

Motility-Induced Phase Separation

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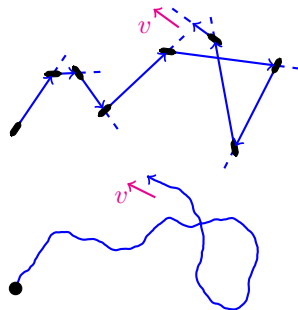
ICFP 2022

Liquid-gas phase separation & self-propulsion

- Phase separation in Equilibrium: Energy vs Entropy
 - Attractive forces: cohesion vs disorder
 - Lowering T : liquid-gas coexistence

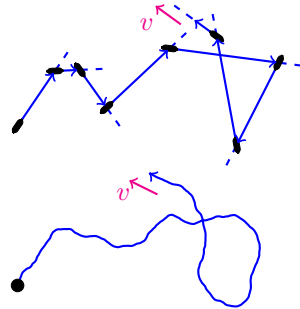
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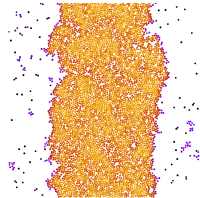
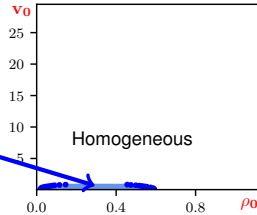


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Lennard-Jones •

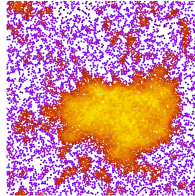
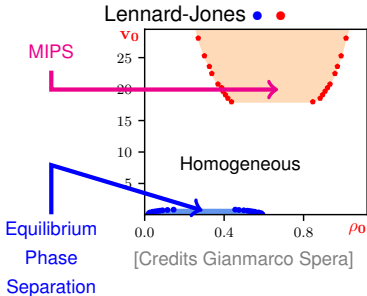
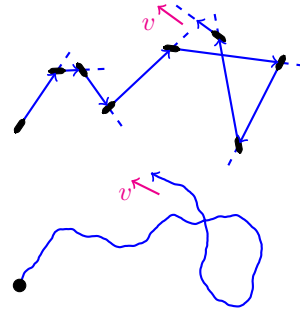


Equilibrium
Phase
Separation

[Credits Gianmarco Spera]

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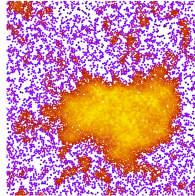
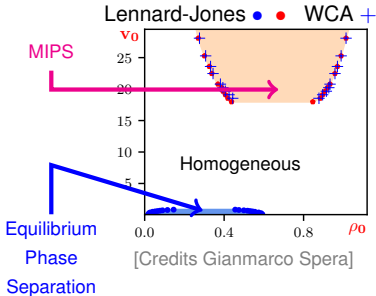
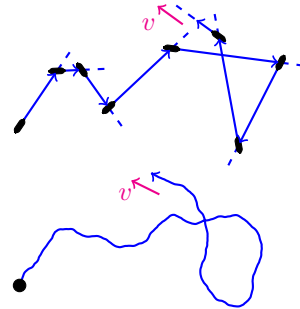
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 - Reentrant transition [Redner *et al.* PRE 2013]



- Motility-Induced Phase Separation (MIPS)

Liquid-gas phase separation & self-propulsion

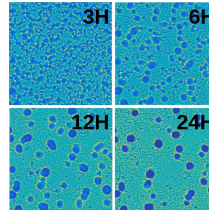
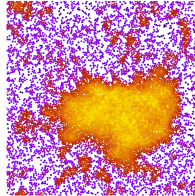
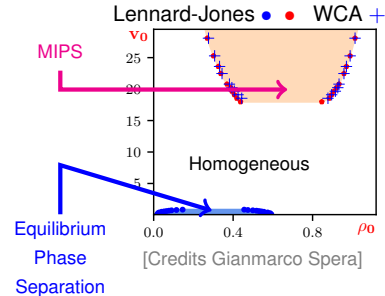
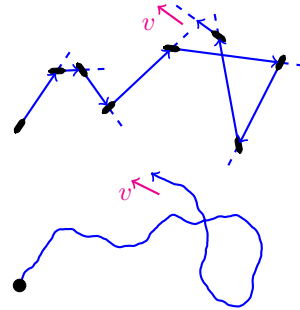
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 - No need of attractive forces!



