Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure

Hui Chen*

January 20, 2008

---

*Sloan School of Management, Massachusetts Institute of Technology. Email: huichen@mit.edu. I am very grateful to the members of my dissertation committee: Monika Piazzesi (Chair), John Cochrane, Doug Diamond, and Pietro Veronesi for constant support and many helpful discussions. I also thank Ravi Bansal, Frederico Belo, George Constantinides, Sergei Davydenko, Darrel Duffie, Gene Fama, Vito Gala, Raife Giovinazzo, Dirk Hackbarth, Lars Hansen, Milt Harris, John Heaton, Andrew Hertzberg, Pete Kyle, Francis Longstaff, Jianjun Miao, Stewart Myers, Robert Novy-Marx, Nick Roussanov, Tano Santos, Martin Schneider, Costis Skiadas, Ilya Strebulaev, Morten Sorensen, Amir Sufi, Suresh Sundaresen, and seminar participants at Carnegie Mellon, Chicago, Columbia, Duke, Emory, Hong Kong University of Science and Technology, Illinois, London Business School, Maryland, Michigan, Massachusetts Institute of Technology, New York University, Rochester, Stanford, Texas-Austin, Toronto, University of California at Los Angeles, University of Southern California, Washington, and the 2007 WFA meetings for comments. Research support from the Katherine Dusak Miller Ph.D. Fellowship in Finance is gratefully acknowledged; any opinions expressed herein are the author's.

Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure

Abstract

Investors demand high risk premia for defaultable claims, because (i) defaults tend to concentrate in bad times when marginal utility is high; (ii) default losses are high during such times. I build a structural model of financing and default decisions in an economy with business-cycle variations in expected growth rates and volatility, which endogenously generate countercyclical comovements in risk prices, default probabilities, and default losses. Credit risk premia in the calibrated model not only can quantitatively account for the high corporate bond yield spreads and low leverage ratios in the data, but have rich implications for firms' financing decisions.

Risks associated with macroeconomic conditions are crucial for understanding asset prices. Naturally, they should also have important implications for corporate decisions. By introducing macroeconomic conditions into firms' financing decisions, this paper provides a risk-based explanation for two puzzles about corporate debt. The first puzzle is the "credit spread puzzle": yield spreads between corporate bonds (especially those with high credit quality) and treasuries are high and volatile relative to the observed default probabilities and recovery rates. The second is the "under-leverage puzzle": firms choose low leverage ratios despite facing seemingly large tax benefits of debt and small costs of financial distress.

To address these puzzles, I build a structural model that endogenizes firms' financing and default decisions over the business cycle. In the model, aggregate consumption and firms' cash flows are exogenous. Their expected growth rates and volatility move slowly over time, which drive the business cycle. Asset prices are determined by a representative household with recursive preferences. Firms choose their capital structure based on the trade-off between tax benefits of debt and deadweight losses of default. Examples of these deadweight losses include legal fees and asset fire sale losses during liquidation. These losses are born by equity-holders ex ante, and they are closely related to the losses that corporate bond investors suffer at default.

The main mechanism of the model is as follows. First, marginal utilities are high in recessions, which means that default losses that occur during such times will affect investors more. Second, recessions are also times when cash flows are expected to grow slower and become more volatile. These factors, combined with higher risk prices at such times, lower the continuation values for equity-holders, making firms more likely to default in recessions. Third, since many firms are experiencing problems in recessions, liquidating assets during such times can be particularly costly, which results in higher default losses. Taken together, the countercyclical variations in risk prices, default probabilities, and default losses raise the present value of expected default losses for bondholders and equity-holders, which leads to high credit spreads and low leverage ratios.

There are two types of shocks in this model: small shocks that directly affect the level of consumption and cash flows, and large shocks that change the conditional moments of growth rates, which drive the business cycle. I model large shocks with a continuous-time Markov chain, which not only yields closed-form solutions for stock and bond prices, but allows for analytical characterization of firms' default policies. Risk prices for small consumption shocks depend on the conditional volatility of consumption growth. Risk prices for large shocks depend on their frequency, size, and persistence. With recursive preferences, investors are concerned with news about future consumption. The arrival of a recession brings bad news of low expected growth rates, which raises marginal utilities. As a result, investors will demand a high premium on securities that pay off poorly in such times.

The calibration strategy is to match the empirical moments of exogenous fundamentals. I use aggregate consumption and corporate profit data from NIPA to calibrate consumption and the systematic components of firms' cash flows. The volatility of firm-specific shocks is calibrated to match the average default probabilities associated with firms' credit ratings. Preference parameters are calibrated to match moments of the equity market and riskfree rate. Default losses are estimated from the time series of aggregate recovery rates. Relative to a benchmark case where consumption and cash flow growth are *i.i.d.* and default losses are constant, the average 10-year Baa-Treasury spread in the calibrated model rises from 57 bp to around 140 bp; the 10-year Baa-Aaa spread rises from 48 bp to around 100 bp; the volatility of Baa-Aaa spread also rises from less than 10 bp to 35 bp. Finally, the average optimal leverage ratio of a Baa-rated firm drops from 67% to around 40%. These values are close to the U.S. data.

< Figure 1 about here >

Figure 1 and 2 provide empirical evidence of business-cycle movements in default rates, credit spreads, and recovery rates. Panel A of Figure 1 plots the historical annual default rates from 1920 to 2006. There are several spikes in default rates, all coinciding with an NBER recession. Panel B of Figure 1 plots the monthly Baa-Aaa spreads from 1920 to 2006. The spreads shoot up in most recessions, but they do not always move in lock steps with default rates,[1] which suggest that other factors, such as recovery rates and risk premia, are also affecting the movements in spreads. Business-cycle variations in the recovery rates are evident in Figure 2. Recovery rates during the three recessions in the sample, 1982, 1990 and 2001, were all lower than the sample average.[2]

< Figure 2 about here >

A model that endogenizes capital structure decisions is well suited to address the puzzles of credit spreads and capital structure for two reasons. First, it helps overcome the difficulty of estimating default probabilities for investment grade firms. By definition, these firms rarely default, which make their credit spreads sensitive to small measurement errors in conditional default probabilities.[3] A structural model explicitly connects conditional default probabilities to the macroeconomic conditions through firms' endogenous decisions, thus rendering more powerful predictions on the magnitude and source of variations in conditional default probabilities over the cycle.

The second advantage of structural model is that it helps identify unobservable default losses for equity-holders (deadweight losses) from observable bond recovery rates. In the model, recovery rates are determined by firm value at default net of default losses. Holding fixed firm value at default, lower recovery rates would imply higher default losses. Since the timing of default and firm value at default are endogenous, the model provides a precise link between recovery rates and default losses. Through this link, I estimate (using the simulated method of moments) default losses and link them to the state of the economy.

I also perform several comparative statics. To see how much the leverage ratio is affected by the distribution of default losses across states, I set default losses to a constant fraction of asset value at default. The resulting leverage ratio is almost as high as in the benchmark case without business cycle variations, which implies that countercyclical default losses are crucial for generating low leverage ratios. Tax benefits are risky because firms can lose part of their tax shield when they generate sufficiently low cash flows, which is more likely in bad times. By changing firms' ability to carry losses, I show that risks in tax benefits have important effects on capital structure as well.

For tractability, I assume that firms cannot issue subordinated debt or buy back debt.[4] Instead, firms in distress can issue equity, which effectively serve the purpose of "super junior" perpetual debt. Not allowing debt buybacks makes it more difficult for firms to avoid default, which could cause a downward bias in the leverage ratio. However, I show that changing equity issuance costs has little effect on the optimal leverage, which suggests that the model is robust to certain relaxations of the financing constraints, such as introducing costly debt buybacks.

The model has rich implications beyond explaining the levels of credit spreads and leverage ratios. First, the model predicts that how much firm cash flows covary with the aggregate economy (both in levels and conditional moments) has important effects on capital structure choices. In particular, controlling for other factors, a firm with procyclical cash flows should have lower leverage than one with countercyclical cash flows.

Second, the model links the likelihood of default and debt restructuring to the expected growth rates and volatility of cash flows. Lower expected growth rates make firms default faster, but wait longer to restructure. Higher volatility increases the option value of default and restructuring, which makes firms wait longer before exercising these options. Again, these effects become stronger when the conditional moments of cash flows are more cyclical.

Third, with time variation in expected growth rates, volatility, and risk premia, there is no longer a one-to-one link between cash flows and market value of assets. An important example of such delinkage is that the optimal default boundaries based on cash flows are countercyclical, but the same boundaries based on asset value are procyclical. As a result, cash flows, market value of assets, and macroeconomic variables such as interest rates and volatility, should all be informative about default probabilities. A related prediction is that, with endogenous leverage, the relation between leverage and risk premium of stocks is ambiguous.

Finally, the model provides an explanation for default and debt issuance waves based on large economic shocks, and it generates a mechanism for "contagion-like" phenomenon: the same large shocks that cause a group of firms to default together will raise the spreads of the other firms.

Related Literature

Huang and Huang (2003) summarize the credit spread puzzle. They calibrate various structural models to match leverage ratios, default probabilities, and recovery rates, as well as key moments in the equity market. They find that these models produce credit spreads well below historical averages. Miller (1977) highlights the under-leverage puzzle: the present value of expected default losses seem disproportionately small compared to tax benefits of debt. For example, Graham (2000) estimates the capitalized tax benefits of debt to be as high as 5% of firm value, much larger than conventional estimates for the values of expected default losses.

This paper is closely related to Hackbarth, Miao, and Morellec (2006) (HMM) and Chen, Collin-Dufresne, and Goldstein (2006) (CCDG). HMM is one of the first papers to show macroeconomic conditions have rich implications for firms' financing policies. Their model assumes investors are risk-neutral, and macroeconomic conditions only lead to jumps in the level of cash flows. Thus, the model cannot address the puzzles of credit spreads and leverage ratios. In this paper, business cycles are driven by the dynamics of risk prices and the conditional moments of cash flows, which allow us to study the impact of macroeconomic risks on firms' financing decisions over the cycle.

CCDG apply consumption-based asset pricing model to study the credit spread puzzle. They do not study how macroeconomic conditions affect firms' financing and default decisions. They show that the strongly countercyclical risk prices generated by the habit formation model (Campbell and Cochrane (1999)), combined with exogenously imposed countercyclical default probabilities, can generate high credit spreads. In contrast, they find the long-run risk model (Bansal and Yaron (2004)) fails to capture the credit spreads because it does not generate sufficiently volatile risk prices. In this paper, I show that the long-run risk model can also generate high credit spreads,[5] and the key is in the comovements among risk prices, default probabilities, and default losses. By endogenizing firms' responses to macroeconomic conditions, the model can better capture such comovements and become less dependent on the variation in risk prices.

Almeida and Philippon (2007) also study the connections between credit spreads and capital structure. They use a reduced-form approach, extracting risk-adjusted default probabilities from observed credit spreads to calculate expected default losses, and find the present value of expected default losses are much larger than traditional estimates. A crucial assumption behind their methodology is that the large credit spreads are indeed due to credit risk. I address this concern by jointly explaining the credit spreads and leverage ratio in a structural model. Moreover, while Almeida and Philippon assume constant default losses, this model demonstrates that countercyclical default losses are crucial for generating low leverage ratio.

Countercyclical default losses can be motivated by Shleifer and Vishny (1992): liquidation of assets is more costly in bad times because other firms in the economy are likely experiencing similar problems. Altman et al. (2005) provide evidence that recovery rates are lower in recessions, and are inversely related to default rates. Acharya et al. (2007) find that recovery rates are significantly lower when the industry of defaulted firm is in distress.

The model's prediction of how defaults depend on market conditions echoes the findings of Pástor and Veronesi (2005) on IPO timing: just as new firms are more likely to exercise their options to go public in good times, existing firms are more likely to exercise their options to default (quit) in bad times. The model's prediction that both cash flows and market value of assets help predict default probabilities is consistent with the empirical finding of Davydenko (2005).

The default risk premium in this model varies significantly over time, and has a large component due to jump risks (large economic shocks). These predictions are consistent with several recent empirical studies using data of corporate bonds and credit derivatives. Longstaff, Mithal, and Neis (2005) show that the majority of the corporate spreads is due to default risk; Driessen (2005) and Berndt et al. (2005) estimate large jump-to-default risk premia in corporate bonds and default swaps; Berndt et al. (2005) also find dramatic time variation in credit risk premia.

Theoretically, this paper provides a novel framework to bring macroeconomic conditions into capital structure models.[6] Most of the existing models view default as an option for equity-holders. Introducing business cycles increases the number of state variables, making the problem untractable. I approximate the dynamics of macroeconomic variables with a finite-state Markov chain, then apply the option pricing technique of Jobert and Rogers (2006). This method reduces a high-dimensional free-boundary problem into a system of ODEs with closed-form solutions.

Finally, this paper contributes to the long-run risk literature, led by Bansal and Yaron (2004), Hansen, Heaton, and Li (2005), and others. Long-run risk models use recursive preferences and predictable components in consumption growth to amplify risk premia for financial claims, which also generate high credit spreads and low leverage ratios in this model. Standard methods to solve these models require approximations of return on wealth or the continuation value around unit elasticity of intertemporal substitution (EIS). The Brownian motion–Markov chain setup in this paper gives closed-form solutions for the prices of stocks, bonds and other claims, which are exact even when EIS is different from 1. Chen (2007) studies the properties of this method in detail.

