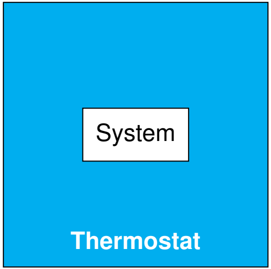
Introduction to Active Matter

Julien Tailleur

MIT-8.08–8.S308 IAP 2025

Equilibrium Statistical Mechanics



- Large thermostat with chaotic dynamics
- Exchange energy with the system
- Drives the system towards thermal equilibrium

$\longrightarrow$ Boltzmann distribution $P_{\text{stat}}(\mathcal{C}) \propto \exp[-\beta E(\mathcal{C})]$

$\longrightarrow$ Standard results of Thermodynamics hold

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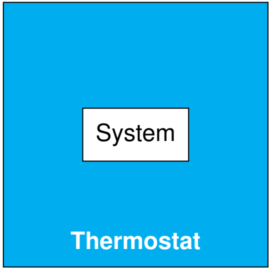

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- Replacing dynamics by statics ⟶ huge simplification
- Give control over wide variety of material (from tooth paste to liquid cristal displays)
- Only a small fraction of our world is in equilibrium (nothing living, moving, driven, dissipative, etc.)

Non-equilibrium physics is like non-elephant biology

Some common definitions

- no steady state
- no Boltzmann weight
- no time-reversal symmetry

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Some examples

Glasses

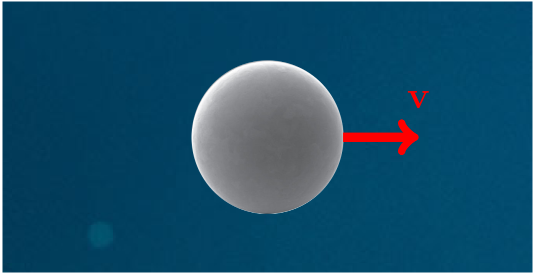
Boundary driven

Active matter

Identify coherent subclasses and say something smart/useful about them!

Active Matter at the microscopic scale

[O'Byrne, Kafri, Tailleur, van Wijland, arXiv:2104.03030, Nat. Rev. Phys., In Press]



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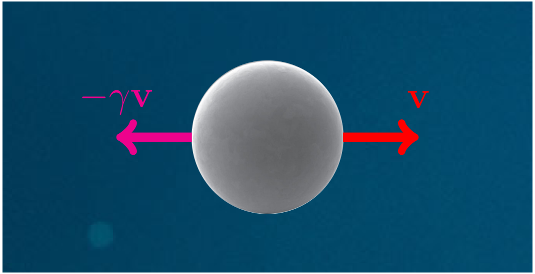

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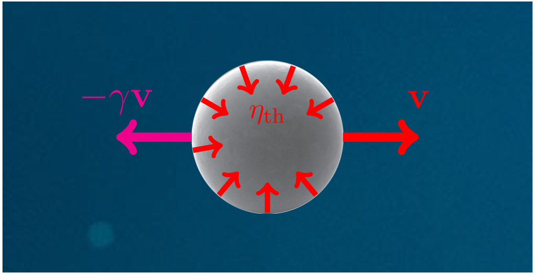

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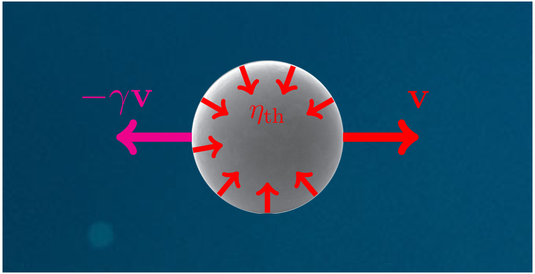

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⟶ Boltzmann weight $P_{\rm s}(x) \propto \exp[-\beta V_{\rm ext}(x)]$

