Comment on *"Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe"* by Ang and Longstaff*

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1 Introduction

Is the source of systemic sovereign credit risk the economic linkages among sovereigns that expose them to the common macroeconomic shocks, or is it the financial linkages that spread and amplify shocks originated from one sovereign to the others? This question is of tremendous importance for economists and policy makers. However, as Hansen (2013) points out, while we have a long list of empirical measures of systemic risk (for example, see the survey of Bisias, Flood, Lo, and Valavanis 2012), we still face significant challenges in identifying, measuring, and even defining systemic risk, which present a major hurdle for testing alternative theories of systemic risk.

Ang and Longstaff take a novel approach to answer the above question. They propose a comparison of the degree of comovement in the sovereign credit default swap (CDS) spreads among the U.S. states against that among the Eurozone countries. The two groups are comparable in several dimensions. Both groups are in a currency union, have comparable debt-to-revenue ratios and CDS spreads on average, and both have similar legal arrangements regarding sovereign default. One might question whether the comovement in sovereign CDS spreads should be classified as *systemic risk* or *systematic risk*. However, this distinction is less important here since the primary goal is to understand what drives the comovement in sovereign CDS spreads.

Based on the assumption that a German (U.S. Federal) default can only occur in conjunction with a systemic shock in the Eurozone (United States), Ang and Longstaff identify the systemic risk exposures of the Eurozone countries (U.S. states) by measuring how strongly their sovereign credit risks comove with the credit risk of Germany (United States). Despite the fact that the U.S. states are arguably more economically and politically integrated than the Eurozone countries, Ang and Longstaff show that there is much more systemic risk among the Eurozone sovereigns than among the U.S. states.

I focus my discussion on the following question: to what extent can the comovement in sovereign CDS spreads be explained by the comovement among the economic fundamentals. To do so, I build a simple structural model of sovereign debt, which takes the degree of comovement in GDP across sovereigns as given, and connects it to the comovement in sovereign CDS spreads. A structural model is important here because it captures the endogenous link between the comovement in GDP and the comovement in CDS spreads, which change nonlinearly as the level of credit risk changes. For example, consider two sovereigns that are highly connected in their economic activities. If the CDS spreads for one of them are low and insensitive to changes in the economic activities, the comovement in the CDS spreads will be close to zero.

I show that the economic linkages as measured by GDP beta are indeed lower among the Eurozone countries than among the U.S. states. After calibrating the model to the observed CDS spreads and adjusting for the sensitivities of CDS spreads to GDP, I find that the economic linkages alone would imply higher systemic risk loadings for the U.S. states than for the Eurozone countries, which is opposite to what Ang and Longstaff find in the data. Interestingly, in a model with only economic linkages, the model-implied CDS betas fit the systemic risk loadings nicely for those Eurozone countries with low default risk, but significantly overshoot the systemic risk loadings for countries with high default risk and for all the U.S. states. I discuss the factors that might explain the failures of the structural model, including the impact of financial linkages and the relation between government revenue and GDP.

2 Economic Linkages

To compare the strength of economic linkages among U.S states and Eurozone countries, I compute the GDP beta for the U.S. states with respect to the United States, and the GDP beta for the Eurozone countries with respect to Germany. Let the annual growth rate of nominal GDP for country (state) i and the benchmark country M in year t be $g_{i,t}$ and $g_{M,t}$. Then, the GDP beta for country (state) i is defined based on the decomposition:

$$g_{i,t} = \beta_i^{GDP} g_{M,t} + e_{i,t}, \tag{1}$$

where $e_{i,t}$ is orthogonal to $g_{M,t}$, and

$$\beta_i^{GDP} = \frac{cov(g_{i,t}, g_{M,t})}{var(g_{M,t})}. \tag{2}$$

This GDP beta measure is connected to the systemic sovereign credit risk loading γ_i proposed by Ang and Longstaff. They assume that the risk-neutral default intensity for country (state) i, $\lambda_{i,t}$, is linked to the benchmark sovereign risk-neutral default intensity λ_t via

$$\lambda_{i,t} = \gamma_i \lambda_t + \xi_{i,t}, \tag{3}$$

where $\xi_{i,t}$ is the sovereign-specific risk-neutral default intensity that is independent of λ_t. Since the risk-neutral default intensity is approximately proportional to the short-term sovereign CDS spread,[1] the systemic risk loading γ_i as defined in (3) is approximately the beta of short-term CDS spreads for country (state) i relative to the benchmark country.