I. Simple Two-Period Example

In this section, I present a simple two-period example to illustrate how comovements among risk prices, default probabilities, and default losses can raise the present value of expected default losses.

< Figure 3 about here >

Suppose that at $t = 1$ the economy can either be in a good state (G) or bad state (B) with equal probability (see Figure 3). The prices of one-period Arrow-Debreu securities that pay \$1 in one of the two states are Q_G and Q_B, respectively. Since marginal utility is high in the bad state, agents will pay more for consumption in that state: $Q_B > Q_G$. A firm issues a one-period bond with face value \$1 at $t = 0$. The probabilities of default in the two states at $t = 1$ are p_G and p_B. Given default, the default losses in the two states are L_G and L_B.

The price of the zero-coupon bond at $t = 0$ is:

$$B = Q_G \left[(1 - p_G) \cdot 1 + p_G \cdot (1 - L_G)\right] + Q_B \left[(1 - p_B) \cdot 1 + p_B \cdot (1 - L_B)\right],$$

which can be rewritten as:

$$B = Q_G + Q_B - \left[Q_G p_G L_G + Q_B p_B L_B\right].$$

This equation says that the price of a defaultable bond is equal to the price of a default-free bond minus the present value of expected losses at default.

As a benchmark, we first assume that the default probabilities and default losses are constant across the two states, equal to their unconditional means: $\bar{p} = (p_G + p_B)/2$ and $\bar{L} = (L_G + L_B)/2$. Next, raise the default probability and default losses in the bad state, but lower their values in the good state, so that the average default probabilities and default losses are unchanged. Such a change shifts the losses to the state with a higher Arrow-Debreu price, which raises the present value of expected default losses. Then, the bond price at $t = 0$ will be lower relative to the benchmark case. This (convexity) effect is stronger the bigger the spread in Q, p, L between the two states.

This simple example treats the Arrow-Debreu prices, default probabilities, and default losses as exogenous. In reality, the comovements among these variables can be difficult to measure in the data, and it is not clear that they will be strong enough to have large effects, because firms could adjust their capital structure over the business cycle to avoid default in those bad states. In this paper, I derive Arrow-Debreu prices from a consumption-based asset pricing model, connect default probabilities to the business cycle through firms' endogenous decisions, and estimate default losses from the data of recovery rates. I then show that the comovements among these quantities can account for a large part of the puzzles of credit spreads and leverage ratios.

II. The Economy

I study an economy with government, firms, and households. The government serves as a tax authority, levying taxes on corporate profit, dividend, and interest income. Firms are financed by debt and equity, and generate cash flows. Households are both the owners and lenders of firms.

A. Preferences and Technology

There is a large number of identical infinitely lived households in the economy. The representative household has stochastic differential utility of Duffie and Epstein (1992a, b), which is a continuous-time version of the recursive preferences of Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1990). I define the utility index over a consumption process c as:

$$U_t = E_t \left(\int_t^{\infty} f\left(c_s, U_s\right) ds \right). \tag{1}$$

The function $f\left(c, v\right)$ is a normalized aggregator of consumption and continuation value in each period. It is defined as:

$$f\left(c, v\right) = \frac{\rho}{1 - \frac{1}{\psi}} \frac{c^{1-\frac{1}{\psi}} - \left(\left(1-\gamma\right) v\right)^{\frac{1-1/\psi}{1-\gamma}}}{\left(\left(1-\gamma\right) v\right)^{\frac{1-1/\psi}{1-\gamma}-1}}, \tag{2}$$

where ρ is the rate of time preference, γ determines the coefficient of relative risk aversion for timeless gambles, and ψ determines the elasticity of intertemporal substitution for deterministic consumption paths.

There are two types of shocks in this economy: small shocks that directly affect output and nominal prices, and large but infrequent shocks that change expected growth rates and volatility. More specifically, a standard Brownian motion W_t^m provides systematic small shocks to the real economy. Large shocks come from the movements of a state variable s. I assume that s_t follows an n-state time-homogeneous Markov chain, and takes values in the set $\{1, \cdots, n\}$. The generator matrix for the Markov chain is $\mathbf{\Lambda} = [\lambda_{jk}]$ for $j, k \in \{1, \cdots, n\}$, which means that the probability of s_t changing from state j to k within time Δ is approximately $\lambda_{jk}\Delta$.

We can equivalently express this Markov chain as a sum of Poisson processes:

$$ds_t = \sum_{k \neq s_{t-}} \delta_k \left(s_{t-}\right) dN_t^{\left(s_{t-}, k\right)}, \tag{3}$$

where

$$\delta_k \left(j\right) = k - j,$$

and $N^{(j,k)}$ $(j \neq k)$ are independent Poisson processes with intensity parameters λ_{jk}. Each jump corresponds to a change of state for the Markov chain.

Let Y_t be the real aggregate output in the economy at time t, which evolves according to the following process:

$$\frac{dY_t}{Y_t} = \theta_m \left(s_t\right) dt + \sigma_m \left(s_t\right) dW_t^m. \tag{4}$$

The state variable s determines the conditional moments θ_m and σ_m, which represent the expected growth rate and volatility of aggregate output. Since s has n states, θ_m and σ_m can each take up to n different values.

In equilibrium, aggregate consumption equals aggregate output, which determines the stochastic discount factor:

Proposition 1 *The real stochastic discount factor follows a Markov-modulated jump-diffusion:*

$$\frac{dm_t}{m_t} = -r \left(s_t\right) dt - \eta \left(s_t\right) dW_t^m + \sum_{s_t \neq s_{t-}} \left(e^{\kappa\left(s_{t-}, s_t\right)} - 1\right) dM_t^{\left(s_{t-}, s_t\right)}, \tag{5}$$

where r is the real riskfree rate; η is the risk price for systematic Brownian shocks W_t^m:

$$\eta(s) = \gamma \sigma_m(s); \tag{6}$$

$\kappa \left(j, k\right)$ is the relative jump size of the discount factor when the Markov chain switches from state j to k; M_t is a matrix of compensated processes,

$$dM_t^{(j,k)} = dN_t^{(j,k)} - \lambda_{jk} dt, \quad j \neq k, \tag{7}$$

where $N_t^{(j,k)}$ are the Poisson processes that move the state variable s_t as in equation (3). The expressions for r and κ are in Appendix A.

Proof. See Appendix A. ∎

The stochastic discount factor is driven by the same set of shocks that drive aggregate output. Small systematic shocks affect marginal utility through today's consumption levels. The risk price for these shocks (equation (6)) rises with risk aversion and local consumption volatility. Large shocks change the state of the economy and cause jumps in the discount factor, even though consumption is perfectly smooth. The relative jump sizes $\kappa(j,k)$ are the risk prices for these shocks.

The reason that a change in the state causes jumps in the discount factor is due to recursive preferences. With such preferences, investors care about the temporal distribution of risk. Their marginal utility not only depends on current consumption, but also news about future consumption. For example, when a recession arrives (caused by a "jump" in the state), it brings the bad news of low expected growth rates. As a result, the marginal utility rises, resulting in a jump of the discount factor. With time-separable preferences, investors would be indifferent to the temporal distribution of risk, and these large shocks would no longer be priced.

Finally, since credit spreads are based on nominal yields and taxes are collected on nominal cash flows, I specify a simple stochastic consumption price index to get nominal prices and quantities. The price index follows the diffusion

$$\frac{dP_t}{P_t} = \pi dt + \sigma_{P,1} dW_t^m + \sigma_{P,2} dW_t^P, \tag{8}$$

where W_t^P is another independent Brownian motion that generates additional shocks to nominal prices. For simplicity, the expected inflation rate π and volatility $(\sigma_{P,1}, \sigma_{P,2})$ are constant. Then, the nominal stochastic discount factor is $n_t = m_t/P_t$, and the nominal interest rate is

$$r^n(s_t) = r(s_t) + \pi - \sigma_P^2 - \sigma_{P,1}\eta(s_t), \tag{9}$$

which is the sum of the real interest rate, expected inflation, and inflation risk premium.

B. Firms

The technology of firm i produces a perpetual stream of real cash flows Y_t^i specified by the process

$$\frac{dY_t^i}{Y_t^i} = \theta^i\left(s_t\right)dt + \sigma_m^i\left(s_t\right)dW_t^m + \sigma_f^i dW_t^i, \tag{10}$$

where θ^i and σ_m^i are firm i's expected growth rate and systematic volatility; W_t^i is an independent standard Brownian motion, which generates idiosyncratic shocks specific to firm i; σ_f^i is firm i's idiosyncratic volatility, which is constant over time. Since operating expenses such as wages are not included in the earnings but are still part of aggregate output, the earnings across firms do not add up to the aggregate real output Y_t.

To capture the heterogeneity in the systematic components of cash flows across firms as well as their links to the aggregate output, I make the following assumptions:

$$\theta^i(s) = a^i(\theta_m(s) - \overline{\theta}_m) + \overline{\theta}_m^i\,, \tag{11a}$$

$$\sigma_m^i(s) = b^i(\sigma_m(s) - \overline{\sigma}_m) + \overline{\sigma}_m^i\,, \tag{11b}$$

where $\overline{\theta}_m$ and $\overline{\sigma}_m$ are the average growth rate and volatility of aggregate output, $\overline{\theta}_m^i$ and $\overline{\sigma}_m^i$ are the average growth rate and systematic volatility of firm i. The coefficients a^i and b^i determine how sensitive firm-level expected growth rate and volatility are to the aggregate economy.

Firms issue bonds and pay taxes on a nominal basis. The nominal cash flow of firm i is denoted $X_t^i = Y_t^i P_t$. An application of the Ito's formula gives:

$$\frac{dX_t^i}{X_t^i} = \theta_X^i\left(s_t\right)dt + \sigma_{X,m}^i\left(s_t\right)dW_t^m + \sigma_{P,2}dW_t^P + \sigma_f^i dW_t^i, \tag{12}$$

with θ_X^i and $\sigma_{X,m}^i$ determined by θ^i and σ_m^i (see Appendix A).

Valuation of Unlevered Firms and Default-free Bonds

If a firm never takes on leverage, its value is simply the present value of expected future cash flows after taxes. The discounting can be done with nominal riskfree rate r^n under the risk-neutral probability measure $\mathcal{Q}$ (see Appendix B for details).

The way the risk-neutral measure adjusts for risks is quite intuitive: it does so by changing the distributions of shocks. Under $\mathcal{Q}$, the expected growth rate of firm i's nominal cash flows becomes:

$$\tilde{\theta}_X^i\left(s_t\right)=\theta_X^i\left(s_t\right)-\sigma_{X,m}^i\left(s_t\right)\left(\eta\left(s_t\right)+\sigma_{P,1}\right)-\sigma_{P,2}^2, \tag{13}$$

where θ_X^i is the expected growth rate under measure $\mathcal{P}$. Cash flows are risky when they are positively correlated with marginal utility, which is accounted for by lowering the expected growth rate under $\mathcal{Q}$.

In addition, the generator matrix for the Markov chain becomes $\tilde{\mathbf{\Lambda}}=\left[\tilde{\lambda}_{jk}\right]$, where the transition intensities are adjusted by the corresponding jump sizes of the stochastic discount factor (see equation (5)):

$$\begin{aligned}\tilde{\lambda}_{jk}&=e^{\kappa(j,k)}\lambda_{jk}\,,\quad j\neq k\\ \tilde{\lambda}_{jj}&=-\sum_{k\neq j}\tilde{\lambda}_{jk}\,.\end{aligned} \tag{14}$$

Bad news about future cash flows are particularly "painful" if they occur at the same time when the economy enters into a recession (marginal utility jumps up). The risk-neutral measure adjusts for such risks by raising the probability that the economy will enter into a bad state, and lowering the probability that it will leave a bad state. For example, if marginal utility jumps up when the economy changes from state j to k, $\kappa(j,k)>0$, then the jump intensity associated with this change of state will be higher under the risk-neutral measure.

The following proposition gives the value of an unlevered firm before taxes.

Proposition 2 *Suppose firm i's cash flows evolve according to (12) and it never levers up. If its current cash flow is X, and the economy is in state s, then the value of the firm (before taxes) is:*

$$V^i\left(X,s\right)=Xv^i\left(s\right). \tag{15}$$

Define a vector $\mathbf{v}^i=\left[v^i\left(1\right),...,v^i\left(n\right)\right]'$, *then*

$$\mathbf{v}^i=\left(\mathbf{r}^n-\tilde{\theta}_X^i-\tilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{1}, \tag{16}$$

where $\mathbf{r}^n = \mathbf{diag}\left([r^n(1), ..., r^n(n)]'\right)$, $\tilde{\theta}^i_X = \mathbf{diag}\left(\left[\tilde{\theta}^i_X(1), ..., \tilde{\theta}^i_X(n)\right]'\right)$, $\tilde{\theta}^i_X(s)$ *is defined in (13),* $\mathbf{1}$ *is an* $n \times 1$ *vector of ones, and* $\tilde{\mathbf{\Lambda}}$ *is the generator of the Markov chain under the risk-neutral measure defined in (14).*

Proof. See Appendix C. ■

Equation (16) is the generalized Gordon growth formula. If there are no large shocks, $v = 1/(r^n - \tilde{\theta})$, where $\tilde{\theta}$ is the constant expected growth rate under the risk-neutral measure. The new feature in Proposition 2 is that the expected growth rate is adjusted by $\tilde{\mathbf{\Lambda}}$, the risk-neutral Markov chain generator, which accounts for possible changes of the state in the future.