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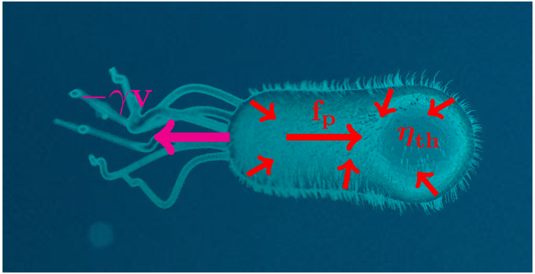

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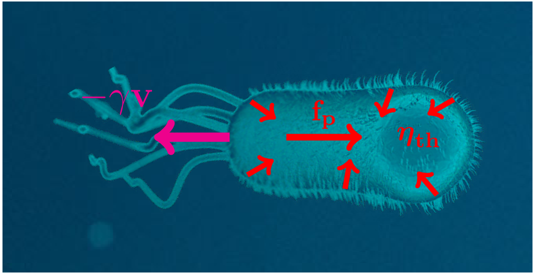

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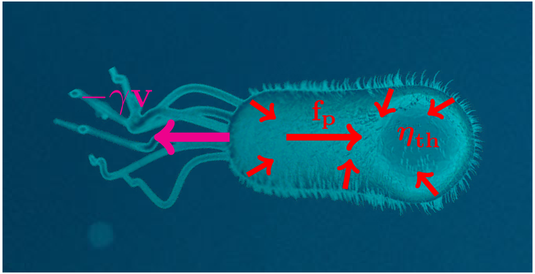

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- Active matter: ⟶ No FDT: system driven out of equilibrium

  ⟶ Dissipation: mean (drag) force from the fluid $\propto \gamma$

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  ⟶ Unknown steady state

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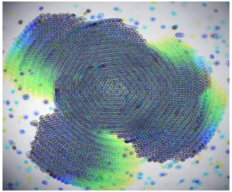
- Already complex at the single-particle level

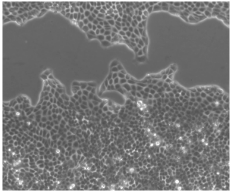
Active Matter at the macroscopic scale

Drive at the microscopic level ⟶ *Strongly out of equilibrium* ⟶ *New physics*



Starfish oocytes
[Fakhri's Lab]

Bird Flocks
[COBBS Lab, Rome]

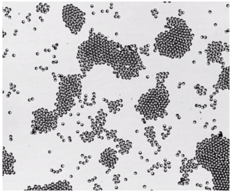


Crawling cells
[Silberzan's lab]

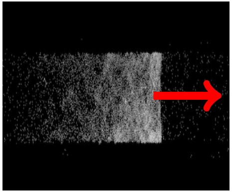
- Biological relevance
- Explore new dynamical phenomenology

Active Matter at the macroscopic scale

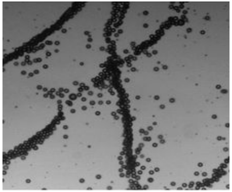
Drive at the microscopic level → *Strongly out of equilibrium* → *New physics*



Clusters without attractive interactions
[van der Linden, PRL 2019]



Solitonic waves
[Bricard , Nature 2013]



Filaments
[Thutupalli, PNAS 2018]

- Biological relevance
- Active Soft Materials

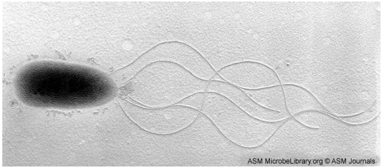
- Explore new dynamical phenomenology
- Build generic framework for Active Matter

Outline of the lectures

- What is an active particle? The standard microscopic models
  - Run-and-Tumble Particles (RTPs)
  - Active Brownian Particles (ABPs)
- Collective behaviors at the macroscopic scale
  - Motility-induced phase separation
  - Transition to collective motion

Run-and-Tumble Particles: a model for swimming bacteria

- *Escherichia coli* – Unicellular Organism $(1\mu m \times 3\mu m)$
- Flagella (few μm long)
- Electron microscopy



Swimming at zero Reynolds

- $Re = \frac{rv}{\nu} \simeq \frac{10^{-6}m\ 10^{-5}m.s^{-1}}{10^{-6}m^2 s^{-1}} \simeq \mathbf{10^{-5}} \longrightarrow$ Inertia irrelevant

