Motility-Induced Phase Separation

Julien Tailleur

8.08/8.S308, IAP 2025

Liquid-gas phase separation & self-propulsion

- Phase separation in Equilibrium: Energy vs Entropy

$\longrightarrow$ Attractive forces: cohesion vs disorder

$\longrightarrow$ Lowering T: liquid-gas coexistence

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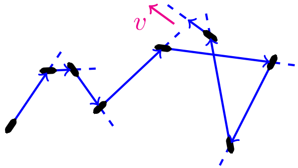
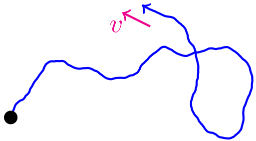


- Active particles: increasing v first vaporises the liquid •



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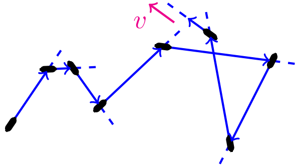

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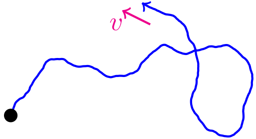

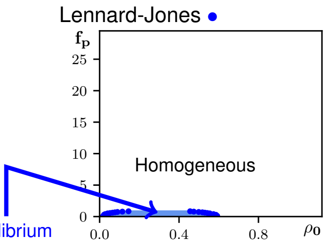

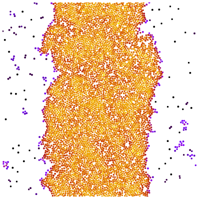

Equilibrium Phase Separation

[Credits Gianmarco Spera]

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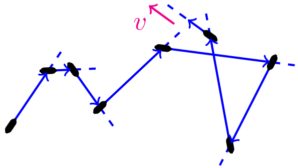
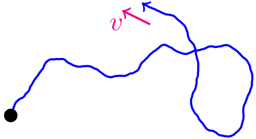
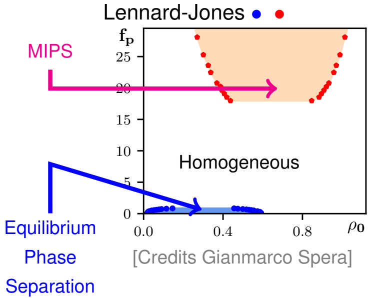


- Active particles: increasing v first vaporises the liquid •
  - ⟶ Reentrant transition [Redner *et al.* PRE 2013]





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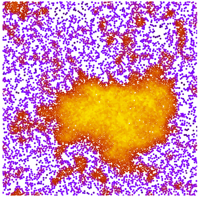

- Motility-Induced Phase Separation (MIPS)

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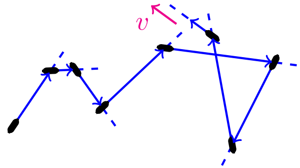

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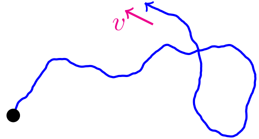

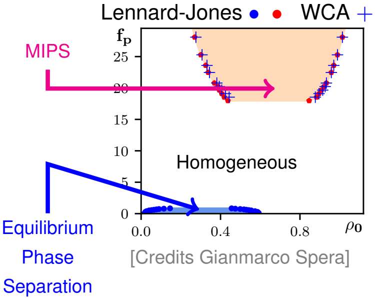

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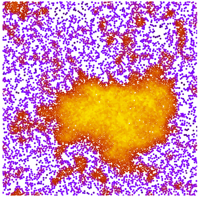

- Motility-Induced Phase Separation (MIPS) ⟶ Cohesive matter without cohesive forces

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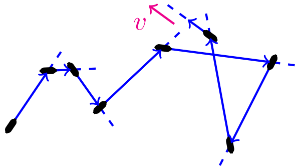

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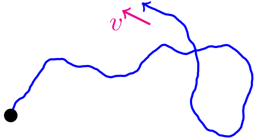