Bad times come with higher risk prices, higher cash flow volatility and lower expected growth rate, all of which lead to a lower risk-neutral growth rate (see equation (13)), hence lower firm value for given cash flows. In addition, real interest rates are countercyclical in this model, which also contributes to lower asset value in recessions. Finally, since the risk-neutral transition probabilities increase the duration of bad times, they push down the asset value in bad times further.

We can obtain the value of a default-free consol bond as a special case of Proposition 2, by treating the bond as a cash flow stream with growth rate and volatility equal to 0.

Corollary 1 *In state* s*, the value of a default-free nominal consol bond with coupon rate* C *(before taxes) is:*

$$B(C, s) = Cb(s), \tag{17}$$

where

$$\mathbf{b} = [b(1), \cdots, b(n)]' = \left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1} \mathbf{1}, \tag{18}$$

and $\mathbf{r}^n$*,* $\tilde{\mathbf{\Lambda}}$ *and* $\mathbf{1}$ *are given in Proposition 2.*

C. Financing Decisions

The setup of firms' financing problems closely follows that of Goldstein, Ju, and Leland (2001) (GJL) and Hackbarth, Miao, and Morellec (2006). Firms make financing and default decisions. Their objective is to maximize equity-holders' value. Since interest expenses are tax deductible, firms lever up with debt to exploit the tax shield. As the amount of debt increases, so does the probability of financial distress, which raises the expected default losses. Thus, firms will lever up to a point where the marginal benefit of debt is zero.

Firms have access to two types of external financing: debt and equity, and they are initially entirely owned by equity-holders. I assume that firms do not hold cash reserves. In each period, a levered firm first uses its cash flow to make interest payments on its debt, then pays taxes, and finally distributes the rest to equity-holders as dividend. The firm faces a "liquidity crunch" whenever its internally generated cash flows fall short of the interest expenses. To finance its debt payments, the firm can issue additional equity. If the "liquidity crunch" becomes too severe and equity-holders are no longer willing to contribute more capital, the firm defaults.

Debt is modeled as a consol bond, i.e., a perpetuity with constant coupon rate C. This is a standard assumption in the literature (see, e.g. Leland (1994), Duffie and Lando (2001)), which helps maintain a time-homogeneous setting. One interpretation for this assumption is that firms commit to a constant financing plan by rolling over finite maturity debt. To simplify the seniority structure of outstanding debt, I assume that all bonds have a pari passu covenant, which gives newly issued bonds equal seniority as old issues.

It is costly to issue debt and equity. Equity issuance costs are a constant fraction e of the proceeds from issuance. For debt, these costs are "quasi-fixed", i.e., they are a fraction q of the amount of debt outstanding after issuance (not the amount newly issued). This assumption can be motivated by two types of issuance costs: 1) underwriting costs, which are proportional to the value of new issues; 2) costs of negotiating with existing debt-holders, which are proportional to the value of old issues. The lumpy costs of debt issuance are consistent with the "stickiness" in firms' financing behavior in practice.[7]

For tractability reason, I follow GJL and HMM by assuming that firms can only adjust debt level upward. In reality, firms in financial distress can reduce their debt by selling part of the assets or entering debt-for-equity swaps. However, Asquith et al. (1994) find that asset fire sale, free-rider problems, and other regulations make such restructurings difficult. Gilson (1997) show that because of high transaction costs, financially distressed firms remain highly leveraged after out of court restructuring. Still, excluding downward restructuring makes it more difficult for firms to avoid default, thus introducing a downward bias in the optimal leverage ratio. I will show that such bias is likely to be small.

Default losses are modeled as a fraction of a firm's unlevered assets at the time of default. To allow these dead-weight losses to vary with economic conditions, I model the fractional default losses $\alpha(s)$ as a function of the state of the economy.

Finally, the tax environment consists of a constant tax rate τ_i for personal interest income, and τ_d for dividend income. A firm's taxable income is equal to cash flow (EBIT) minus interest expenses. Positive taxable income is taxed at rate τ_c^+, while negative taxable income is taxed at a lower rate τ_c^-. The assumption of two different corporate tax rates is a crude way to model "partial loss offset". The US tax laws allow firms to carry net operating losses backward and forward for a limited number of years, which means a firm can lose part of the tax shield when earnings are low.

I study firms' financing decisions in two settings: a static setting where firms only issue debt once at time 0 and make no adjustment afterwards, and a dynamic setting where firms can make subsequent adjustments to their debt levels.

Static Financing Decisions

The static problem is to choose an amount of debt and a default policy that maximize the value of equity *right before issuance*, E_U, which in turn is equal to the expected present value of the firm's cash flows, plus the tax benefits of debt, minus default losses and debt/equity issuance costs:

$$\max_{\{C,\mathcal{T}_D\}} E_U\left(C, \mathcal{T}_D, \chi_0\right), \tag{19}$$

where C is the coupon rate of perpetual debt issued at time 0, $\mathcal{T}_D$ is a stopping time that determines when the firm defaults, and χ_0 contains all the state variables at time 0.

Dynamic Financing Decisions

In the dynamic problem, firms can issue additional debt after time 0, which I refer to as "upward restructuring". Now, in addition to the initial coupon rate and default policy, a firm also needs to decide when to increase its debt level, and by how much. Thus, the firm's problem becomes:

$$\max_{\{C,\mathcal{T}_D,\{\mathcal{T}_U\},\{C_{\mathcal{T}_U}\}\}} E_U\left(C,\mathcal{T}_D,\{\mathcal{T}_U\},\{C_{\mathcal{T}_U}\},\chi_0\right), \tag{20}$$

where $\{\mathcal{T}_U\}$ is a series of stopping times that determines when the firm restructures, and $\{C_{\mathcal{T}_U}\}$ are the new coupon rates at each restructuring point.

III. Static Financing Decisions

The static financing problem is solved in three steps. First, I calculate debt and equity values for any given amount of debt outstanding and default boundaries. Second, I determine the optimal default boundaries for given amount of debt outstanding. Third, I find the optimal amount of debt by maximizing the value of equity before debt issuance.

Since only one firm is under consideration, I will temporarily drop the superscript i for firms. After debt with coupon C is issued at $t = 0$, the default policy is the solution to an optimal stopping problem, characterized by a set of default boundaries, X_D^k for state k, $k = 1, \cdots, n$. A firm defaults if its cash flow falls below the boundary X_D^k while the economy is in state k. Since it is not optimal to default when the value of equity is above zero, these default boundaries will never be higher than the coupon rate. Moreover, I can always re-order the states such that:

$$X_D^1 \le X_D^2 \le \cdots \le X_D^n \le C. \tag{21}$$

The default boundaries and coupon rate divide the relevant range for cash flows into $n+1$ regions: $\mathcal{D}_k = [X_D^k, X_D^{k+1})$ for $k < n$, $\mathcal{D}_n = [X_D^n, C)$ and $\mathcal{D}_{n+1} = [C, +\infty)$. In regions $\mathcal{D}_1$ through $\mathcal{D}_{n-1}$, the firm faces immediate default threats. For example, suppose the economy is currently in state 1, which has the lowest default boundary. If cash flow is in region $\mathcal{D}_{n-1}$, then it is below the default boundary in state n, but above the boundary for the current state. The firm will not default now, but if a big shock suddenly changes the state from 1 to n, it will default immediately. In region $\mathcal{D}_n$, there is no immediate danger of default. However, as is in region $\mathcal{D}_1$ through $\mathcal{D}_{n-1}$, the firm faces a liquidity crunch because it does not have enough internal cash flow to cover interest payments. Finally, $\mathcal{D}_{n+1}$ is the normal region (no default threats or liquidity problems).

A. Debt and Equity Value

Debt and equity are contingent claims whose values depend on a firm's cash flows as well as the state of the economy. They belong to a general class of corporate contingent claims, with dividend rate $F\left(X_t, s_t\right)$ for as long as the firm is solvent, and a payment $H\left(X_{\mathcal{T}_D}, s_{\mathcal{T}_D}\right)$ at the time of default.

For debt, "dividend" is the after-tax coupon rate. With strict priority, default payment is equal to the residual value of the firm at default net of taxes and default losses.[8]

$$\begin{aligned} F\left(X, s\right) &= \left(1-\tau_i\right) C \\ H\left(X, s\right) &= \left(1-\tau_c^+\right)\left(1-\tau_d\right)\left(1-\alpha(s)\right) V(X, s) \end{aligned} \tag{22}$$

where V is the value of unlevered firm given in Proposition 2.

For equity, dividend rate is positive when cash flow exceeds interest expenses. If cash flows are less than interest, the firm faces a liquidity crunch. As long as the present value of future dividend income exceeds their debt obligation, equity-holders will contribute additional capital through costly equity issuance. The issuance costs are a fraction e of the proceeds. At default, equity-holders recover nothing.

$$\begin{aligned} F\left(X, s\right) &= \left(1-\tau_d\right)\left(1-\tau_c^+\right)\left(X-C\right)^+ - \frac{1-\tau_c^-}{1-e}\left(X-C\right)^- \\ H\left(X, s\right) &= 0 \end{aligned} \tag{23}$$

Proposition 3 *Suppose a firm has a consol bond outstanding with coupon rate C and a default policy characterized by a set of default boundaries $\left(X_D^1, \cdots, X_D^n\right)$, which satisfy the ordering of (21). Then, the value of debt is:*

$$\begin{aligned} \mathbf{D}(X; C) &= \sum_{j=1}^{2k} w_{k,j}^D \mathbf{g}_{k,j} X^{\beta_{k,j}} + \xi_k^D X + \zeta_k^D, \quad X \in \mathcal{D}_k, \quad k = 1, \cdots, n-1 \\ \mathbf{D}(X; C) &= \sum_{j=1}^{2n} w_{n,j}^D \mathbf{g}_{n,j} X^{\beta_{n,j}} + \left(1-\tau_i\right) C \mathbf{b}, \quad X \in \mathcal{D}_n \cup \mathcal{D}_{n+1} \end{aligned} \tag{24}$$

The coefficients $\mathbf{g}$, β, (w^D, ξ^D, ζ^D) are given in Appendix D.

Proof. See Appendix D. ■

The proposition specifies the value of debt in each of the $n+1$ regions. In the first $n-1$ regions, the firm is already in default for some of the states, and the value of debt in those states will be 0. In regions $\mathcal{D}_n$ and $\mathcal{D}_{n+1}$, the firm is alive in all n states, and bond-holders receive coupon C. Fixing the amount of debt outstanding, as X increases, the firm gets further away from bankruptcy. In the limit, the firm is free of default risk:

$$\lim_{X\uparrow+\infty} \mathbf{D}(X;C) = (1-\tau_i)\,C\mathbf{b},$$

where $\mathbf{b}$ is the value of a consol with unit coupon rate (see Corollary 1). This intuition suggests that the coefficients $w^D_{n,j}$ associated with those exponents $\beta_{n,j}$ that are positive will be zero. The remaining term $\sum_j w^D_{n,j}\mathbf{g}_{n,j}X^{\beta_{n,j}}$ determines the value of expected losses at default.

Proposition 4 *For a given coupon rate C and a set of default boundaries $\left(X^1_D,\cdots,X^n_D\right)$, the value of equity is:*

$$\begin{aligned} \mathbf{E}(X;C) &= \sum_{j=1}^{2k} w^E_{k,j}\mathbf{g}_{k,j}X^{\beta_{k,j}} + \xi^E_k X + \zeta^E_k\,, X\in\mathcal{D}_k\,,\quad k=1,\cdots,n \\ \mathbf{E}(X;C) &= \sum_{j=1}^{2n} w^E_{n+1,j}\mathbf{g}_{n,j}X^{\beta_{n,j}} + (1-\tau_d)(1-\tau_c^+)\left(X\mathbf{v}-C\mathbf{b}\right),\quad X\in\mathcal{D}_{n+1} \end{aligned} \tag{25}$$

The coefficients $\mathbf{g}$ and β are given in Proposition 3; (w^E, ξ^E, ζ^E) are given in Appendix D.

Proof. See Appendix D. ■

When cash flows are sufficiently large, the firm no longer needs to issue equity to finance debt payments, and default probability becomes negligible. In the limit, the value of equity should be equal to the value of future cash flows net of perpetual coupon payments and taxes:

$$\lim_{X\uparrow+\infty} \mathbf{E}(X;C) = (1-\tau_d)(1-\tau_c^+)\left(X\mathbf{v}-C\mathbf{b}\right),$$

where $\mathbf{v}$ is the value to cash flow ratio given in Proposition 2. This intuition suggests that, in region $\mathcal{D}_{n+1}$, all the coefficients $w^E_{n+1,j}$ associated with positive exponents $\beta_{n,j}$ are equal to zero. The remaining term $\sum_j w^E_{n+1,j}\mathbf{g}_{n,j}X^{\beta_{n,j}}$ gives the option value of default.