Figure 1 plots the time series of the annual growth rates of nominal GDP from 1971 to 2011. Nominal GDP data for the U.S. states are from the Bureau of Economic Analysis. GDP data for the Eurozone countries are from the OECD. The two panels show clear comovement in GDP within each group of sovereigns. Among the Eurozone countries, the comovement appears to be stronger in the first half of the sample than the second half, when Germany had slower growth than the other Eurozone countries for extended periods following the reunification. The opposite is true for the U.S. sates, which have comoved more strongly in the second half of the sample.

Ang and Longstaff's data of sovereign CDS spreads cover the period from 2008 to 2011. To strike a balance between the need to use more recent data and having a sample size that is not too small, I compute the GDP beta based on the data from 1991-2011. In Figure 1, I plot the GDP betas β_i^{GDP} along with the systemic credit risk loadings γ_i estimated by Ang and Longstaff. The average GDP beta for the Eurozone countries (ex Germany) in this period is 0.58, whereas the average GDP beta for the U.S. states is 1.14. In contrast, the average of the systemic

[1]Under the assumption of a constant recovery rate, the risk-neutral default intensity will be proportional to the instantaneous CDS spread.

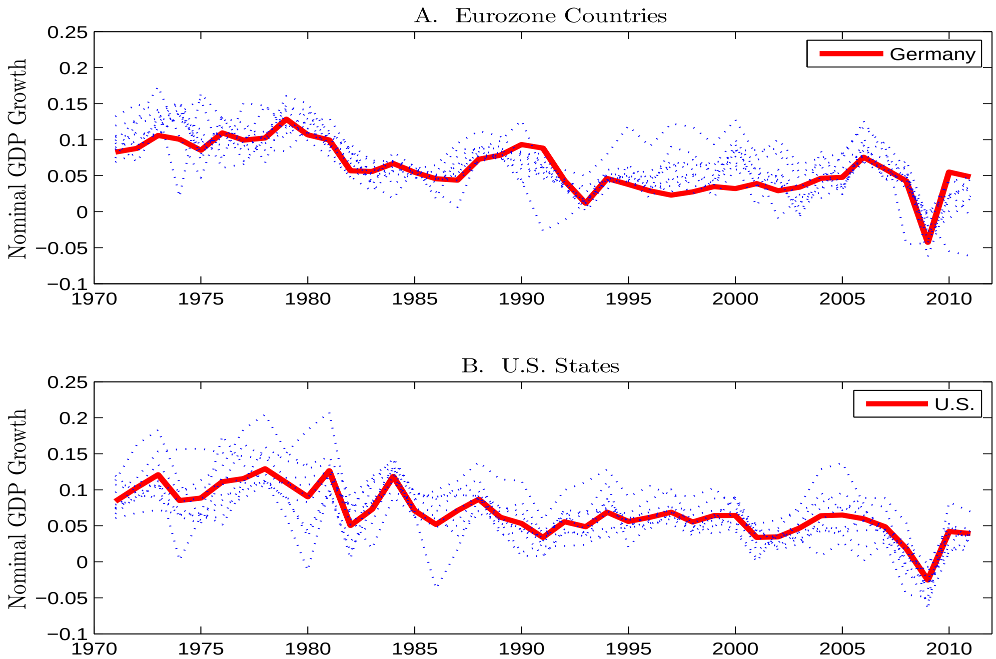

Figure 1: **Nominal GDP growth rates for the Eurozone countries and U.S. states.** The solid lines highlight the growth rates for the benchmark sovereigns: Germany and U.S.

sovereign risk loadings among the Eurozone countries is 1.60, compared to 0.72 for the U.S. states.