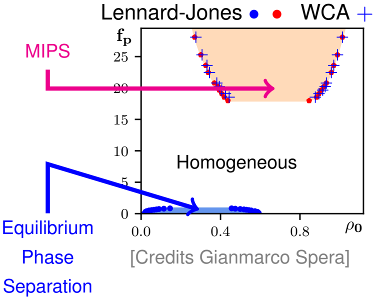

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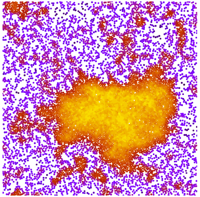
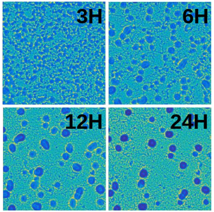
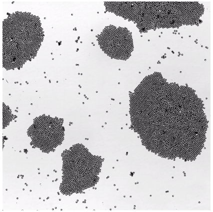




[Liu *et al.* PRL 2019]

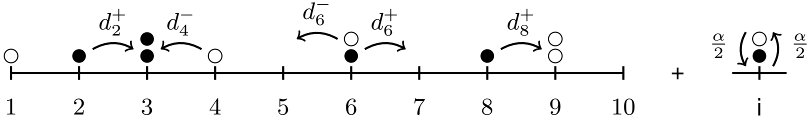


[Van Der Linden *et al.* PRL 2019]

- Motility-Induced Phase Separation (MIPS) ⟶ Cohesive matter without cohesive forces

Phase coexistence without cohesive forces

- Lattice-gas model of run & tumble particles (RTP)
  [Thompson et al. JSTAT 2011, Soto & Golestanian, PRE 2014]



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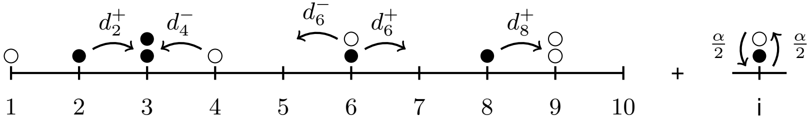
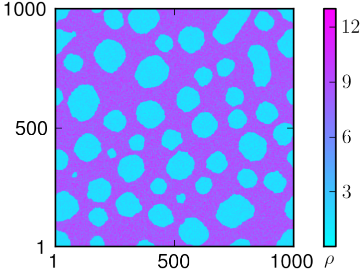
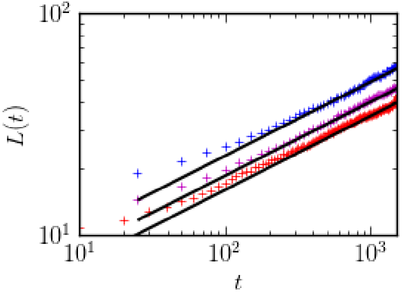
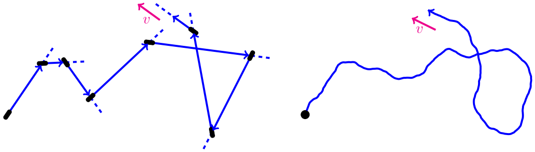


- Exclusion: $d_i^\pm = v_0(1 - \frac{n_{i\pm1}}{n_M})$





- Quorum-sensing self-propelled particles with tumbles/rot. diff.



$$\dot{\mathbf{r}}_\mathbf{i} = v[\tilde{\rho}(\mathbf{r}_\mathbf{i})]\mathbf{u}(\theta_i); \qquad \tilde{\rho}(\mathbf{r}_\mathbf{i}) = \sum_j K(|\mathbf{r}_\mathbf{i} - \mathbf{r}_\mathbf{j}|)$$

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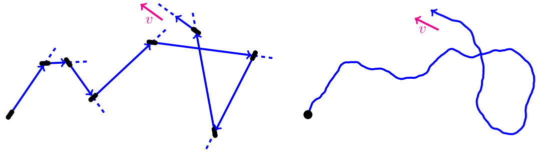