B. Optimal Default Boundaries and Capital Structure

The optimal default boundaries for a given amount of debt outstanding satisfy the smooth-pasting conditions:

$$\left.\frac{\partial}{\partial X}E\left(X,k;C\right)\right|_{X=X_D^k}=0,\quad k=1,...,n. \tag{26}$$

Given the pricing formula for equity in Proposition 4, the n smooth-pasting conditions translate into a system of nonlinear equations, which is solved numerically.

< Figure 4 about here >

Default can be triggered by small shocks or large shocks. In the case of small shocks, the state of the economy does not change, but a series of small negative shocks drives the cash flow below the default boundary of the current state. Alternatively, cash flow can still be above the current boundary, but a sudden change of the state causes the default boundary to jump above the cash flow, leading the firm to default immediately. Figure 4 illustrates these two types of defaults.

The second type of default generates default waves: firms with cash flows between two default boundaries can default at the same time when a large shock arrives. The model of Hackbarth, Miao, and Morellec (2006) generates a similar feature, but with a very different mechanism. In their model, default waves occur when aggregate cash flow levels jump down; in this model, default waves are caused by large changes in expected growth rates, volatility, and risk prices. These distinctions can be tested in the data.

The optimal amount of debt to issue at time 0 is determined by the coupon rate that maximizes the value of equity right before issuing debt. This value is equal to the sum of equity and debt right after issuance minus debt issuance costs, which are a fraction q of debt value. Thus, the value of equity right before debt issuance is:

$$E_U\left(X_0,s_0;C\right)=E\left(X_0,s_0;C\right)+\left(1-q\right)D\left(X_0,s_0;C\right), \tag{27}$$

and the optimal coupon rate in state s_0 is:

$$C^*\left(X_0,s_0\right)=\arg\max_C E_U\left(X_0,s_0;C\right). \tag{28}$$

IV. Dynamic Financing Decisions

While the static model captures many of the effects that macroeconomic conditions have on capital structure and credit spreads, it has one obvious limitation: firms cannot adjust their debt levels. Since firms anticipate their cash flows to grow, not allowing debt issuance in the future can bias the initial leverage ratio upward. Moreover, the model generates nonstationary leverage ratios, so we cannot use it to study firms' financing behavior over time. In this section, I address these concerns by adding the option of upward restructuring into the problem.

Consider a firm's problem at time 0 as defined in (20). Suppose the initial state is s_0. After the firm chooses a coupon rate $C(s_0)$, it needs a default policy and a restructuring policy, which indirectly depend on the initial state through C. The former is described by a set of default boundaries $\left(X_D^1(s_0), ..., X_D^n(s_0)\right)$; the latter is described by a set of upward restructuring boundaries $\left(X_U^1(s_0), ..., X_U^n(s_0)\right)$ and the corresponding new coupon rates.

One can show that the ordering of the default boundaries does not depend on the initial state. Thus, I assume that:

$$X_D^1(s_0) \leq X_D^2(s_0) \leq \cdots \leq X_D^n(s_0).$$

However, there is no guarantee that the restructuring boundaries will have the same order. To accommodate arbitrary orderings, I define a function $u(\cdot)$ that maps the order of restructuring boundaries across states into the indices for the states. For example, $u(i)$ denotes the state with the i^{th} lowest restructuring boundary. Then, by definition,

$$X_U^{u(1)}(s_0) \leq X_U^{u(2)}(s_0) \leq \cdots \leq X_U^{u(n)}(s_0).$$

For reasonable parameters, the default and restructuring boundaries are sufficiently apart such that $X_D^n(s_0) < X_U^{u(1)}(s_0)$.

To facilitate notation, I denote the regions dividend by default and restructuring boundaries as follows. The first n regions are the same as in the static case: there are the default regions, $\mathcal{D}_k = \left[X_D^k(s_0), X_D^{k+1}(s_0)\right)$ for $k = 1, \cdots, n-1$, and region $\mathcal{D}_n = [X_D^n(s_0), C(s_0))$, where there is no immediate threat of default, but the firm is in liquidity crunch. Next, in region

$\mathcal{U}_1 = \left[C(s_0), X_U^{u(1)}(s_0)\right]$, there is no liquidity crunch, either. Finally, there are $n-1$ restructuring regions, $\mathcal{U}_k = \left(X_U^{u(k-1)}(s_0), X_U^{u(k)}(s_0)\right]$ for $k = 2, \cdots, n$, where a change of state can trigger immediate restructuring.

After adding upward restructuring, both debt and equity can be viewed as a contingent claim that pays dividend $F(X_t, s_t)$ until default or upward restructuring, whichever comes first, making payment $H(X_{\mathcal{T}_D}, s_{\mathcal{T}_D})$ at default, and payment $K(X_{\mathcal{T}_U}, s_{\mathcal{T}_U})$ at restructuring.

A. Scaling Property

Since firms have infinite horizon, they face essentially the same problem at each restructuring point. Cash flow level will be higher than at the previous restructuring point, and the economy might be in a different state. Thus, the optimal capital structure problem is recursive and can be formulated into a dynamic programming problem. The problem can be further simplified to a static problem using the scaling property, which is stated next.

Lemma 1 *Conditional on the state of the economy, the optimal coupon rate, default and restructuring boundaries, value of total debt outstanding, and value of equity are all homogeneous of degree 1 in cash flow.*

This is a generalized version of the scaling property used by Goldstein, Ju, and Leland (2001). The intuition is as follows.[9] If the state is the same, the firm at two adjacent restructuring points faces almost identical problems, except that cash flow levels are different. The log-normality of cash flows and proportional costs of debt and equity issuance guarantee that if cash flow has doubled, it is optimal to double the amount of debt and the default/restructuring boundaries.

The scaling property only holds after conditioning on the state. The following example illustrates how we can apply it when the state changes. Suppose the economy is in state 1 at time 0, and a firm chooses its coupon rate and default/restructuring boundaries. The rest of the $n-1$ states can be viewed as "shadow states", which also have their own coupon rates and boundaries. Next, the firm decides to restructure at time t in state 2, with cash flow X. Then, the scaling factor is X/X_0. Because the state has changed, we cannot apply the scaling factor to the coupon rate and default/restructuring boundaries at time 0 to get their new values. Instead, we need to scale up their “shadow values” in state 2.

< Figure 5 about here >

Figure 5 plots a sample path of cash flows for a firm in the dynamic problem. The firm enjoys strong growth in early periods. Cash flows rise and hit the restructuring boundaries twice, causing the firm to raise more debt. When the firm restructures, both the default and restructuring boundaries scale up proportionally. In two occasions, the economy moves into a bad state (shaded area), causing both default and restructuring boundaries to jump. The firm’s luck reverses after 20 years. Its cash flow declines until hitting the default boundary, and the firm defaults.

B. Debt and Equity

For debt, the “dividend rate” and default payment are the same as in the static model (see equation (22)). After restructuring, debt from previous issues gets diluted by new issues. Because of the pari passu covenant, the restructuring payment for old debt will be equal to a fraction of total debt outstanding after restructuring:

$$K\left(X,s;C(X_0,s_0)\right)=\frac{C(X_0,s_0)}{C(X,s)}D\left(X,s;C(X,s)\right)=\frac{C(X_0,s_0)}{C\left(X_0,s\right)}D\left(X_0,s;C\left(X_0,s\right)\right), \tag{29}$$

where $C(X,s)$ denotes the optimal coupon rate for cash flow X in state s, $D(\cdot;C)$ denotes the value of debt for given coupon C. The first equality follows from the pari passu covenant, the second from the scaling property.

When two consecutive restructurings occur in the same state, the value of old debt after restructuring will be exactly the same as its value at the previous restructuring point. If the firm does not issue new debt, the value of old debt will be higher than its original value as cash flow rises. We can measure the dilution effect by comparing the value of old debt after restructuring with the value of debt assuming no restructuring is allowed.

For equity, dividend rate and default payment are also the same as in the static model (see equation (23)). Restructuring payment is equal to the proceeds from new debt issuance (net of issuance costs), plus the equity claim after restructuring:

$$K\left(X,s;C\left(X_0,s_0\right)\right)=\left(1-q\right)D\left(X,s;C\left(X,s\right)\right)-D\left(X,s;C\left(X_0,s_0\right)\right)+E\left(X,s,C\left(X,s\right)\right). \tag{30}$$

The following proposition gives the value of debt and equity for given coupon rates and default/restructuring boundaries.

Proposition 5 *Let cash flow at time* 0 *be* X_0. *Suppose the firm will issue consol bond with coupon rate* $C(s_0)$ *in state* s_0*, its default policy is characterized by a set of default boundaries* $\left(X_D^1(s_0),\cdots,X_D^n(s_0)\right)$*, and its restructuring policy characterized by a set of restructuring boundaries* $\left(X_U^1(s_0),\cdots,X_U^n(s_0)\right)$. *Then, following the initial state* s_0*, the value of debt is:*

$$\begin{aligned}
\mathbf{D}\left(X;s_0\right)&=\sum_{j=1}^{2k}\bar{w}_{k,j}^D(s_0)\mathbf{g}_{k,j}X^{\beta_{k,j}}+\bar{\xi}_k^D(s_0)X+\bar{\zeta}_k^D(s_0),\ X\in\mathcal{D}_k,\ k=1,\cdots,n-1\\
\mathbf{D}\left(X;s_0\right)&=\sum_{j=1}^{2n}\bar{w}_{n,j}^D(s_0)\mathbf{g}_{n,j}X^{\beta_{n,j}}+(1-\tau_i)C(s_0)\mathbf{b},\ X\in\mathcal{D}_n\cup\mathcal{U}_1\\
\mathbf{D}\left(X;s_0\right)&=\sum_{j=1}^{2(n-k+1)}\bar{w}_{n+k,j}^D(s_0)\mathbf{g}_{n+k,j}X^{\beta_{n+k,j}}+\bar{\zeta}_{n+k}^D(s_0),\ X\in\mathcal{U}_k,\ k=2,\cdots,n
\end{aligned} \tag{31}$$

The value of equity is:

$$\begin{aligned}
\mathbf{E}\left(X;s_0\right)&=\sum_{j=1}^{2k}\bar{w}_{k,j}^E(s_0)\mathbf{g}_{k,j}X^{\beta_{k,j}}+\bar{\xi}_k^E(s_0)X+\bar{\zeta}_k^E(s_0),\ X\in\mathcal{D}_k,\ k=1,\cdots,n\\
\mathbf{E}\left(X;s_0\right)&=\sum_{j=1}^{2(n-k+1)}\bar{w}_{n+k,j}^E(s_0)\mathbf{g}_{n+k,j}X^{\beta_{n+k,j}}+\bar{\xi}_{n+k}^E(s_0)X+\bar{\zeta}_{n+k}^E(s_0),\ X\in\mathcal{U}_k,\ k=1,\cdots,n
\end{aligned} \tag{32}$$

The coefficients $\mathbf{g}$, β, $(\bar{w}^D,\ \bar{\xi}^D,\ \bar{\zeta}^D)$, *and* $(\bar{w}^E,\ \bar{\xi}^E,\ \bar{\zeta}^E)$ *are given in Appendix E.*

Proof. See Appendix E. ■

The solutions for the value debt and equity always consist of two parts: one that is linear in X (including the constant case), and the other that is nonlinear in X. The linear part of the solution is equal to the value of future dividend payments ignoring the possibility of default or restructuring, plus the expected value of residual payment due to immediate default/restructuring. The nonlinear part adjusts for the option value associated with default and restructuring.

Next, we need to look for the optimal default/restructuring boundaries for *each initial state*. The default boundaries satisfy the smooth-pasting conditions:

$$\frac{\partial}{\partial X} E\left(X, k; s_0\right)\bigg|_{X=X_D^k(s_0)} = 0, \quad k = 1, ..., n \tag{33}$$

which translate into a set of nonlinear equations as in the static case. Similarly, the restructuring boundaries satisfy:

$$\frac{\partial}{\partial X} E\left(X, u\left(k\right); s_0\right)\bigg|_{X \uparrow X_U^{u(k)}(s_0)} = \frac{\partial}{\partial X_U^{u(k)}\left(s_0\right)} K\left(X_U^{u(k)}\left(s_0\right), u\left(k\right); s_0\right), \quad k = 1, ..., n \tag{34}$$

These conditions need to be evaluated numerically.

Finally, the total value of the firm before levering up is the sum of the value of equity and debt net of issuance costs (same as in (27)). With the help of the scaling property, I am able to compute the value of debt and equity as functions of the complete set of coupon rates $\{C(1), ..., C(n)\}$. Thus, instead of solving the dynamic problem, I can search for the optimal coupon rates by maximizing the total value of the firm in an arbitrary initial state:

$$\left(C^*(1), \cdots, C^*(n)\right) = \underset{C(1),\cdots,C(n)}{\arg\max}\ E_U\left(X_0, s_0; C(1), \cdots, C(n)\right). \tag{35}$$

V. Results

This section examines the quantitative performance of the model. I first calibrate the model parameters using data on aggregate consumption, corporate profit, moments of the equity market, recovery rates, and default rates for firms with different credit ratings. Next, I calculate the optimal leverage ratios and credit spreads in the model. Since the credit spreads of consols in the model are not directly comparable with those of finite maturity bonds, I also compute the spreads of hypothetical 10-year coupon bonds with a given rating by imposing the same default probabilities and recovery rates as firms with the same credit rating.