In addition to comparing the average degree of economic linkage in the two groups of sovereigns, we get additional information by checking whether the individual sovereign GDP betas line up in the same way as their systemic risk loadings. As Figure 2 shows, the rankings of β^{GDP} and γ are often different. Greece has the largest systemic risk loading but the lowest GDP beta among the Eurozone countries. Nevada has the highest GDP beta among the U.S. states, while California has the highest systemic risk loading.

We should be cautious in directly comparing the GDP betas against the systemic risk loadings. Even in the case where the only source of comovement in CDS spreads are the comovement in GDP, we still need to translate the beta of GDP to the beta of CDS spreads. The relation between the two betas can be highly nonlinear. Fixing the GDP beta, we can get a wide range of variations in the CDS beta by changing the level of sovereign credit risk. It is difficult to capture this nonlinear relation within the reduced-form model framework used in this paper.

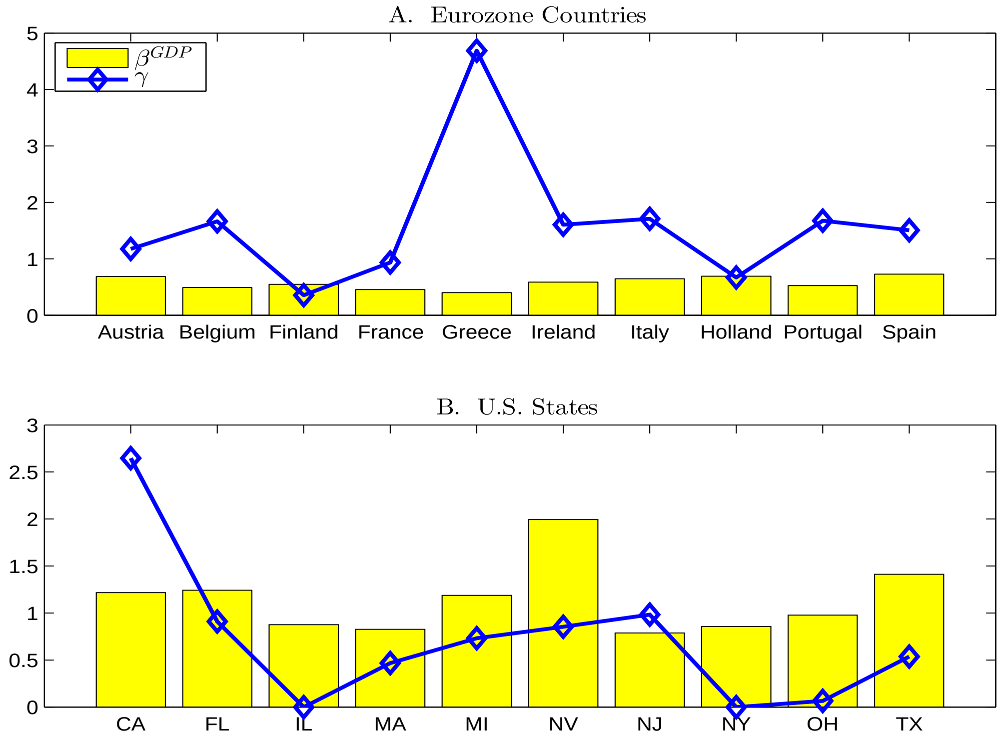

Figure 2: **GDP Beta vs. Systemic Risk Loading γ**

Next, I build a simple structural model to connect the comovement in GDP growth and the comovement in sovereign CDS spreads.

