Finally, we need to an opacity. Two commonly used examples are

$$\begin{aligned} \bar{\kappa} &= \kappa_{\rm es} \qquad \text{(electron scattering)} \\ &= \kappa_0 \rho T^{-7/2} \qquad \text{(Kramer's law)}\ . \end{aligned} \tag{635}$$

7. **Equation of state.**
   Two choices are typically used:

$$\begin{aligned} P &= \frac{\rho}{\mu} k_B T \qquad \text{or} \\ &= \frac{4\sigma}{3c} T^4\ , \end{aligned} \tag{636}$$

   depending on whether you think gas pressure or radiation pressure is more important. Which do you use? That's a question of phenomenology: you try both, and see which describes your data better.

To summarize, the α-disk model uses the following set of coupled equations:

$$T_{\rm eff} = \left(\frac{3G}{8\pi}\right)^{1/4} M^{1/4} \dot{M}^{1/4} f \tag{637}$$

$$PH = \alpha^{-1} \frac{\sqrt{GM}}{2\pi} \dot{M} r^{-3/2} f^4 \tag{638}$$

$$T^4 = \frac{3}{4} \bar{\kappa} \bar{\rho} H T^4_{\rm eff} \tag{639}$$

$$P = \frac{GM}{r^3} \rho H^2 \tag{640}$$

$$= \rho k_B T / \mu \qquad \text{or} \qquad \frac{4\sigma}{3c} T^4 \tag{641}$$

with a choice for the $\bar{\kappa}$ we use. Once the opacity and the equation of state are specified, we can build remarkably (well, relatively) simple solutions for the whole thing.

23.7 *Observations of Accretion*

The single simplest signature of accretion onto a compact object is the luminosity. If a disk is optically thin, a single proton falling through it and onto the central object releases

$$\Delta E = \frac{GMm_p}{R} \approx 0.2 m_p c^2 \tag{642}$$

(for a neutron star). If all this went into thermal energy, we'd then expect from

$$\Delta E \sim k_b T \tag{643}$$

to find

$$T \sim \frac{GMm_p}{k_b R} \sim 100 \text{ MeV} \tag{644}$$

(again, for a neutron star).

Based on this simple estimate, we should expect accreting neutron stars to be gamma-ray sources. But in fact, disks are typically quite optically thick. Emitted photons are scattered and reprocessed many times, so that the luminosity comes out at lower energies. Things saturate at the Eddington luminosity (Eq. 455); when this is set equal to the Stefan-Boltzmann law

$$\frac{4\pi c G M m_p}{\sigma_T} = 4\pi R^2 \sigma_{SB} T_{\text{eff}}^4 \tag{645}$$

the result is that

$$k_b T_{\text{eff}} \approx \text{ keV} \tag{646}$$

for neutron stars – i.e., at X-ray energies.

To map the disk itself in more detail, we can use spectroscopy. A rotating disk exhibits a gradient of radial velocities along its surface, determined by both the disk's Keplerian velocity profile and by the viewing angle. This turns the emission from your favorite, single emission line in the disk into a double-peaked shape, with each velocity component corresponding to a particular velocity. Especially when combined with the orbital motion of the disk in the binary system, and sometimes also with eclipses of the disk by the secondary star, one can infer quite a detailed picture of the disk in question.

Different types of stars exhibit different types of behavior when undergoing accretion:

– **Normal stars** (i.e., not compact remnants). These add mass from one member of the binary to the other. As discussed in Sec. 23.4, these can have the odd situation that the more evolved member of the binary is less massive. This is because it swelled up, overflowed its Roche lobe, and dumped mass onto its companion. One such example is the eclipsing binary Algol.

– **White dwarfs.** These are particularly interesting, since they exhibit variable behavior – these are the so-called **cataclysmic variables.** There are a few types:

  1. *Dwarf nova:* an instability in $\dot{M}$ leads to sudden, occasional brightenings of the source.
  2. *Classical nova:* sufficient H accumulates on the WD's surface until it gets hot and dense enough to initiate fusion. In a degenerate medium, this initiates flash burning: the whole layer fuses very rapidly.

