater extent; on the contrary, for the single agent, a less precise signal leads to more refined multiplicty, which gets very close to uniqueness for large* ϵ*. We notice that the difference* $t_S - t_R$ *is more sensitive to changes in the noise for the single agent than it is for the clique of two agents.*

5.2 Characterization in the Case of Cliques of Equal Size

In the special case of M equally sized cliques, we can write down closed-form expressions for t_R and t_S. Notice that each clique has size n/M.

Corollary 1. *A strategy profile survives IESDS if and only if it satisfies the following. Each agent i in clique c chooses R if $\frac{\sum_{j\in c} x_j}{n/M} < t_R$, and S if $\frac{\sum_{j\in c} x_j}{n/M} > t_S$, where*

$$t_R = \frac{1}{M}\left(h\left(\frac{n-n/M+1}{n}\right) + \sum_{r=1}^{M-2} h\left(\frac{n-r(n/M)-(n/M)+1}{n}\right) + h\left(\frac{1}{n}\right)\right),$$

and

$$t_S = \frac{1}{M}\left(h(1) + \sum_{r=1}^{M-2} h\left(\frac{n-r(n/M)}{n}\right) + h\left(\frac{n/M}{n}\right)\right).$$

Example 5. *We showcase the characterization for the simple network of Figure 2. The thresholds are given by*

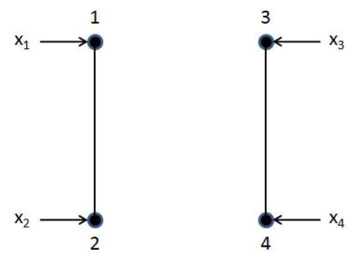

Figure 2: A simple network with cliques of equal sizes.

$$t_R = \frac{1}{2}\left(h\left(\frac{3}{4}\right) + h\left(\frac{1}{4}\right)\right)$$
$$t_S = \frac{1}{2}\left(h(1) + h\left(\frac{1}{2}\right)\right).$$

Notice that $t_R \leq \frac{1}{4}\left(h(1) + h\left(\frac{3}{4}\right) + h\left(\frac{2}{4}\right) + h\left(\frac{1}{4}\right)\right) \leq t_S$, where $\frac{1}{4}\left(h(1) + h\left(\frac{3}{4}\right) + h\left(\frac{2}{4}\right) + h\left(\frac{1}{4}\right)\right)$ is the threshold pertaining to the network of 4 disconnected agents. Thus, there exist equilibria involving a threshold which is larger than that for the disconnected network (i.e., the society is "braver") as well as equilibria involving a threshold which is smaller than that for the disconnected network (i.e., the society is "less brave"). In particular, more communication can make society either more or less brave.

6. The Case of Asymptotically Many Agents

We have seen that for a finite number of agents, links induce multiplicity of strategy profiles that survive IESDS. The natural question that arises is under what conditions on the network there exists asymptotically a unique strategy profile that survives IESDS, and thus a unique Bayesian Nash equilibrium. In this section we provide sufficient conditions for uniqueness in the case of an asymptotically large number of agents.

We consider a growing sequence of graphs G_k, $k = 1, 2, \ldots$, with $G_k \subseteq G_{k+1}$. The graph G_k consists of $g(k)$ cliques of equal size $f(k) \geq 1$, for a total of $n(k) = f(k)g(k)$ nodes. We assume that the function f is nondecreasing in k. For example, the graph could grow by adding more cliques of the same size, or by merging existing cliques to form larger cliques. For any k, the strategy profiles that survive IESDS are described by the thresholds in Corollary 1. We denote these thresholds by t_R^k and t_S^k to indicate explicitly the dependence on k.

The proposition that follows shows that if the number of cliques grows to infinity (equivalently, if the clique size grows sublinearly with the number of agents), then t_R^k and t_S^k converge to a common value, as k increases. Thus, loosely speaking, in the limit, there is an essentially unique strategy that survives IESDS, and therefore a unique Bayesian Nash equilibrium.