For target credit spreads, I use the estimates of Duffee (1998). In his sample, the average credit spread of a Baa-rated bond in the industrial sector with 10 years to maturity is 148 bp, while the average Baa-Aaa spread is 101 bp.[10] I calculate the volatility of Baa-Aaa spreads using the Moody's data, which is 40 bp in the sample from 1920 to 2006. For leverage ratio, Chen, Collin-Dufresne, and Goldstein (2006) estimate the average market leverage for Baa firms to be 44%.

A. Calibration

I calibrate the Marked chain that controls the conditional moments of consumption growth to be consistent with the long-run risk model of Bansal and Yaron (2004), which are in turn calibrated to the annual consumption data from 1929 to 1998. Appendix F provides the details of the calibration. I choose 9 states for the Markov chain, maintaining the tractability of the model while allowing for more flexible dynamics in the conditional moments of consumption than a 2-state model.[11] Simulations show that the Markov chain captures the main properties of consumption well. Some of the median statistics from simulations (with data estimates reported in parentheses) are: average annual growth rate 1.81% (1.80%), volatility 2.64% (2.93%), first order autocorrelation 0.42 (0.49), second order autocorrelation 0.18 (0.15).

< Table I about here >

Table I reports the pricing implications of the Markov chain model. The equity premium is based on the same levered up series of aggregate consumption as in Bansal and Yaron (2004). With $\gamma = 7.5$, the model generates moments that are largely consistent with the data. Changing γ to 10 raises the Sharpe ratio and lowers the price-dividend ratio. In both cases, the model requires a tiny subjective discount factor to keep the risk-free rate low. Moreover, the model predicts that short term interest rates are higher in good times, and that the real yield curve is downward sloping on average. This result is consistent with the findings of Piazzesi and Schneider (2006). I use $\gamma = 7.5$ as the benchmark case in this paper.

There are 5 parameters for a firm's cash flow process (equation (10)). I fix the long-run average growth rates of firms' cash flows to be the same as that of aggregate consumption. For a Baa-rated firm, I calibrate the multipliers a_i, b_i, and the average systematic volatility $\overline{\sigma}_m^i$ to fit the moments of the aggregate corporate profits for nonfinancial firms. I calibrate the idiosyncratic volatility σ_f^i

to match the 10-year default probability of Baa-rated firms (4.9%).

Because there are very few Aaa-rated nonfinancial firms, it is difficult to calibrate the cash flow process for a "typical" Aaa firm. Instead, I make the following assumption: the values of a_i, b_i, $\overline{\sigma}_m^i$ and σ_f^i for an Aaa-rated firm are scaled down by the same factor from their values for a Baa firm to match the 10-year default rate for Aaa-rated firms (0.6%). This assumption makes Aaa cash flows less volatile, and their conditional moments less cyclical. Thus, the Aaa firm in the model not only serves as a benchmark to compute credit spreads, but also illustrates the effect of cash flow cyclicality on the capital structure.

I use the tax rate estimates of Graham (2000), which take into account the effect that tax benefits of debt at the corporate level are partially offset by individual tax disadvantages of interest income. I set τ_c^- to 0.2, which makes the net tax benefit of debt negative when firms have negative earnings.

< Table II about here >

Finally, inflation parameters are calibrated using the price index for nondurables and services from NIPA. Costs of debt and equity issuance are from Altinkihc and Hansen (2000) estimates of the underwriting fees for straight bond and seasoned equity offerings. Table II summarizes the calibrated parameters.

The Cyclicality of Recovery Rates and default losses

There are direct and indirect costs for firms going through financial distress. Examples of direct costs include litigation expenses and losses due to asset liquidation. Examples of indirect costs include losses of customers, human capital, growth options, etc. In a model with business cycles, not only the average levels, but the distribution of default losses over different states of the economy matters for capital structure and credit spreads.

Shleifer and Vishny (1992) suggest that liquidation of assets will be particularly costly in recession, when many firms are in distress. In other words, default losses are countercyclical. However, default losses are difficult to measure, because it is difficult to distinguish between losses due to financial and economic distress (see Andrade and Kaplan (1998)). The clustering of defaults in bad times makes it even more difficult to measure the variation of default losses across states. As a result, most structural models assume that default losses are a constant fraction of the value of assets at default. This fraction is often set to the estimates by Andrade and Kaplan (1998), a number between $10 \sim 20\%$.

However, this approach is problematic for several reasons. The estimates of Andrade and Kaplan (1998) are relative to the *pre*-distress value of a firm, which are likely to be significantly higher than the firm value at default. Moreover, it is unclear how well these estimates represent the default losses of a typical firm. On the one hand, Leland (1998) argues that firms choosing to undergo highly leveraged buyouts might have lower default losses than others. On the other hand, the distress periods of many firms in their sample coincide with the 1990-91 recession. If default losses are higher in bad times, then the estimates from this sample might be higher than average.

I use a new approach to estimate default losses. In the structural model, recovery value of corporate bonds is equal to asset value at default net of default losses and taxes. Unlike default losses, bond recovery rates are observable, and have a relatively long time series (Moody's average recovery rates series starts in 1982). Since the model determines default boundaries (and asset value at default) endogenously, one can compute the implied default losses for any given recovery rate. Thus, the structural model makes it possible to identify the time variations in default losses from the variations of recovery rates.

< Figure 6 about here >

Figure 2 shows that recovery rates are lower in recessions. Figure 6 provides further evidence that recovery rates covary with macroeconomic variables: GDP, industrial production, consumption, and price-earnings ratio. I evaluate these relations formally with regressions. Altman et al. (2005) find that default rates explain a large fraction of the variations in recovery rates, while macro variables appear to have little explanatory power. However, default rates are themselves strongly affected by macroeconomic conditions: Table III Panel A shows that growth rates of industrial production, GDP, price-earnings ratio, and consumption all have significant explanatory power. The signs of the coefficients are as expected: lower growth rates in industrial production, GDP, price-earnings ratio and consumption are associated with higher default rates. Moreover, squared consumption growth also enters into the regressions significantly. It captures the nonlinear relationship between default rates and consumption growth: default rates rise more rapidly when consumption growth becomes negative.

< Table III about here >

In Table III Panel B, the univariate regression of recovery rates on default rates confirms the finding of Altman et al. (2005). A regression with only macro variables (PE, g and g^2) can explain 42% of the variation in recovery rates. This number increases to 50% when the riskfree rate is included. In a two-stage regression (last column of the table), the residuals from the regression of default rates on the other macro variables still have significant explanatory power for recovery rates, suggesting that default rates do contain information about recovery rates not captured by the macro variables.

In light of the regression results, I model default losses as a function of the expected growth rate $\theta_m(s)$ and volatility $\sigma_m(s)$ of aggregate consumption:

$$\alpha(s) = a_0 + a_1\theta_m(s) + a_2\theta_m^2(s) + a_3\sigma_m(s). \tag{36}$$

I estimate the coefficients for Baa and Aaa firms separately using the simulated method of moments (SMM). The target moments are: mean and volatility of recovery rate, plus the correlations between recovery rate and the following variables: default rate, price-dividend ratio, and realized consumption growth. The model predicted moments are computed by simulating the dynamic capital structure model. Table IV Panel A reports the target moments. These moments are based on Moody's recovery rates. The average recovery rate for all corporate bonds between 1982-2005 is \$41.1 per \$100 par, with standard deviation \$9.4. These numbers do not apply to debt instruments such as bank loans or mortgages, whose recovery rates are higher and less volatile (the average recovery rate of senior secured bank loans is \$64.2 in the sample). To be conservative, I assume that 70% of debt have recovery rates similar to corporate bonds, and the rest similar to bank loans. Panel B reports the SMM estimates of the coefficients in equation (36).

< Table IV about here >

How does the model identify countercyclical variations in default losses? While asset values drop in recessions, they do not drop as much as recovery rates. Moreover, firms tend to default at higher cash flow levels in recessions, which further dampens the variations in firm value at default. Then, default losses must be higher in recessions in order for the model to fit low recovery rates at those times.

B. Capital Structure and Credit Spreads

To illustrate the difficulty for standard structural models to generate reasonable credit spreads and leverage ratios, I first study the benchmark case of the model. In this case, I shut down business-cycle variations in aggregate consumption, cash flows, and default losses, setting all variables to their unconditional means, with two exceptions: the default loss coefficient α, and the idiosyncratic volatility of cash flows, σ_f. I use them to match the average recovery rate and 10-year default probability of a Baa and Aaa-rated firm.

< Figure 7 about here >

Figure 7 reports results for a wide range of recovery rates. Credit spreads are surprisingly insensitive to changes in recovery rates, while the optimal leverage ratio rises with the recovery rate. The latter is intuitive: as recovery rates rise, default losses drop, which makes firms take on more debt. Higher leverage tends to raise default probability, and we need to lower idiosyncratic volatility to offset the effect. So, as recovery rates become higher, default becomes more systematic, offsetting the effects of recovery rates on spreads. The expected excess returns for equity appear to be high, which is because these firms are highly-levered, making their dividend processes volatile. The rise in expected excess return with recovery rates is again due to rising leverage.

For Baa firms, with a relative risk aversion of 7.5, and a recovery rate of 48%, the model generates a 57 bp credit spread for a 10-year bond, far short of the average spread in the data (148 bp). The model predicts a leverage ratio of 67%, significantly higher than the average leverage of Baa firms (44%). The interest coverage, measured as the ratio of cash flow to interest expenses, is 0.7, while the value in the data is around 3. These discrepancies highlight the dual puzzles of credit spreads and leverage ratio. The puzzles get worse as recovery rate becomes higher.

One can not resolve the puzzles simply by raising risk aversion. While higher risk aversion raises spreads, it increases the expected excess return on equity dramatically. Moreover, a higher risk aversion actually increases market leverage. It does lead to lower debt level, as measured by interest coverage. However, higher risk aversion makes equity value drop more than debt, resulting in higher market leverage.

< Table V about here >

Table V reports the results from the static capital structure model with variations in macroeconomic conditions. To facilitate the comparison with the benchmark case, I first consider the case that leaves out partial loss offset and equity issuance costs. A firm can issue debt in any state. Rather than reporting the results for each state, the table reports for each variable the average values across all states, along with their standard deviations. In order to match the 10-year default probabilities, the idiosyncratic volatilities of the Baa and Aaa firms are lowered to 0.169 and 0.117 (from the values in Table II).

The static model already makes considerable progress in explaining credit spreads and leverage ratios. It raises the average credit spread of a 10-year Baa-rated bond from 57 to 141 bp, while the average credit spread between Baa and Aaa-rated bonds is 98 bp. The market leverage drops from 66.7% in the benchmark model to 50.4%. The levered firm has an expected excess return of 9.3%. The value of the net tax benefits, which is computed as the percentage increase in firm value when it takes on optimal leverage, is about 5.3%, about half of the value in the benchmark.

Since Aaa firms in the model have safer cash flows, they can choose higher leverage ratios and still have lower default probability. This effect is very pronounced in the benchmark model, both in terms of market leverage and net tax benefits. Comparing Aaa and Baa firms in Panel A and Panel B of Table V, we see that macroeconomic risks have more significant impacts on Aaa firms. Since Aaa firms in the model have low volatility, their defaults are more likely caused by systematic and persistent shocks to their growth rates, which make their capital structure more affected by credit risk premia.

The last column of Table V reports the risk premium on equity. In the benchmark model, Aaa firms have much higher leverage than Baa firms, so much so that the risk premium on their equity is higher than that of Baa firms. With business cycles, the leverage of Aaa firms is still higher, but the risk premium on their equity has fallen below that of Baa firms. These results highlight the ambiguous relation between leverage and equity risk premium with endogenous leverage.

Table V Panel C reports the results of four comparative static exercises. Case I considers risky tax benefits by modeling partial loss offset (setting $\tau_c^- = 0.2$). Case II considers costly equity issuance. Case III reports the full model, where both of the above features are included. With partial loss offset, the optimal leverage drops to 45.9%. Moreover, the model predicts a 10-year default probability of only 3.9%. If we recalibrate the model to match the default rate of 4.9%, the leverage ratio will drop further. In contrast, the effect of equity issuance costs on capital structure appears to be very small, suggesting that we will obtain similar results if we relax the constraints for downside restructuring by introducing large adjustment costs.

Case IV sets default losses to a constant proportion of asset value at default (by fixing α to its unconditional mean). Leverage ratio jumps back to 65.6%, almost as high as in the benchmark case. Moreover, the 10-year default probability is too high. If we recalibrate the model to bring down the default probability, the leverage ratio will become even higher. The results of Case IV highlight the central role that countercyclical default losses play in explaining the leverage puzzle. When α is constant, procyclical variation in asset value leads to lower default losses in bad times, which actually generates a negative risky premium on the defaultable claim for equity-holders, causing leverage ratio to become much higher.