3. *Supernova Ia:* It is possible for a white dwarf to accrete enough mass to go over the Chandrasekhar mass limit and collapse. As described in Sec. 19.6, this is one (though probably not the dominant) pathway to forming SNe Ia.

– **Neutron stars.** These are seen in X-rays and have two two categories.

1. *High-mass X-ray binaries* (HMXBs): In this case the companion to the neutron star is a young, massive O or B star. The NS has a strong magnetic field, and X-ray flux is seen to pulsate. Most of these will evolve into NS-NS binaries.

2. *Low-mass X-ray binaries* (LMXBs): The companion to the NS is $\lesssim 1 M_\odot$, and so the system is considerably older. As we saw in Fig. 48 and Sec. 21.5, the NS's magnetic field is thus somewhat weaker. As with the HMXBs, the X-ray flux is seen to pulsate in some cases. Most of these LMXBs will likely evolve into millisecond pulsars.
   Before they become ms pulsars, LMXBs also exhibit two types of behavior. "Type I" show a sharp rise in luminosiry, followed by a slow decline; this indicates the thermonuclear detonation of accreted material on the NS, similar to cataclysmic variables. "Type II" show more frequent, lower-amplitude variations indicative of instabilities in the accretion flow – i.e., sudden changes in $\dot{M}$.

In either case, the magnetic field is crucial for understanding the pulsations. A strong $\vec{B}$ field channels the accreting material to the NS's magnetic poles, and one gets lots of emission from these "hot spots," with the emission modulated by the NS spin.

We can make a rough estimate of the $\vec{B}$ strength needed to channel the incoming accretion flow by examining the radius $r_{\rm mag}$ at which the magnetic field's energy density equals the kinetic energy density of the accretion flow. If we assume that the neutron star's magnetic field is approximately a dipole, then $B \propto r^{-3}$. If the magnetic field at the pole is B_p, then

$$(647)\quad u_B = \frac{B^2}{8\pi} = \frac{B_p^2}{8\pi}\left(\frac{R}{r}\right)^6 .$$

We then define

$$(648)\quad \eta = \frac{v_{\rm rad}}{v_{\rm Kepler}} \ll 1$$

And then consider energy density balance:

(649)
$$\frac{B_p^2}{8\pi}\left(\frac{R}{r}\right)^6 = u_{\text{kinetic}} \quad \text{(K.E. per mass of infalling gas)}$$
(650)
$$= \frac{1}{2}\rho\left(v_{\text{Kepler}}^2 + v_{\text{rad}}^2\right)$$
(651)
$$\approx \frac{1}{2}\rho v_{\text{Kepler}}^2$$
(652)
$$\approx \frac{GM\rho}{2r_{\text{mag}}}$$

The mass inflow rate of the disk (which subtends a solid angle Ω_{disk}) is

(653) $\dot{M} = \rho v_r A_{\text{disk}} = \rho\eta v_K r_{\text{mag}}^2 \Omega_{\text{disk}}$.

Solving for ρ, we find

(654) $\rho = \dfrac{\dot{M}}{\eta v_K r_{\text{mag}}^2 \Omega}$,

We substitute this expression for ρ along with $v_K = \sqrt{\frac{GM}{r}}$ into our expression for u_K at the disk's inner edge.

(655) $u_K = \dfrac{\dot{M} v_K}{8\pi\eta r_{\text{mag}}^2 \Omega} = \dfrac{\dot{M}\,(GM)^{1/2}}{2\eta r_{\text{mag}}^{5/2}\Omega}$.

Equating u_K and u_B, and solving for r_{mag} we find,

(656) $r_{\text{mag}} \propto \left(\dfrac{B_p^2 R_*^6}{\dot{M}\sqrt{M}}\right)^{2/7}$

(where R_* is the size of the compact object). It turns out that if $r_{\text{mag}} > R_*$, then the accretion flow is significantly affected by $\vec{B}$ and becomes magnetically channeled.

– **Black Holes:** These have no surface, and so no pinned $\vec{B}$ fields, no pulsations, and no bursts. These characteristics are found in all sources with $M \gtrsim 3M_\odot$. What we see instead is messy hydrodynamics in strong (relativistic) gravity. The disk itself is still magnetized; the field gets "wrapped up" by the black hole spin, producing jets.