6.1 Sufficient Conditions for Asymptotic Uniqueness

Proposition 5 (Asymptotic uniqueness). *Suppose that* $\lim_{m\to\infty} g(m) = \infty$. *Then,*

$$\lim_{k\to\infty} t_R^k = \lim_{k\to\infty} t_S^k = \int_0^1 h(x)dx.$$

Proof. By Corollary 1, we have

$$\begin{aligned}
\lim_{k\to\infty} t_S^k &= \lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=1}^{g(k)} h\left(\frac{jf(k)}{n(k)}\right) \\
&= \lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=1}^{g(k)} h\left(\frac{j}{g(k)}\right) \\
&= \int_0^1 h(x)dx.
\end{aligned}$$

Again by Corollary 1, we have

$$\lim_{k\to\infty} t_R^k = \lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=0}^{g(k)-1} h\left(\frac{jf(k)+1}{n(k)}\right).$$

Notice that for all k,

$$\lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=0}^{g(k)-1} h\left(\frac{jf(k)}{n(k)}\right) \leq \lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=0}^{g(k)-1} h\left(\frac{jf(k)+1}{n(k)}\right) \leq \lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=1}^{g(k)} h\left(\frac{jf(k)}{n(k)}\right).$$

We showed above that

$$\lim_{k\to\infty} \frac{1}{g(k)} \sum_{j=1}^{g(k)} h\left(\frac{jf(k)}{n(k)}\right) = \int_0^1 h(x)dx.$$

Similarly, we have

$$\begin{aligned}
\lim_{k\to\infty} \frac{1}{g(k)} \sum_{k=0}^{g(k)-1} h\left(\frac{jf(k)}{n(k)}\right) &= \lim_{k\to\infty} \frac{1}{g(k)} \left(\sum_{k=0}^{g(k)} h\left(\frac{jf(k)}{n(k)}\right) - h(1)\right) \\
&= \lim_{k\to\infty} \frac{1}{g(k)} \sum_{k=1}^{g(k)} h\left(\frac{j}{g(k)}\right) \\
&= \int_0^1 h(x)dx.
\end{aligned}$$

By a standard sandwich argument, it follows that

$$\lim_{k\to\infty} t_R^k = \int_0^1 h(x)dx.$$

□

We note that we can use a similar argument to establish that if the growth of each clique is linear in n, then we have multiplicity: the thresholds satisfy

$$\lim_{k\to\infty} t_R^k < \lim_{k\to\infty} t_S^k.$$

6.2 Interpretation

We believe that Proposition 5 extends to the case of unions of cliques of unequal sizes, when the fastest-growing clique grows sublinearly with the total number of agents n. We also note that the case, in Proposition 5, of sublinearly growing cliques leads to the same asymptotic equilibrium analysis as the case of disconnected agents. Loosely speaking, in general we can view the properties of equilibria for the case of asymptotically many agents as being governed by two competing effects: the network is growing, and the sharing of information among agents is also growing. An intuitive explanation why the equilibrium analysis for the two aforementioned sequences of networks is asymptotically the same, and yields uniqueness, is the following: the two sequences of networks, in the limit of large n, are informationally equivalent; precisely, for both sequences of networks the growth of information sharing is insignificant compared to the growth of the network, and this gap induces a unique equilibrium. In turn, we can view uniqueness of equilibria as predictability of individual behavior.

On the other hand, we conjecture that for the case of unions of disconnected cliques, when the fastest-growing clique grows linearly with the total number of agents n, there are asymptotically infinitely many strategy profiles that survive IESDS. The intuitive interpretation is that the growth of the sharing of information among agents is comparable to the growth of the network; the excess in communication is what breaks uniqueness (and predictability of individual behavior).

If our conjectures hold, there are no sequences of networks that are unions of disconnected cliques for which uniqueness of Bayesian Nash equilibrium is obtained asymptotically, at a unique threshold other than $\int_0^1 h(x)dx$. If this is so, we cannot come up with sequences of networks for which the unique threshold shifts, signifying a societal shift in favor of or against taking a risky action. (A shift of the threshold to higher values would signify that rational individuals are willing to play the risky action over a signal space that includes higher observations, corresponding to higher realizations of the fundamentals. A shift of the threshold to lower values would signify that rational individuals are only willing to play the risky action over a signal space that is limited to lower observations, corresponding to lower realizations of the fundamentals.)

7. Current Work

Having addressed the case of unions of disconnected cliques, we are currently developing results for more general topologies. We have already identified non-trivial network topologies that yield uniqueness in the case of finitely many agents, and have been developing results that classify more general finite networks into a class of networks that induce uniqueness and a class that induce multiplicity of Bayesian Nash equilibria. We thus come closer to a complete characterization of topological conditions for uniqueness versus multiplicity. We have been also studying a generalization of the model presented in this paper, according to which agents do not observe each other's idiosyncratic signals, but instead observe information coming from different sources. This generalization subsumes both undirected and directed networks.

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