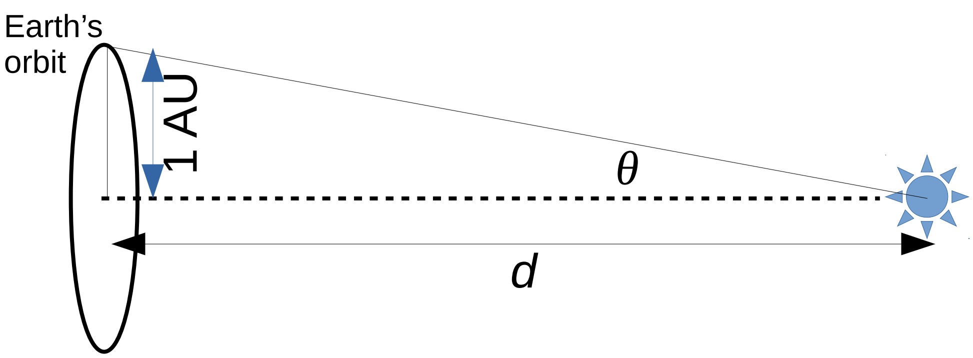
Lecture 2

- Space is Big.
- Last time we talked about mass scales; today we'll talk about **size scales**:
  - Bohr radius → $p^2/2m \sim hbar^2 / (2\ m\ a^2) \sim e^2/a$ → 5.3e-9 cm
  - Rearth = 6.3e8 cm = (20000 / pi) km
  - Rsun = 7e10 cm
  - 1 AU = 1.5e13 cm ~ 8 light-seconds
  - 1 parsec = 1 pc = 3e18 cm = 3.26 light years (note that l.y. are ~never used in astrophysics)

- **Parsec** is the fundamental unit of distance; it is ~the typical distance between stars (though that's just a coincidence). It is observationally defined:

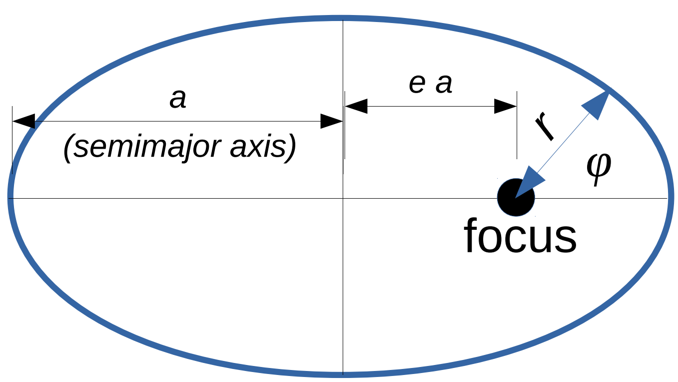


  - Over one year, the Earth's displacement is 2 AU and an object at distance *d* changes apparent position by 2θ, where
    $\tan\ \theta = 1\ AU / d$, or
    $d = 1\ AU / \ \theta$, or
    $d / 1\ pc\ = 1\ arcsec / \ \theta$
  - Nearest star: 1.3 pc
    To our galactic center: 8 kpc (kiloparsecs)
    To the nearest big galaxy: 620 kpc !!

- **Cosmic Distance Ladder:**
  - Distance is a key concept in astrophysics – e.g. the revolution currently underway thanks to ESA's Gaia mission (measuring parallax for billions of objects with sub-milliarcsec precision)
  - "Distance ladder" refers to the bootstrapping of distance measurements, from nearby stars to the furthest edges of the observable universe.
  - Within solar system: light travel time. Radar, spacecraft communication, etc.
  - Parallax: the first rung outside the Solar system. Measured by Gaia (2nd data release) for ~1 billion stars across ~half of the Galaxy. A revolution is underway!