2.1 A Simple Structural Model

Consider a group of n countries (or states) indexed by i, with $i = 1, ..., n$. Let country 1 be the reference country for the definition of GDP beta and CDS beta. The nominal GDP for country i, $Y_{i,t}$, follows a Geometric Brownian motion with average growth rate $\mu_{Y,i}$ and volatility σ_i,

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu_{Y,i} dt + \sigma_i dW_{i,t}, \tag{4}$$

where $W_{i,t}$ is a standard Brownian motion. GDP growth rates are correlated across countries through the correlations in the Brownian shocks, with

$$cov_t \left(dW_{i,t}, dW_{j,t} \right) = \rho_{ij} dt. \tag{5}$$

The GDP beta of country i with respect to country 1 is defined as

$$\beta_i^{GDP} = \frac{cov_t\left(\frac{dY_{i,t}}{Y_{i,t}}, \frac{dY_{1,t}}{Y_{1,t}}\right)}{var_t\left(\frac{dY_{1,t}}{Y_{1,t}}\right)} = \rho_{i1}\frac{\sigma_i}{\sigma_1}. \tag{6}$$

I assume that the sovereign debt for country i is a consol bond with coupon rate C_i. Government revenue (net of the structural component of government spending), $X_{i,t}$, is perfectly correlated with GDP but is more volatile,

$$\frac{dX_{i,t}}{X_{i,t}} = \mu_{X,i}dt + \alpha\sigma_i dW_{i,t}, \tag{7}$$

where α is a constant that amplifies the volatility of government revenue relative to GDP. The assumption that α is the same for all countries implies that revenue beta will be identical to GDP beta. Default occurs whenever the government revenue falls below its interest expense, $X_{i,t} < C_i$. In addition, debtholders are assumed to recover zero value at default.

For pricing, I assume that markets are complete, and there is an international stochastic discount factor Λ_t that follows the process

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \eta dW_{1,t}, \tag{8}$$

with constant riskfree rate r and price of risk η. The fact that only $W_{1,t}$ affects the discount factor means that the only systematic shocks that are priced in the model are the shocks to the reference country.

One can price the sovereign debt using standard risk-neutral pricing techniques (see, e.g., Leland, 1994). Denote the value of country i's sovereign debt at time t as $D_i(X_{i,t})$, which is equal to the present value of future coupon payments until the time of default τ,

$$D_i(X_{i,t}) = E_t^{\mathbb{Q}}\left[\int_t^\tau e^{-rs}C_i\right], \tag{9}$$

where $\mathbb{Q}$ denotes the risk-neutral probability measure corresponding to Λ_t. Then, D_i satisfies the following ordinary differential equation:

$$rD_i(x) = C_i + D_i'(x)\widehat{\mu}_i x + \frac{1}{2}D_i''(x)\alpha^2\sigma_i^2 x^2, \tag{10}$$

where $\widehat{\mu}_i$ is the risk-neutral growth rate of government revenue:

$$\widehat{\mu}_i \equiv \mu_{X,i} - \alpha\rho_{i1}\sigma_i\eta = \mu_{X,i} - \alpha\beta_i^{GDP}\sigma_1\eta. \tag{11}$$

The boundary condition at default is

$$D_i\left(C_i\right) = 0. \tag{12}$$

An additional boundary condition is that debt becomes riskfree as x goes to infinity.

The solution for D_i is:

$$D_i\left(x\right) = \frac{C_i}{r}\left(1 - \left(\frac{x}{C_i}\right)^{b_i}\right), \tag{13}$$

where

$$b_i = \frac{1}{\alpha^2\sigma_i^2}\left[-\left(\widehat{\mu}_i - \frac{\alpha^2\sigma_i^2}{2}\right) - \sqrt{\left(\widehat{\mu}_i - \frac{\alpha^2\sigma_i^2}{2}\right)^2 + 2r\alpha^2\sigma_i^2}\right]. \tag{14}$$

The term $\left(\frac{x}{C_i}\right)^{b_i}$ in Equation (13) is the value of an Arrow-Debreau security that pays \$1 at the time of default and zero at all other times (conditional on the current revenue x). Thus, Equation (13) states that the value of a sovereign debt is equal to the present value of a riskless consol, C_i/r, minus the present value of the losses at default.

Next, the credit spread for country i's consol is given by

$$S_i\left(X_{i,t}\right) = \frac{C_i}{D_i\left(X_{i,t}\right)} - r = \frac{r}{1 - \left(\frac{X_{i,t}}{C_i}\right)^{b_i}} - r, \tag{15}$$

which is a function of the ratio of government revenue to interest expense (interest coverage) for country i, $X_{i,t}/C_i$. Moreover, the credit spread of the consol will be equal to the spread of an infinite-maturity CDS contract.