< Table VI about here >

Table VI reports the results for the dynamic model. The full model presented here consider both partial loss offset and equity issuance costs. On average, optimal leverage for Baa-rated firm drops to 37.2%, reflecting the correction of the “upward bias” of leverage ratio in the static model. Interest coverage rises to 2.5, and net tax benefits rise to 6.0%. Credit spreads of the consol and 10-year coupon bond in the dynamic model do not differ much from their values in the static model. This is because both models are calibrated to have default probabilities and recovery rates matching the data. Compared to the static model, the differences in leverage ratio between Aaa and Baa firms become bigger in the dynamic model. Since Baa firms have more risky cash flows, they become more conservative in choosing initial leverage when they have the option to restructure in the future.

The comparative statics for Baa-rated firms confirm the results in the static case. The model predicted leverage again rise significantly when default losses are no longer countercyclical. Removing equity issuance costs raises leverage mildly to 41.8%. While giving firms cheaper access to equity will not necessarily correct for the bias due to firms' inability to reduce their debt, the small effect on leverage ratio suggests that this bias will be small as long as the costs of reducing debt are sufficiently high.

The standard deviations for credit spreads reported in Table VI do not measure the volatility of credit spreads for firms with certain credit ratings. They measure the deviation in credit spreads across optimally levered firms in different states. Because of the lumpy adjustment costs, a firm does not always adjust its capital structure immediately following a large shock. The firm's leverage ratio and credit spread will change, but not necessarily the credit rating, because rating agencies assign ratings through the cycle. Thus, the lumpiness of a firm's capital structure can lead to high volatilities in credit spreads.[12] I compute the volatility of spreads between a Baa and Aaa-rated bond, both issued in the normal state (with medium expected growth rate and medium volatility), using the variation of the spreads across different states. This way, I get the volatility of the 10-year Baa-Aaa spread to be 35.2 bp (40 bp in the data).

< Figure 8 about here >

To see how the optimal leverage ratios, default boundaries, recovery rates, and default losses vary over the business cycle, I simulate the state of the economy for 100 years, and plot the corresponding values of the above variables in Figure 8. Recessions, marked with shades in the plots, are periods when the expected growth rates are negative. Those recessions with high (low) volatility are the most (least) severe, and are marked with the darkest (lightest) shades. The optimal leverage ratios are lower in recessions, and they appear more sensitive to the movements in volatility than in expected growth rates. The default boundaries are higher in recessions, and they appear more sensitive to changes in the expected growth rates. Recovery rates are lower in recessions, especially when the volatility is high. Default losses, specified as percentages of *pre-*

distress firm value, are rather low outside of recessions. They rise significantly in recessions, topping out at 17% in a most severe recession, which is still below the upper bound estimated by Andrade and Kaplan (1998).

The countercyclical default boundaries in Figure 8 implies that equity-holders will voluntarily default earlier (at higher cash flow levels) in recessions. This feature, combined with the fact that low expected growth rates and high uncertainty in bad times make it more likely for a firm to enter into distress (by lowering cash flow), generates high default probabilities in recessions.

Why do firms choose higher default boundaries in bad times? Equity-holders of a levered firm hold a perpetual compound option (see Geske (1977)). At any point, they can either make debt payments and retain their claim on future dividend and the option to default, or forfeit the firm's future cash flows in exchange for the waiver of debt obligations. In bad times, higher risk premia lower the present value of future cash flows. This is the "discount rate effect". Expected growth rates are also lower in bad times, which not only lower the present value of future cash flows directly, but raise the probability of default and the probability that the firm loses part of its tax shield. This is the "cash flow effect". Both effects reduce the continuation value for equity-holders, making them default earlier. Finally, high volatility makes the option to default more valuable, which tends to defer default. This is the "volatility effect". In the model, the discount rate and cash flow effects dominate the volatility effect, making firms default earlier in recessions.

< Figure 9 about here >

Finally, to illustrate the countercyclical default rates and the clustering of defaults, I simulate 1000 identical firms over 50 years, and record the timing of defaults. These firms experience the same aggregate shocks, but have different outcome due to idiosyncratic shocks. Figure 9 plots the default counts and corresponding annual default rates for a typical simulation. During this simulation, the economy experiences 3 states – high growth & median uncertainty, low growth & median uncertainty, and low growth & high uncertainty (at the end of 50 years). Most of the defaults occur in the latter two states where growth rate is low. The simulation nicely replicates the countercyclical default rates in the data, and we see the dramatic increase in default rate when the economy moves into the "worst state" – low growth and high uncertainty. In the graph of default counts, the two highest spikes occur right at the time when the economy moves from a high growth state into a low growth one. These are examples of default clustering: firms default at the same time due to the sudden increase of default boundary. The phenomenon resembles the "contagion effect": credit spreads shoot up sharply following a wave of defaults. However, it is the same large shocks that cause a group of firms to default together while raising the spreads of other firms.

VI. Concluding Remarks

Since corporate defaults tend to concentrate in bad times when marginal utility is high, and default losses are particularly high during such times, investors will demand high risk premia for holding defaultable claims, including corporate bonds and levered firms. In this paper, I formally study these comovements in a structural model, and show that credit risk premia can quantitatively account for the high credit spreads and low leverage ratios in the data. The model also has a rich set of predictions about how business cycles affect financing decisions and the pricing of corporate securities.

To highlight the effects of macroeconomic conditions and risk premia on firms' financing decisions, I study a very simple trade-off model, which abstracts away from many important decisions that firms face, such as cash reserves, investments, dividend, agency costs, downside restructuring, strategic debt services, etc. It will be interesting to see how macroeconomic conditions interact with these different features in corporate finance.

The model provides an important role for macro variables in the determination of credit spreads, which is consistent with the empirical findings of Collin-Dufresne et al. (2001) and Elton et al. (2001). On the other hand, there is a large body of research that use default spreads to predict returns for stocks and bonds (Cochrane (2006) surveys these studies). This model provides a theoretical basis for including credit spread information in such studies: unlike stocks, credit spreads (especially those with high credit quality) are less exposed to small cash flow shocks, but are more sensitive to risk prices in bad states. These characteristics can make changes in credit spreads a good proxy for variations in risk prices.

Appendix

A Proof of Proposition 1

To get the stochastic discount factor, we first need to solve for the value function of the representative household. In equilibrium, the representative household consumes aggregate output, which is given in (4). Thus, I directly define the value function of the representative agent as:

$$J\left(Y_t, s_t\right) = E_t\left[\int_0^\infty f\left(Y_{t+s}, J_{t+s}\right) ds\right]. \tag{A.1}$$

The Hamilton-Jacoby-Bellman equation in state i is:

$$0 = f\left(Y, J\left(Y, i\right)\right) + J_c\left(Y, i\right) Y \theta_m\left(i\right) + \frac{1}{2} J_{cc}\left(Y, i\right) Y^2 \sigma_m^2\left(i\right) + \sum_{j \neq i} \lambda_{ij}\left(J\left(Y, j\right) - J\left(Y, i\right)\right). \tag{A.2}$$

There are n such differential equations for the n states. Thus, by using a Markov chain to model the expected growth rate and volatility, we replace a high-dimensional partial differential equation with a system of ordinary differential equations. As long as the number of states for the Markov chain is not too large, the ODE system will be relatively easy to handle.

I conjecture that the solution for J is:

$$J\left(Y, s\right) = \frac{\left(h\left(s\right) Y\right)^{1-\gamma}}{1-\gamma}, \tag{A.3}$$

where h is a function of the state variable s. Substituting J into the differential equations above, we get a system of nonlinear equations for h:

$$\begin{aligned} 0 = {} & \rho \frac{1-\gamma}{1-\delta} h(i)^{\delta-\gamma} + \left[\left(1-\gamma\right)\theta_m\left(i\right) - \frac{1}{2}\gamma\left(1-\gamma\right)\sigma_m^2\left(i\right) - \rho\frac{1-\gamma}{1-\delta}\right] h\left(i\right)^{1-\gamma} \\ & + \sum_{j \neq i} \lambda_{ij}\left(h\left(j\right)^{1-\gamma} - h\left(i\right)^{1-\gamma}\right), \qquad i = 1, \cdots, n \end{aligned} \tag{A.4}$$

where $\delta = 1/\psi$, the inverse of the intertemporal elasticity of substitution. These equations can be solved quickly using a nonlinear equation solver, even in the case when the number of states is fairly large.

Duffie and Epstein (1992) and Duffie and Skiadas (1994) show that the stochastic discount factor is equal to:

$$m_t = e^{\int_0^t f_v\left(c_u, J_u\right) du} f_c\left(c_t, J_t\right). \tag{A.5}$$

Plugging J and Y into (A.5) gives:

$$m_t = \exp\left(\int_0^t \frac{\rho\left(1-\gamma\right)}{1-\delta}\left[\left(\frac{\delta-\gamma}{1-\gamma}\right) h\left(s_u\right)^{\delta-1} - 1\right] du\right) \rho h\left(s_t\right)^{\delta-\gamma} Y_t^{-\gamma}. \tag{A.6}$$

Applying Ito's formula with jumps (see, e.g., Appendix F Duffie (2001)) to m, we get:

$$\frac{dm_t}{m_t} = -r(s_t)\,dt - \eta(s_t)\,dB_t + \sum_{s_t \neq s_{t-}} \left(e^{\kappa\left(s_{t-},s_t\right)} - 1\right) dM_t^{\left(s_{t-},s_t\right)}, \tag{A.7}$$

where

$$\begin{aligned} r(i) &= -\frac{\rho(1-\gamma)}{1-\delta}\left[\left(\frac{\delta-\gamma}{1-\gamma}\right) h(i)^{\delta-1} - 1\right] + \gamma\theta_m(i) \\ &\quad -\frac{1}{2}\gamma(1+\gamma)\,\sigma_m^2(i) - \sum_{j\neq i}\lambda_{ij}\left(e^{\kappa(i,j)} - 1\right), &\text{(A.8a)} \\ \eta(i) &= \gamma\sigma_m(i), &\text{(A.8b)} \\ \kappa(i,j) &= (\delta-\gamma)\log\left(\frac{h(j)}{h(i)}\right). &\text{(A.8c)} \end{aligned}$$

B The Risk-neutral Measure

Let $(\boldsymbol{\Omega}, \mathfrak{F}, \mathcal{P})$ be the probability space on which the Brownian motions and Poisson processes in the model are defined. Let the corresponding information filtration be $(\mathfrak{F}_t)$.

The nominal stochastic discount factor is:

$$n_t = \frac{m_t}{P_t}. \tag{B.1}$$

Applying Ito's formula to n_t, we get the dynamics of the nominal stochastic discount factor n_t,

$$\frac{dn_t}{n_t} = -r^n(s_t)\,dt - \eta^m(s_t)\,dW_t^m - \eta^P dW_t^P + \sum_{s_t\neq s_{t-}} \left(e^{\kappa\left(s_{t-},s_t\right)} - 1\right) dM_t^{\left(s_{t-},s_t\right)}, \tag{B.2}$$

where the nominal risk-free rate is

$$r^n(s_t) = r(s_t) + \pi - \sigma_{P,1}\eta(s_t) - \sigma_P^2, \tag{B.3}$$

and the risk prices for the two Brownian motions are

$$\begin{aligned} \eta^m(s_t) &= \eta(s_t) + \sigma_{P,1}, &\text{(B.4)} \\ \eta^P &= \sigma_{P,2}. &\text{(B.5)} \end{aligned}$$

We can define the risk-neutral measure $\mathcal{Q}$ associated with the nominal stochastic discount factor n_t (equation (B.2)) by specifying the density process ξ_t,

$$\xi_t = E_t\left[\frac{dQ}{dP}\right],$$

which evolves according to the following process:

$$\frac{d\xi_t}{\xi_t} = -\eta^m(s_t)\,dW_t^m - \eta^P dW_t^P + \sum_{s_t\neq s_{t-}} \left(e^{\kappa\left(s_{t-},s_t\right)} - 1\right) dM_t^{\left(s_{t-},s_t\right)}. \tag{B.6}$$

Applying the Girsanov theorem, we get the new standard Brownian motions under $\mathcal{Q}$, $\tilde{W}^m$ and $\tilde{W}^P$, which solve:

$$d\tilde{W}_t^m = dW_t^m + \eta^m(s_t)\,dt, \tag{B.7}$$

$$d\tilde{W}_t^P = dW_t^P + \eta^P dt. \tag{B.8}$$

The Girsanov theorem for point processes (see Elliott (1982)) gives the new jump intensity of the Poisson process under $\mathcal{Q}$:

$$\tilde{\lambda}_{jk} = E\left[e^{\kappa(j,k)}\right]\lambda_{jk} = e^{\kappa(j,k)}\lambda_{jk}, \quad j \neq k \tag{B.9}$$

which adjusts the intensity of the Poisson processes under measure $\mathcal{P}$ by the expected jump size of the density ξ_t. Finally, the diagonal elements of the generator has to be reset to make each row sum up to zero,

$$\tilde{\lambda}_{jj} = -\sum_{k\neq j}\tilde{\lambda}_{jk}. \tag{B.10}$$

These two equations characterize the new generator matrix $\tilde{\mathbf{\Lambda}}$ under $\mathcal{Q}$.