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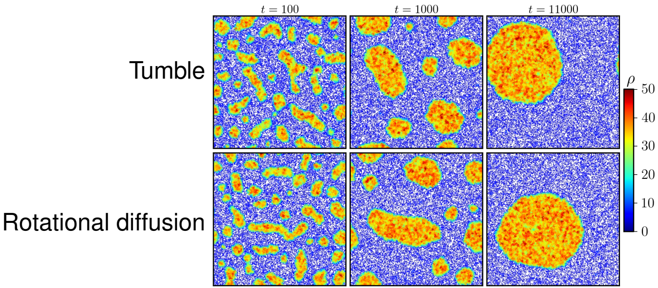
- If $v(\rho)$ decreases as ρ increases ⟶ Phase separation

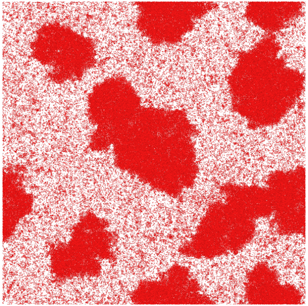


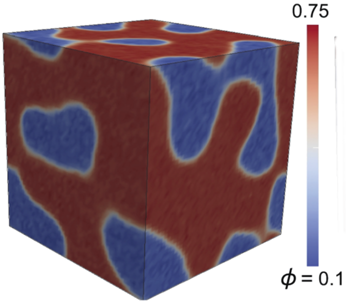
[Cates, JT, EPL 2013; Solon, Cates, JT, EPJST 2015]

- Self-propelled particles with pairwise forces

  [Fily & Marchetti PRL 2012, Redner *et al.* PRL 2013, Stenhammar *et al.* PRL 2013, Bialké *et al.* PRL 2013, etc.]

$$\dot{\mathbf{r}}_{\mathbf{i}} = v\mathbf{u}(\theta_i) - \mu \sum_j F_{ij}(\mathbf{r}_{\mathbf{i}} - \mathbf{r}_{\mathbf{j}}) + \sqrt{2D_t}\eta_i; \qquad \dot{\theta}_i = \sqrt{2D_r}\xi_i$$





- Not-so-self-propelled particles with pairwise forces
  [Fodor *et al.* PRL 2016, Martin *et al.* PRE 2021]

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- Particles powered by Ornstein-Uhlenbeck process $\langle \mathbf{v}_\mathbf{i}(t)\mathbf{v}_\mathbf{i}(0)\rangle = \frac{D}{\tau}e^{-t/\tau}$

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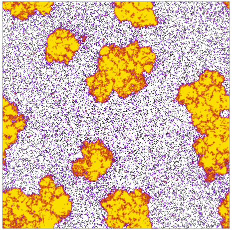
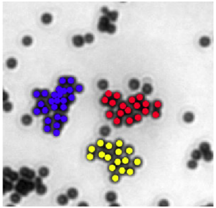
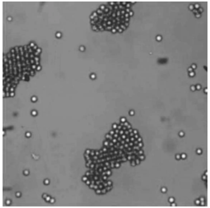


- Very generic in toy models of active systems

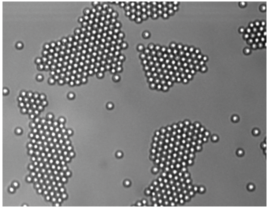
- Experiments: Active colloids (role of attractive interactions?)



[Theurkhauff *et al.* PRL 2012]



[Buttinoni *et al.* PRL 2013]



[Palacci *et al.* Science 2014]

- Enhanced tendency to clustering

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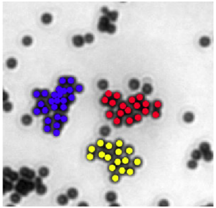

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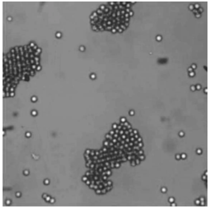

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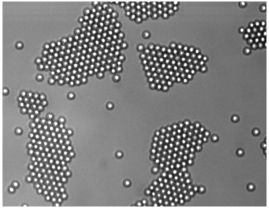

[Palacci *et al.* Science 2014]

- Enhanced tendency to clustering

- Real life far too messy (hydrodynamics, chemically mediated interactions, systems slow & dilute, aligning interactions, etc.)

- Idea: understand simple systems first before moving to more complicated situations.