By applying Ito's Lemma to $S_{i,t}$, I express the CDS beta of country i with respect to the reference country 1, $\beta_{i,t}^{CDS}$, as

$$\beta_{i,t}^{CDS} = \frac{cov(dS_{i,t}, dS_{1,t})}{var(dS_{1,t})} = \frac{S_i'(X_{i,t})\, X_{i,t}}{S_1'(X_{1,t})\, X_{1,t}} \beta_i^{GDP}. \tag{16}$$

Equation (16) demonstrates intuitively the connection between GDP beta and CDS beta. If country i is remote from default, its credit spreads will be low and insensitive to changes in its revenue or GDP, i.e., $S_i'(X_{i,t})$ is close to 0, which implies that $\beta_{i,t}^{CDS}$ is close to 0 even when β_i^{GDP} is large. Alternatively, if a country's credit spreads are highly sensitive to changes in its GDP, which is true when credit risk is high, $\beta_{i,t}^{CDS}$ can be multiple times larger than β_i^{GDP}.

This amplification channel is missing from the comparison of the GDP beta and the systemic risk loading in Figure 2. To compute the model-implied CDS beta, we need to calibrate this model to the country and state data. In particular, the model should be able to fit the observed sovereign credit spreads. Suppose the current CDS spread for country i is s_i. Computing the derivative $S_i'(X_{i,t})$ and using the result from Equation (15), we get

$$S_i'(X_{i,t})\, X_{i,t} = \frac{C_i^2}{D_i^2(X_{i,t})} \frac{b_i}{r} \left(\frac{X_{i,t}}{C_i}\right)^{b_i} = s_i\,(s_i + r)\,\frac{b_i}{r}.$$

Plugging this expression into (16), and ignoring the heterogeneity in the risk-neutral growth rates and volatilities of government revenues across different sovereigns (so that $b_i = b_1$), we get a simple relation between GDP beta and CDS beta:

$$\beta_{i,t}^{CDS} = \frac{s_i\,(s_i + r)}{s_1\,(s_1 + r)} \beta_i^{GDP}\,. \tag{17}$$

To take the model to the data, I use the average sovereign CDS spreads from Ang and Longstaff's sample (reported in Table 2) and assume that the CDS spreads are constant across maturities. The resulting model-implied CDS betas are plotted in Figure 3 along with the systemic risk loadings. While connected, the CDS beta is different from the systemic risk loading in that the former is based on the CDS spreads for a consol while the latter is based on short-term CDS spreads.

As Panel A shows, among the Eurozone countries, the model-implied CDS beta β^{CDS} (dash-cross line) tracks the systemic risk loading γ (solid-diamond line)