C Proof of Proposition 2

The risk-neutral dynamics of the log nominal cash flow $x_t^i = log(X_t^i)$ for firm i is:

$$dx_t^i = \left(\tilde{\theta_X^i}(s_t) - \frac{1}{2}\sigma_X^i(s_t)^2\right)dt + \sigma_X^i(s_t)\,d\tilde{W}_t^i. \tag{C.1}$$

where $\tilde{\theta}_X^i$ is the risk-neutral growth rate,

$$\tilde{\theta}_X^i(s_t) = \theta_X^i(s_t) - \sigma_{X,m}^i(s_{t-})\,\eta^m(s_{t-}) - \sigma_{P,2}\eta^P,$$

$\sigma_X^i(s_t)$ is the total volatility of cash flow,

$$\sigma_X^i(s_t) = \sqrt{\left(\sigma_{X,m}^i(s_t)\right)^2 + \sigma_{P,2}^2 + \left(\sigma_f^i\right)^2}, \tag{C.2}$$

and $\tilde{W}_t^i$ is a new Brownian motion that aggregates all the shocks for firm i,

$$d\tilde{W}_t^i = \frac{\sigma_{X,m}^i(s_t)}{\sigma_X^i(s_t)}d\tilde{W}_t^m + \frac{\sigma_{P,2}}{\sigma_X^i(s_t)}d\tilde{W}_t^P + \frac{\sigma_f^i}{\sigma_X^i(s_t)}dW_t^i. \tag{C.3}$$

The total value of firm i's cash-flows before taxes is:

$$V^i\left(X_t^i, s_t\right) = \mathbb{E}_t^Q\left[\int_t^\infty \exp\left(-\int_t^\tau r^n(s_u)\,du\right)X_\tau^i d\tau\right]. \tag{C.4}$$

Let $\mathbf{V}^i(x) = \left[V^i(x,1), ..., V^i(x,n)\right]'$ be the vector of firm i's asset values in n states. The Feynman-Kac formula implies that $\mathbf{V}^i$ satisfies the following system of ODEs:

$$\mathbf{r}^n\mathbf{V}^i = \left(\tilde{\theta}^i_X - \frac{1}{2}\mathbf{\Sigma}^i_X\right)\mathbf{V}^i_x + \frac{1}{2}\mathbf{\Sigma}^i_X\mathbf{V}_{xx} + \tilde{\mathbf{\Lambda}}\mathbf{V}^i + e^x \cdot \mathbf{1}, \tag{C.5}$$

where $\mathbf{r}^n = \mathbf{diag}\left(\left[r^n\left(1\right), \cdots, r^n\left(n\right)\right]'\right)$, $\tilde{\theta}^i_X = \mathbf{diag}\left(\left[\tilde{\theta}^i_X\left(1\right), \cdots, \tilde{\theta}^i_X\left(n\right)\right]'\right)$, $\mathbf{1}$ is an $n \times 1$ vector of ones, and $\mathbf{\Sigma}^i_X = \mathbf{diag}\left(\left[\sigma^i_X(1)^2, \cdots, \sigma^i_X(n)^2\right]'\right)$. Solving the ODE subject to the boundary conditions:

$$\lim_{x\uparrow+\infty}\mathbf{V}^i(x)e^{-x} < \infty\,, \quad \lim_{x\downarrow-\infty}\mathbf{V}^i(x) = \mathbf{0} \tag{C.6}$$

we get:

$$\mathbf{V}^i(x) = e^x\mathbf{v}^i,$$

with

$$\mathbf{v}^i = \left(\mathbf{r}^n - \tilde{\theta}^i_X - \tilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{1}. \tag{C.7}$$

D Proof of Proposition 3 and 4

To simplify notation, I temporarily drop the superscripts denoting individual firms. Start with a corporate contingent claim $J\left(x_t, s_t\right)$, which pays a dividend rate $F\left(x_t, s_t\right)$ for as long as the firm is solvent, and a default payment $H\left(x_\tau, s_\tau\right)$ when default occurs at time τ. Let $\mathbf{F}(x)$ be an $n \times 1$ vector of dividend rate across n states, and $\mathbf{H}(x)$ an $n \times 1$ vector of the default payments. I also define an $n \times n$ diagonal matrix $\mathcal{A}$. Its ith diagonal element $\mathcal{A}^i$ is the infinitesimal generator for any C^2 function $\phi(x)$ in state i, where x is the log nominal cash flow specified in (C.2):

$$\mathcal{A}^i\phi\left(x\right) = \left(\tilde{\theta}_X\left(i\right) - \frac{1}{2}\sigma^2_X\left(i\right)\right)\frac{\partial}{\partial x}\phi\left(x\right) + \frac{1}{2}\sigma^2_X\left(i\right)\frac{\partial^2}{\partial x^2}\phi\left(x\right). \tag{D.1}$$

When cash flow X is in the region $\mathcal{D}_k = [X^k_D, X^{k+1}_D)$ (for $k < n$), the firm will already be in default in all states $s > k$. Thus, the security will only be " alive" in the first k states. Define sets $\mathcal{I}_k = \{1, ..., k\}$ and their complements $\mathcal{I}^c_k = \{k+1, ..., n\}$. When $X \in \mathcal{D}_k$, the claims that are not in default yet are $\mathbf{J}_{[\mathcal{I}_k]}$, which satisfy the following system of ordinary differential equations:

$$\mathcal{A}_{[\mathcal{I}_k,\mathcal{I}_k]}\mathbf{J}_{[\mathcal{I}_k]} + \mathbf{F}_{[\mathcal{I}_k]} + \tilde{\mathbf{\Lambda}}_{[\mathcal{I}_k,\mathcal{I}_k]}\mathbf{J}_{[\mathcal{I}_k]} + \tilde{\mathbf{\Lambda}}_{\left[\mathcal{I}_k,\mathcal{I}^c_k\right]}\mathbf{H}_{\left[\mathcal{I}^c_k\right]} = \mathbf{r}^n_{[\mathcal{I}_k,\mathcal{I}_k]}\mathbf{J}_{[\mathcal{I}_k]}. \tag{D.2}$$

This equation states that, under the risk-neutral measure, the instantaneous expected return of a claim in any state should be equal to the riskfree rate in the corresponding state. A sudden change of the state can lead to abrupt changes in the value of the claim. It could also lead to immediate default, in which case the default payment is realized. These explain the last two terms on the LHS of the equation.

In regions $\mathcal{D}_n$ and $\mathcal{D}_{n+1}$, the firm is alive in all states. Sudden change of the state will not cause default. Thus, the ODE becomes:

$$\mathcal{A}\mathbf{J} + \mathbf{F} + \tilde{\mathbf{\Lambda}}\mathbf{J} = \mathbf{r}^n\mathbf{J}. \tag{D.3}$$

The homogeneous equation in region $\mathcal{D}_k$ can be written as:

$$\mathcal{A}_{[\mathcal{I}_k,\mathcal{I}_k]}\mathbf{J}_{[\mathcal{I}_k]} + \left(\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_k,\mathcal{I}_k]} - \mathbf{r}^n_{[\mathcal{I}_k,\mathcal{I}_k]}\right)\mathbf{J}_{[\mathcal{I}_k]} = \mathbf{0}, \tag{D.4}$$

which is a quadratic eigenvalue problem. Jobert and Rogers (2006) show its solution takes the following form:

$$\mathbf{J}\left(x\right)_{[\mathcal{I}_k]} = \sum_{j=1}^{2k} w_{k,j}\mathbf{g}_{k,j}\exp\left(\beta_{k,j}x\right), \tag{D.5}$$

where $\mathbf{g}_{k,j}$ and $\beta_{k,j}$ are solutions to the following standard eigenvalue problem:

$$\begin{bmatrix} 0 & \mathbf{I} \\ -\left(2\mathbf{\Sigma}_X^{-1}\left(\tilde{\mathbf{\Lambda}} - \mathbf{r}^n\right)\right)_{[\mathcal{I}_k,\mathcal{I}_k]} & -\left(2\mathbf{\Sigma}_X^{-1}\tilde{\theta}_X - \mathbf{I}\right)_{[\mathcal{I}_k,\mathcal{I}_k]} \end{bmatrix} \begin{bmatrix} \mathbf{g}_k \\ \mathbf{h}_k \end{bmatrix} = \beta_k \begin{bmatrix} \mathbf{g}_k \\ \mathbf{h}_k \end{bmatrix}, \tag{D.6}$$

where $\mathbf{I}$ is an $n \times n$ identity matrix, $\mathbf{r}^n$, $\tilde{\theta}_X$ and $\mathbf{\Sigma}_X$ are defined in (C.5). The coefficients $w_{k,j}$ will be different for different securities. Barlow, Rogers, and Williams (1980) show that there are exactly n eigenvalues with negative real parts, and n with positive real parts.

The remaining tasks are to find particular solutions for the inhomogeneous equations, and solve for the coefficients $w_{k,j}$ through the boundary conditions, which depend on the specific type of security under consideration.

D.1 Debt

Let $D\left(x,s\right)$ be the total value of corporate debt outstanding when the firm has log cash flow x and the economy is in state s. The dividend rate and default payment are given in equation (22).

When $X \in \mathcal{D}_k$ $(k = 1, \cdots, n-1)$, for those states $i \in \mathcal{I}_k$, it follows from equation (D.2) that:

$$\begin{aligned} r^n(i)D\left(x,i\right) &= \mathcal{A}^i D\left(x,i\right) + \tilde{\lambda}_{i,1}D\left(x,1\right) + \cdots + \tilde{\lambda}_{i,k}D\left(x,k\right) \\ &\quad + \tilde{\lambda}_{i,k+1}H\left(x,k+1\right) + \cdots + \tilde{\lambda}_{i,n}H\left(x,n\right) + \left(1-\tau_i\right)C. \end{aligned} \tag{D.7}$$

Stacking up $D(X,s)$ in a vector, $\mathbf{D}(X) = \left[D\left(X,1\right), \cdots, D\left(X,n\right)\right]'$. We first get the following solution to the homogeneous equations,

$$\mathbf{D}\left(x\right)_{[\mathcal{I}_k]} = \sum_{j=1}^{2k} w^D_{k,j}\mathbf{g}_{k,j}\exp\left(\beta_{k,j}x\right), \quad k < n, \tag{D.8}$$

where $\mathbf{g}_{k,j}$ and $\beta_{k,j}$ are characterized in the eigenvalue problem (D.6).

The inhomogeneous equation has the additional term that is linear in e^x:

$$\begin{aligned} &\tilde{\lambda}_{i,k+1}V_B\left(x,k+1\right) + \cdots + \tilde{\lambda}_{i,n}V_B\left(x,n\right) + \left(1-\tau_i\right)C \\ = &\left(1-\tau_c^+\right)\left(1-\tau_d\right)\sum_{j=k+1}^{n}\tilde{\lambda}_{ij}\left(1-\alpha(j)\right)v\left(j\right)e^x + \left(1-\tau_i\right)C. \end{aligned}$$

It is easy to verify that a particular solution is:

$$\mathbf{D}\left(x\right)_{[\mathcal{I}_k]} = \xi_k^D(\mathcal{I}_k)e^x + \zeta_k^D(\mathcal{I}_k), \tag{D.9}$$

where

$$\begin{aligned} \xi_k^D(\mathcal{I}_k) &= \left(1-\tau_c^+\right)\left(1-\tau_d\right)\left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}} - \tilde{\theta}_X\right)^{-1}_{[\mathcal{I}_k,\mathcal{I}_k]}\left(\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_k,\mathcal{I}_k^c]}\left(\alpha \odot v\right)_{[\mathcal{I}_k^c]}\right) \\ \zeta_k^D(\mathcal{I}_k) &= \left(1-\tau_i\right)C\left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_k,\mathcal{I}_k]}\mathbf{1}_k \end{aligned} \tag{D.10}$$

The symbol $\odot$ denotes element-by-element multiplication; $\xi_k^D(\mathcal{I}_k^c)$ and $\zeta_k^D(\mathcal{I}_k^c)$ are equal to zero.