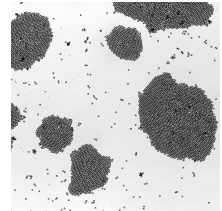
- Motility-Induced Phase Separation (MIPS) → Cohesive matter without cohesive forces

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[Liu *et al.* PRL 2019]



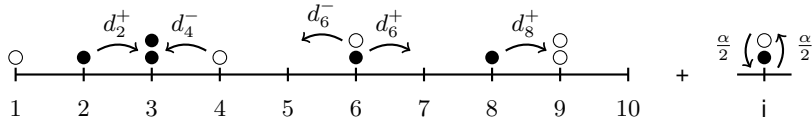
[Van Der Linden *et al.* PRL 2019]

- Motility-Induced Phase Separation (MIPS) → Cohesive matter without cohesive forces

Phase coexistence without cohesive forces

- **Lattice-gas model** of run & tumble particles (RTP)

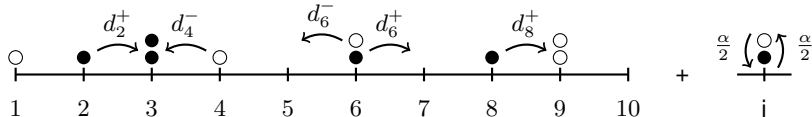
[Thompson et al. JSTAT 2011, Soto & Golestanian, PRE 2014]



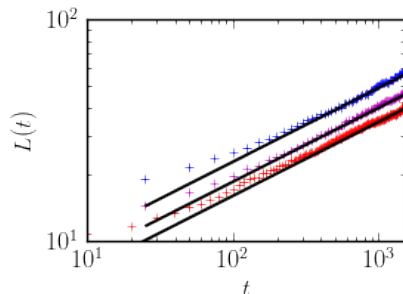
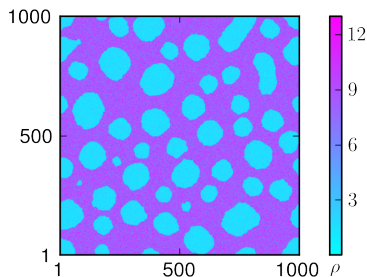
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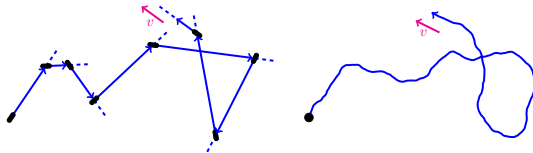
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- Exclusion: $d_i^\pm = v_0(1 - \frac{n_{i\pm 1}}{n_M})$

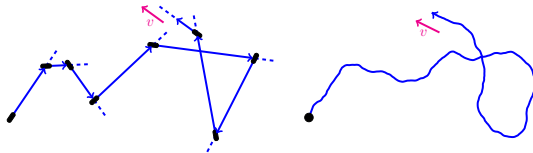


- Quorum-sensing self-propelled particles with tumbles/rot. diff.