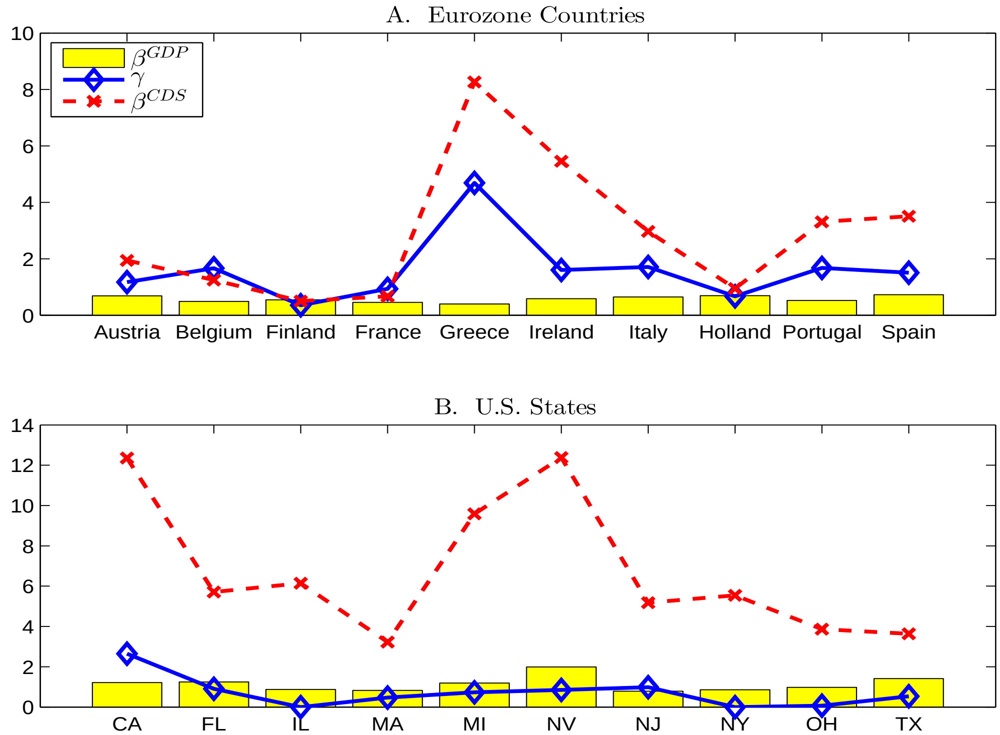

Figure 3: **Model-Implied CDS Beta vs. Systemic Risk Loading** γ

reasonably well for those countries with low credit risk (Austria, Belgium, Finland, France, Netherlands). However, for the high CDS-spread countries (Greece, Ireland, Italy, Portugal, Spain), the model-implied CDS beta β^{CDS} significantly overshoots the systemic risk loading γ. The model-implied CDS betas are also too high compared to the systemic risk loadings for the U.S. states (see Panel B).

Moreover, since the level of CDS spreads for the U.S. states is comparable to that of the Eurozone countries on average, the model applies a similar multiplier to the U.S. state GDP betas as it does the Eurozone country GDP betas. The result is that the model-implied CDS betas β^{CDS} are higher on average for the U.S. states than for the Eurozone countries (comparing the dash-cross lines in Panels A and B), while the opposite is true in the data. These results show that the comovement in GDP cannot by itself account for the comovement in sovereign CDS spreads.

Why are the model-implied CDS betas so much higher than the systemic risk loadings for the U.S. states? Ang and Longstaff point out that the size of state governments are much smaller than the size of the governments of the Eurozone countries. Thus, one possibility is that while the average level of CDS spreads for a U.S. state is high, their sensitivity to state GDP is low because state government revenue (net of any non-discretionary spending) is not as highly correlated with its GDP as in the case of the Eurozone countries. It would be interesting to check whether this is indeed the case.

Another possible reason for the failure of this structural model is that it attributes the sovereign credit spread entirely to the ratio of government revenue to interest expense of the country (state). If the high sovereign CDS spreads are not due to a low interest coverage now, but rather the potential need to bail out distressed banks or the risks with rolling over a large amount of government debt that is maturing, then this model will be under-estimating the interest coverage and hence over-estimating the sensitivity of the CDS spreads to GDP. In other words, financial linkages and frictions can simultaneously account for part of the level of the sovereign CDS spreads and the comovement in the spreads. They can potentially explain why the model-implied CDS beta is too high for those high-risk Eurozone countries.

3 Concluding Remarks

What's next? Absent from the above discussions are direct measures of the financial linkages among the Eurozone countries and the U.S. states. There is anecdotal evidence of significant cross holdings of peripheral Eurozone country sovereign debt by banks in different Eurozone countries. Careful investigation of how such financial linkages are connected to the degree of comovement in the sovereign CDS spreads will further improve our understanding of the source of systemic sovereign credit risks.

Broadening the notion of systemic risk can also generate new insights for future research of systemic risk. One can imagine that a rise in the systemic risk in the Eurozone leads to the price of German bonds to rise, not fall, due to "flight-to-quality" in the markets. Such possibilities are ruled out by the assumption that the Eurozone systemic risk is the same as the risk of German default.

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