- Standard candles: if Luminosity L is known, then observed flux F gives distance:
  $F = L / 4 \pi d^2 \quad \rightarrow \quad d = (L / 4 \pi F)^{1/2}$
  Most important types:
  - Cepheid variables - giant pulsating stars, period varies with absolute magnitude
  - Type Ia Supernovae – exploding stars (probably white dwarf)
  - Neither of these are truly standard – only "standardizable" (which is almost as good)
  - Other types (for other galaxies):
    - Tully-Fischer: $L \sim V^2$ (rotational velocity of spiral galaxies)
    - Faber-Jackson: $L \sim \sigma^2$ (velocity dispersion of elliptical galaxies)
- Hubble's Law – for very distant Galaxies.
  - The universe is expanding at a nearly-constant rate (more on that in 8.902). Roughly, it expands evenly everywhere, so a distant galaxy's apparent velocity is $v = H_0\, d$. So, $d = v / H_0$.
  - Note that this doesn't work for nearby galaxies like Andromeda (which is moving toward us due to gravity dominating over cosmic expansion).

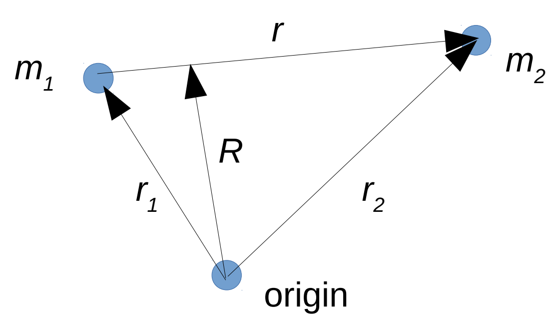
- **The two-body problem** ← see Ch. 2 of Murray & Dermott
  - The motion of two bodies about one another due to their mutual gravity.
    - Planets orbiting stars
    - Stars orbiting each other
    - Objects orbiting white dwarf; neutron star; black hole
  - Certain quantities can be measured very precisely, enabling precise measurements of masses and sizes of bodies. E.g., binary pulsars (neutron stars): masses measured to within $10^{-3}\ M_{sun}$ (~0.1%)
  - Goal here: Go through the gravitational two-body problem with an eye on features that are observationally testable, and on features specific to the $1/r^2$ nature of gravity. Many "details" of the real world push us away from exact $1/r^2$ – e.g. physical sizes, non-spherical shapes, general relativity
  - Key behavior we will use:
    - (1) Bodies move in elliptical trajectors (**Kepler's 1st Law**)



$$r(\varphi) = \frac{a(1-e^2)}{1+e\cos\varphi}$$

    - **2nd Law:** Motion sweeps out equal areas in equal times: $dA/dt = \frac{1}{2} r^2\, d\varphi/dt$ = constant
    - **3rd Law:** $a^3/P^2 \sim (M_{tot})$ ← our eventual goal. P=orbital period, M_{tot} = total system mass

- To fully describe two pointlike bodies in 3D, we need 6 position components and 6 velocity components. (If they are not pointlike we need even more – in this case we use, e.g., Euler angles to define bodies' orientations.) Regardless, we need to reduce this to make things tractable!
- FIRST, go from 2 bodies to 1 … we can do this for any central force (not just $1/r^2$). This means converting potential: $V(r_1, r_2) = V(|r_2 - r_1|)$ ← since potential depends only on relative position



- $\vec{r} = \vec{r}_2 - \vec{r}_1$ (eq 1)
  $\vec{R} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$ (position of center of mass)
  $\ddot{\vec{R}} = 0$ IF no external forces operate on the system.
- We can always set ***Rdot*** = 0 by choosing an intertial reference frame, and we can choose our origin so that ***R*** = 0 too.
  That means that $m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$ – combining with (eq 1) above shows that
  $\vec{r}_2 = \frac{\vec{r}}{1 + m_2/m_1}$ and $\vec{r}_1 = -\frac{m_2}{m_1} \vec{r}_2$
- … Which gives us the motions and velocities of both bodies in terms of the single variable *r*. So, we've reduced the 2-body problem to a one-body problem.