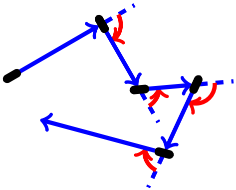
MIPS: from phase separation to pattern formation

Julien Tailleur

M2-ICFP

Run-and-tumble bacteria

[Berg & Brown, Nature, 1972]



- Run ●: straight line, velocity $v \simeq 20\,\mu\mathrm{m.s}^{-1}$, duration $\tau_\mathrm{r} \simeq 1s$
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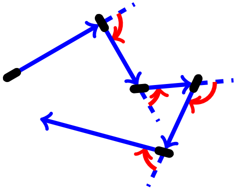

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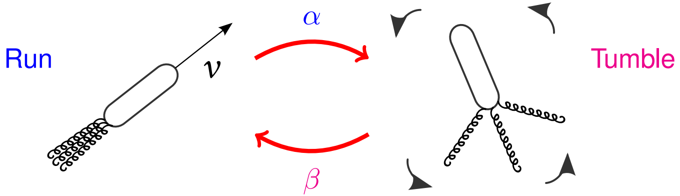

Position-dependent motility

- Non-uniform speed $v(\mathbf{r})$, tumbling rate $\alpha(\mathbf{r}) = {\tau_{\rm r}}^{-1}(\mathbf{r})$, swimming rate $\beta(\mathbf{r}) = {\tau_{\rm t}}^{-1}(\mathbf{r})$

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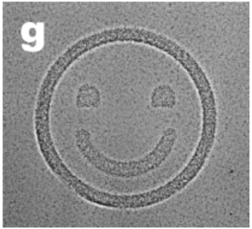
- 'Painting with bacteria'

[Frangipane *et al.*, e-life (2018)]



[Arlt *et al.*, Nat. Com. (2018)]

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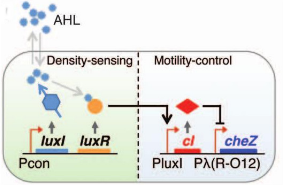
[Liu et al., Science (2011)]

- Bacteria produce diffusing molecules that regulates protein expressions

- cheZ favors swimming over tumbling



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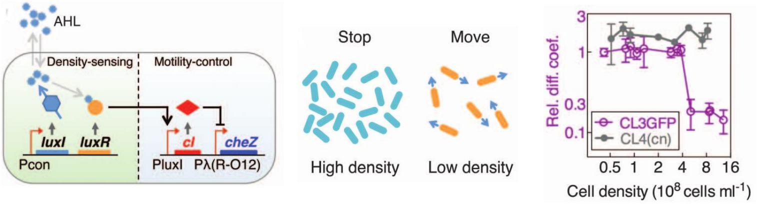

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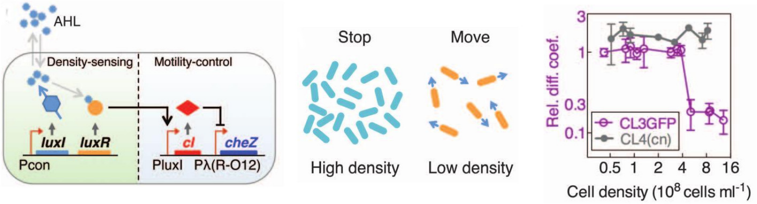

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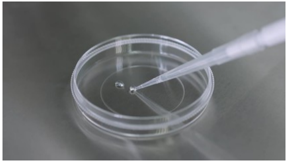

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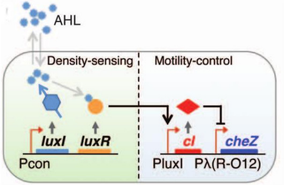

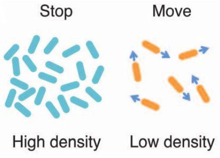

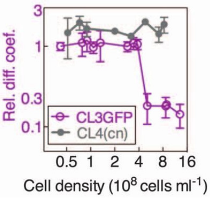

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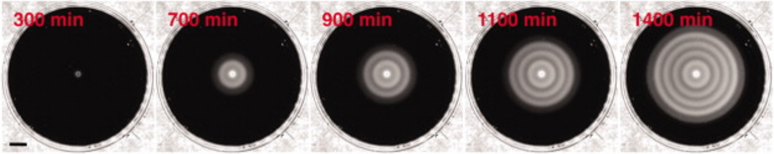
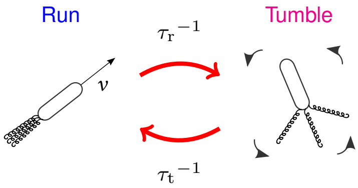


Motility-Induced Phase Separation (MIPS)

- Particle are less motile at high density (QS interactions)
- Bacteria accumulate where they are less motile $\rho \propto \frac{\tau_{\mathrm{r}}(\mathbf{r})+\tau_{\mathrm{t}}(\mathbf{r})}{\tau_{\mathrm{r}}(\mathbf{r})v(\mathbf{r})}$