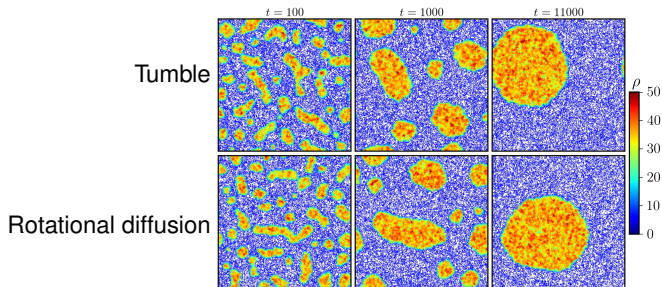
$$\dot{\mathbf{r}}_i = v[\tilde{\rho}(\mathbf{r}_i)]\mathbf{u}(\theta_i); \quad \tilde{\rho}(\mathbf{r}_i) = \sum_j K(|\mathbf{r}_i - \mathbf{r}_j|)$$

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- If $v(\rho)$ decreases as ρ increases $\longrightarrow$ Phase separation

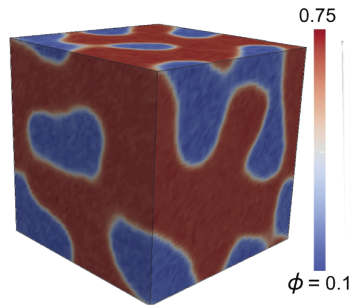
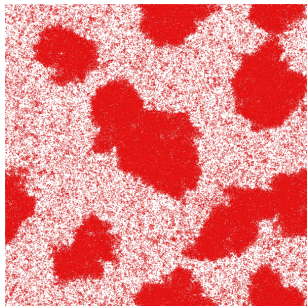


[Cates, JT, EPL 2013; Solon, Cates, JT, EPJST 2015]

- Self-propelled particles with pairwise forces

[Fily & Marchetti PRL 2012, Redner *et al.* PRL 2013, Stenhammar *et al.* PRL 2013, Bialké *et al.* PRL 2013, etc.]

$$\dot{\mathbf{r}}_i = v\mathbf{u}(\theta_i) - \mu \sum_j F_{ij}(\mathbf{r}_i - \mathbf{r}_j) + \sqrt{2D_t}\eta_i; \quad \dot{\theta}_i = \sqrt{2D_r}\xi_i$$



- Not-so-self-propelled particles with pairwise forces

[Fodor *et al.* PRL 2016, Martin *et al.* PRE 2021]

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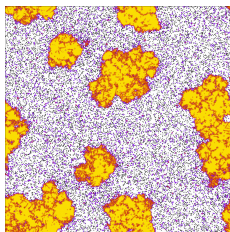
- Particles powered by Ornstein-Uhlenbeck process $\langle \mathbf{v}_{\mathbf{i}}(t) \mathbf{v}_{\mathbf{i}}(0) \rangle = \frac{D}{\tau} e^{-t/\tau}$

- Not-so-self-propelled particles with pairwise forces

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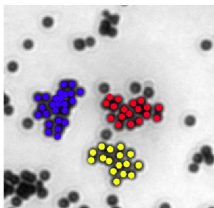
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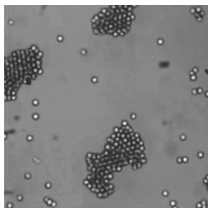


- Very generic in toy models of active systems

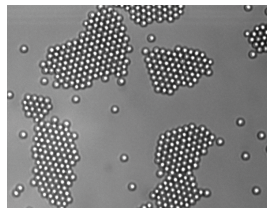
- Experiments: Active colloids (role of attractive interactions?)



[Theurkhauff *et al.* PRL 2012]



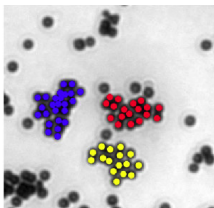
[Buttinoni *et al.* PRL 2013]



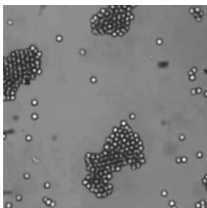
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- Enhanced tendency to clustering

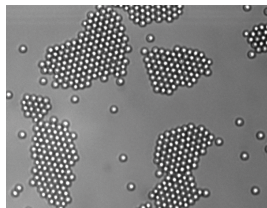
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- Enhanced tendency to clustering
- Real life **far too messy** (hydrodynamics, chemically mediated interactions, systems slow & dilute, aligning interactions, etc.)
- Idea: **understand simple systems first** before moving to more complicated situations.