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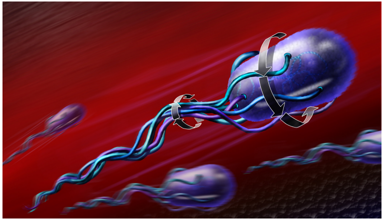
- Scallop theorem [Purcell (1976)] •
- Flagella rotating counter-clockwise

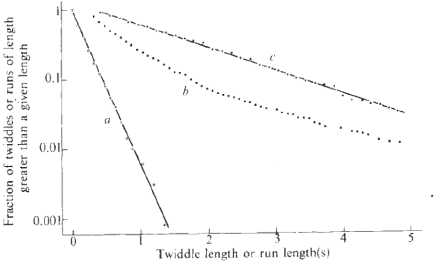


Run and Tumbles

- All the flagella rotate counter-clockwise $\longrightarrow$ run •

Run and Tumbles

- All the flagella rotate counter-clockwise ⟶ run •
- One or more flagella change direction ⟶ tumble •
- Internal process (Poisson distributed, Berg 1972)



Schematic trajectory [Berg & Brown, Nature, 1972]

- Run: straight line (velocity $v \simeq 20\,\mu\mathrm{m.s}^{-1}$)
- Tumble: change of direction (rate $\nu \simeq 1\,\mathrm{s}^{-1}$, duration $\tau \simeq 0.1s$)

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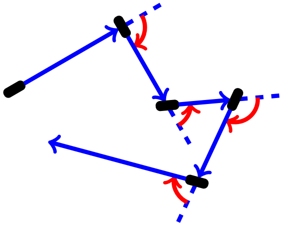

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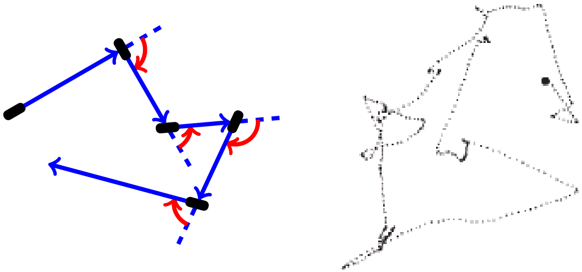
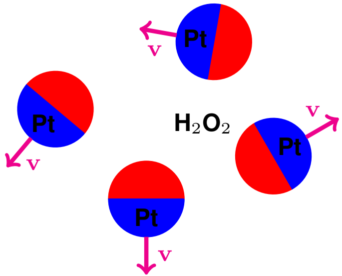


- Diffusion at large scale
  - Run-and-Tumble $D = \frac{v^2}{d\nu(1+\nu\tau)} \sim 100\,\mu m^2.s^{-1}$
  - Brownian Motion $D_{col} = \frac{kT}{6\pi\eta r} \sim 0.2\mu m^2.s^{-1}$

Active Brownian Particles to model self-propelled colloids

- Colloids with asymmetric coating
- Self [diffusio-] phoresis
- Self propulsion $\mathbf{v}$



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- Rotational diffusion crucial

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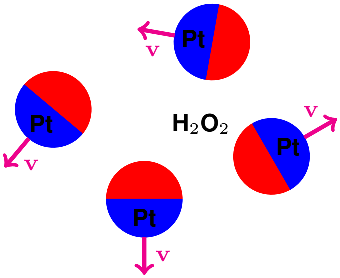

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- Light-controlled [Palacci *et al.* Science **339**, 936 (2013)] •

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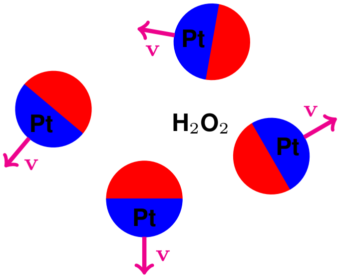

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$$\dot{\mathbf{r}} = v_0 \mathbf{u}(\theta); \qquad \dot{\theta} = \sqrt{2D_r}\eta \quad \longrightarrow \quad D_{\text{eff}} = \frac{v_0^2}{2D_r}$$