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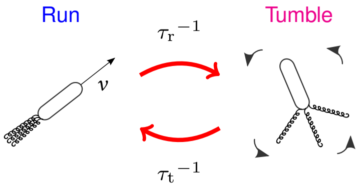
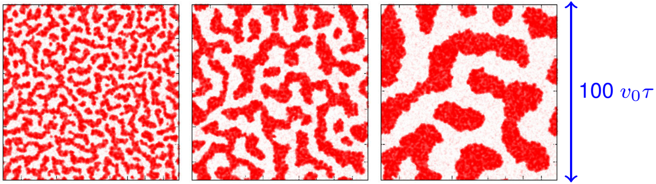


→ Particle accumulate at high density: instability

  - Microscopic simulations [A. Curatolo]: slower at high density

$t = 10^3\tau$ $t = 10^4\tau$ $t = 10^5\tau$ $\tau \equiv {\tau_r}^0 + {\tau_t}^0$

$v'(\rho) < 0$



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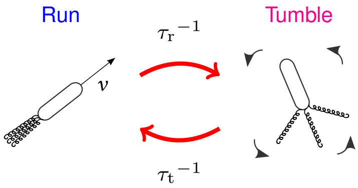

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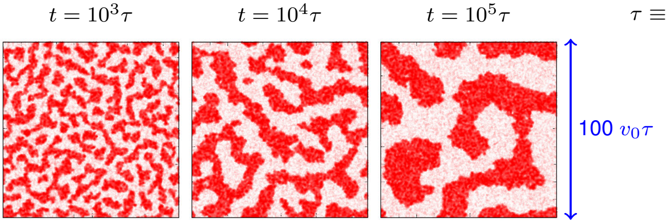

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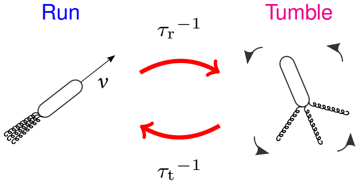

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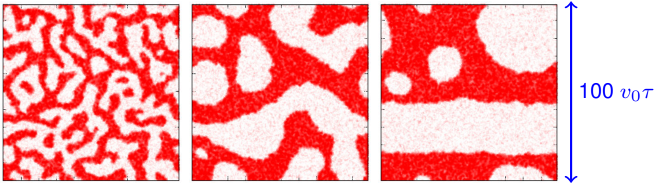

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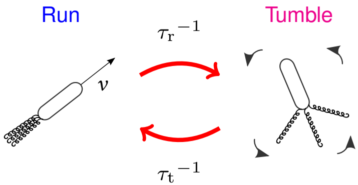

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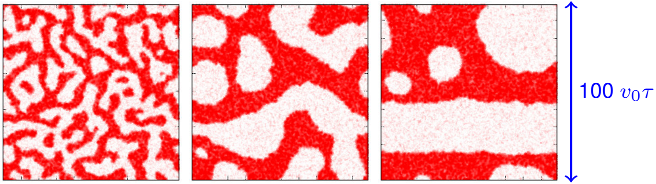

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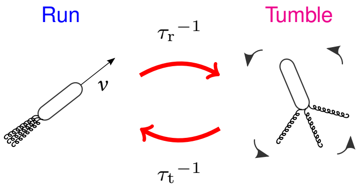

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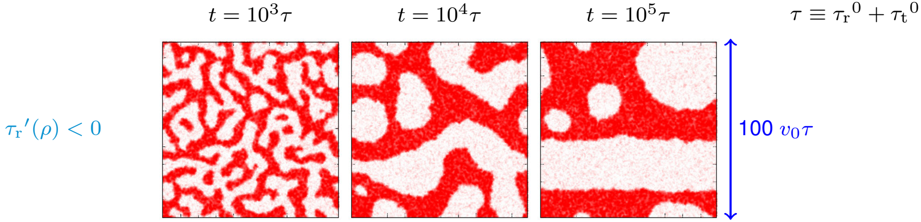

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- Slow coarsening: population dynamics !

Pattern formation in bacterial colonies: a simple mechanism

- Large-scale description of run & tumble dynamics: $\dot{\rho} = \nabla \cdot \left[\frac{v^2 \beta}{d\alpha(\beta+\alpha)} \nabla \rho + \frac{v\rho}{d\alpha} \nabla \frac{v\beta}{\alpha+\beta} \right]$

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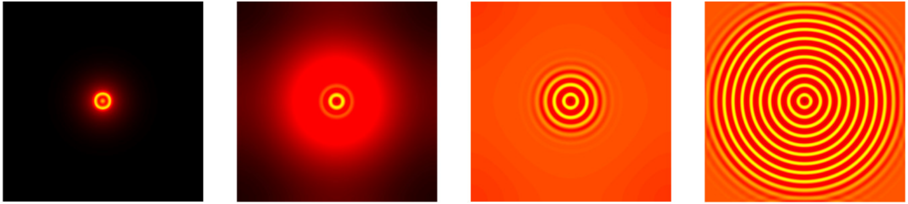

$\longrightarrow$ Qualitatively accounts for the experiments

- Motility-induced phase separation + population dynamics $\longrightarrow$ Selects patterns of fixed size

  [Cates *et al.* PNAS 2010; Liu *el al.* Science 2011]