In the region $\mathcal{D}_n \cup \mathcal{D}_{n+1} = [X_D^n, +\infty)$, a sudden change of state will not lead to immediate default. Thus, the extra term in the inhomogeneous equation no longer depends on x. The total value of debt satisfies

$$r^n(i)D\left(x,i\right) = \mathcal{A}^i D\left(x,i\right) + \tilde{\lambda}_{i,1}D\left(x,1\right) + \cdots + \tilde{\lambda}_{i,n}D\left(x,n\right) + \left(1-\tau_i\right)C. \tag{D.11}$$

Now the solution to the homogeneous equation is:

$$\mathbf{D}\left(x\right) = \sum_{j=1}^{n} w_{n,j}^D \mathbf{g}_{n,j}\exp\left(\beta_{n,j}x\right), \tag{D.12}$$

and a particular solution in this region is:

$$D\left(x,i\right) = \zeta_n^D\left(i\right), \tag{D.13}$$

where

$$\zeta_n^D = \left(1-\tau_i\right)C\mathbf{b}. \tag{D.14}$$

Next, I specify the boundary conditions that determine the coefficients $w_{k,j}^D$. Debt value should be finite as x goes to infinity. To exclude any explosive terms, we need to set $w_{n,j}^D$ associated with all the $\beta_{n,j}$ with positive real parts (n of them) to zero. Then, the value of debt as cash flow gets large approaches that of a perpetuity without default risk:

$$\lim_{X\to+\infty}\mathbf{D}\left(X\right) = \left(1-\tau_i\right)C\mathbf{b}. \tag{D.15}$$

Another set of boundary conditions specify the value of debt at the n different default boundaries:

$$D\left(X_D^i, i\right) = H\left(X_D^i, i\right), \quad i = 1,\cdots,n. \tag{D.16}$$

Because the payoff function F and terminal payoff H are bounded and piecewise-continuous in X, while the discount rate r is constant in each state, an application of Theorem 4.9 (Karatzas and Shreve 1991, page 271) shows that $D\left(X,s\right)$ must be piecewise C^2 with respect to X over the region where it is defined, $[X_D^s, +\infty)$. Thus, for any

$i \in \mathcal{I}_{n-1}$, we need to ensure that $D(X,i)$ is C^0 and C^1 at the boundaries $X_D^{i+1}, \cdots, X_D^n$:

$$\begin{aligned}
\lim_{X\uparrow X_D^k} D(X,i) &= \lim_{X\downarrow X_D^k} D(X,i), \quad k = i+1, ..., n \\
\lim_{X\uparrow X_D^k} D_X(X,i) &= \lim_{X\downarrow X_D^k} D_X(X,i), \quad k = i+1, ..., n
\end{aligned}$$

There are $2n^2$ of unknown coefficients for $\left\{w_{k,j}^D\right\}$. The continuity of D and its derivatives at the different default boundaries also give us $2n^2$ conditions. So we can solve for $\left\{w_{k,j}^D\right\}$ from a system of linear equations.

D.2 Equity

Solving for E is similar to solving for D, except for the different payoffs and boundary conditions. The dividend rate and payment upon default are given in (23). By definition, cash flow falls short of the interest expense in all regions except $\mathcal{D}_{n+1}$.

Define $\mathbf{E}(x) = [\mathbf{E}(x,1), \cdots, \mathbf{E}(x,n)]'$. The solutions to the homogeneous equations in regions $\mathcal{D}_k$ $(k = 1, \cdots, n)$ are

$$\mathbf{E}(x)_{[\mathcal{I}_k]} = \sum_{j=1}^{2k} w_{k,j}^E \mathbf{g}_{k,j} \exp(\beta_{k,j} x). \tag{D.17}$$

The additional term for the inhomogeneous equations in these regions are the amount of equity injection needed to meet coupon payment, and the issuance costs:

$$\frac{1-\tau_c^-}{1-e}(C-X).$$

It is easy to verify that a particular solution is:

$$\mathbf{E}(x)_{[\mathcal{I}_k]} = \xi_k^E(\mathcal{I}_k) e^x + \zeta_k^E(\mathcal{I}_k). \tag{D.18}$$

The coefficients $\xi_k^E(i)$ and $\zeta_k^E(i)$ will be zero for $i \in \mathcal{I}_k^c$, because the firm is already in default in those states. For $i \in \mathcal{I}_k$,

$$\begin{aligned}
\xi_k^E(\mathcal{I}_k) &= -\frac{1-\tau_c^-}{1-e}\left(\mathbf{r}^n - \tilde{\theta}_X - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_k,\mathcal{I}_k]} \mathbf{1}_k \\
\zeta_k^E(\mathcal{I}_k) &= \frac{1-\tau_c^-}{1-e} C\left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_k,\mathcal{I}_k]} \mathbf{1}_k
\end{aligned} \tag{D.19}$$

In $\mathcal{D}_{n+1}$, the homogeneous equation is identical to that in the region $\mathcal{D}_n$. Thus, the solution shares the same $\mathbf{g}$ and β,

$$\mathbf{E}(x) = \sum_{j=1}^{2n} w_{n+1,j}^E \mathbf{g}_{n,j} \exp(\beta_{n,j} x). \tag{D.20}$$

The additional term for the inhomogeneous equation in region $\mathcal{D}_{n+1}$ is the dividend payment

$$(1-\tau_d)(1-\tau_c^+)(X-C),$$

and it is easy to verify that a particular solution is:

$$E\left(x,i\right)=\xi_{n+1}^{E}\left(i\right)e^{x}+\zeta_{n+1}^{E}\left(i\right), \tag{D.21}$$

where

$$\begin{aligned}\xi_{n+1}^{E}&=(1-\tau_d)(1-\tau_c^{+})\left(\mathbf{r}^{n}-\tilde{\theta}_X-\tilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{1}_n=(1-\tau_d)(1-\tau_c^{+})\mathbf{v}\\ \zeta_{n+1}^{E}&=-(1-\tau_d)(1-\tau_c^{+})C\left(\mathbf{r}^{n}-\tilde{\mathbf{\Lambda}}\right)^{-1}\mathbf{1}_n=-(1-\tau_d)(1-\tau_c^{+})C\mathbf{b}\end{aligned} \tag{D.22}$$

The boundary conditions for E are also similar to those for debt. As X becomes large, a firm becomes essentially free of default risk, which makes the value of $\mathbf{E}$ converge to the difference between a claim on the cash flow stream and a riskfree perpetuity:

$$\lim_{X\to+\infty}\mathbf{E}\left(X\right)=(1-\tau_d)(1-\tau_c^{+})\left(X\mathbf{v}-C\mathbf{b}\right). \tag{D.23}$$

To satisfy this boundary condition, we need to set $w_{n+1,j}^{E}$ associated with all the $\beta_{n,j}$ with positive real parts.(n of them) to zero in region $\mathcal{D}_{n+1}$. The rest of the boundary conditions follow from the requirement that the value of E is zero at default, and that E is C^0 and C^1 at $X_D^{i+1},\cdots,X_D^{n}$ and C for $i=1,...,n-1$.

E Proof of Proposition 5

Recall that after adding the option of upward restructuring, we have the following default/restructuring boundaries, $\left(X_D^1,\cdots,X_D^n,X_U^{u_1},\cdots,X_U^{u_n}\right)$. To simplify notation, I have suppressed the dependence of the boundaries on the initial state. The default regions are $\mathcal{D}_k=\left[X_D^k,X_D^{k+1}\right)$ for $k=1,\cdots,n-1$; the region without default or restructuring are $\mathcal{D}_n=[X_D^n,C)$ and $\mathcal{U}_1=\left[C,X_U^{u(1)}\right]$; the restructuring regions are $\mathcal{U}_k=\left(X_U^{u(k-1)},X_U^{u(k)}\right]$ for $k=2,\cdots,n$.

I use index set $\mathcal{I}_{n+k}=\left\{u\left(k\right),\cdots,u\left(n\right)\right\}$ to denote states where the firm has not yet restructured, with its compliment $\mathcal{I}_{n+k}^{c}=\left\{u\left(1\right),\cdots,u\left(k-1\right)\right\}$ denoting the states where restructuring has occurred.

In regions $\mathcal{D}_k$ ($k<n$), the equation governing a general corporate contingent claim J is identical to those in the static case (see equation (D.2)). The same is true in $\mathcal{D}_n$ and $\mathcal{U}_1$, where the firm will neither default nor restructure because of a sudden change of state. Thus, I will focus on the restructuring regions.

In region $\mathcal{U}_k$ ($k\geq 2$), the firm has not restructured yet in those states in $\mathcal{I}_{n+k}$:

$$\mathcal{A}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\mathbf{J}_{[\mathcal{I}_{n+k}]}+\mathbf{F}_{[\mathcal{I}_{n+k}]}+\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\mathbf{J}_{[\mathcal{I}_{n+k}]}+\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}^{c}]}\mathbf{K}_{[\mathcal{I}_{n+k}^{c}]}=\mathbf{r}^{n}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\mathbf{J}_{[\mathcal{I}_{n+k}]}. \tag{E.1}$$

Notice that the restructuring payments K appear in the equation, which specify the value of the claim when a change of state triggers restructuring.

The homogeneous equation in region $\mathcal{D}_k$ ($k\leq n$) and $\mathcal{U}_1$ is the same as in the static model, with solution given by (D.5). The homogeneous equation in region $\mathcal{U}_k$ ($k=2,\cdots,n$) can be written as:

$$\mathcal{A}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\mathbf{J}_{[\mathcal{I}_{n+k}]}+\left(\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}-\mathbf{r}^{n}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\right)\mathbf{J}_{[\mathcal{I}_{n+k}]}=\mathbf{0}. \tag{E.2}$$

Its solution takes the following form

$$\mathbf{J}\left(x\right)_{\left[\mathcal{I}_{n+k}\right]} = \sum_{j=1}^{2(n-k+1)} w_{n+k,j}\mathbf{g}_{n+k,j}\exp\left(\beta_{n+k,j}x\right), \tag{E.3}$$

where $\mathbf{g}_{n+k,j}$ and $\beta_{n+k,j}$ are solutions to the following standard eigenvalue problem:

$$\begin{bmatrix} 0 & \mathbf{I} \\ -\left(2\mathbf{\Sigma}_X^{-1}\left(\tilde{\mathbf{\Lambda}} - \mathbf{r}^n\right)\right)_{\left[\mathcal{I}_{n+k},\mathcal{I}_{n+k}\right]} & -\left(2\mathbf{\Sigma}_X^{-1}\tilde{\theta}_X - \mathbf{I}\right)_{\left[\mathcal{I}_{n+k},\mathcal{I}_{n+k}\right]} \end{bmatrix} \begin{bmatrix} \mathbf{g}_{n+k} \\ \mathbf{h}_{n+k} \end{bmatrix} = \beta_{n+k} \begin{bmatrix} \mathbf{g}_{n+k} \\ \mathbf{h}_{n+k} \end{bmatrix}, \tag{E.4}$$

$\mathbf{I}$ is an $n \times n$ identity matrix, $\mathbf{r}^n$, $\tilde{\theta}_X$ and $\mathbf{\Sigma}_X$ are defined in (C.5). Again, the coefficients $w_{n+k,j}$ will be different for different securities.

E.1 Debt

Suppose the coupon rates that the firm chooses in the different states at time 0 are $\mathbf{C}(X_0) = [C(1), \cdots, C(n)]'$. Let $D\left(x, s; s_0\right)$ be the value of debt after debt is issued, where s_0 represents the dependence of debt value on coupon chosen at time 0. The dividend rate, default payment, and restructuring payment are specified in (22) and (29). Following the same derivations in the general case, we get:

$$\begin{aligned}
\mathbf{D}(X; s_0)_{[\mathcal{I}_k]} &= \sum_{j=1}^{2k} \bar{w}_{k,j}^D(s_0)\mathbf{g}_{k,j}X^{\beta_{k,j}} + \bar{\xi}_k^D(\mathcal{I}_k; s_0)X + \bar{\zeta}_k^D(\mathcal{I}_k; s_0), \quad X \in \mathcal{D}_k, \quad k = 1, \cdots, n-1 \\
\mathbf{D}(X; s_0)_{[\mathcal{I}_n]} &= \sum_{j=1}^{2n} \bar{w}_{n,j}^D(s_0)\mathbf{g}_{n,j}X^{\beta_{n,j}} + \bar{\zeta}_n^D(\mathcal{I}_n; s_0), \quad X \in \mathcal{D}_n \cup \mathcal{U}_1 \\
\mathbf{D}(X; s_0)_{\left[\mathcal{I}_{n+k}\right]} &= \sum_{j=1}^{2(n-k+1)} \bar{w}_{n+k,j}^D(s_0)\mathbf{g}_{n+k,j}X^{\beta_{n+k,j}} + \bar{\xi}_{n+k}^D(\mathcal{I}_{n+k}; s_0)X + \bar{\zeta}_{n+k}^D(\mathcal{I}_{n+k}; s_0), \quad X \in \mathcal{U}_k, \quad k = 2, \cdots, n
\end{aligned} \tag{E.5}$$

Next, I specify the particular solutions and boundary conditions that determine the unknown values $\bar{w}, \bar{\xi}, \bar{\zeta}$.

When $X \in \mathcal{D}_k$ $(k < n)$, the particular solutions are the same as in the static case:

$$D(x, i; s_0) = \bar{\xi}_k^D(i; s_0)e^x + \bar{\zeta}_k^D(i; s_0), \quad i \in \mathcal{I}_k, \tag{E.6}$$

where $\bar{\xi}_k^D$ and $\bar{\zeta}_k^D$ are given in (D.10). Similarly, when $X \in \mathcal{D}_n \cup \mathcal{U}_1$, a particular solution is:

$$D(x, i; s_0) = \bar{\zeta}_n^D(i; s_0), \quad i \in \mathcal{I}_n \tag{E.7}$$

where $\bar{\zeta}_n^D$ is given in (D.14).

When $X \in \mathcal{U}_k$ $(k = 2, \cdots, n)$, for $i \in \mathcal{I}_{n+k}$,

$$\begin{aligned} r^n(i)D(x,i;s_0) &= \mathcal{A}^i D(x,i;s_0) + \tilde{\lambda}_{i,u(1)}K(x,u(1)) + \cdots + \tilde{\lambda}_{i,u(k-1)}K(x,u(k-1)) \\ &\quad + \tilde{\lambda}_{i,u(k)}D(x,u(k);s_0) + \cdots + \tilde{\lambda}_{i,u(n)}D(x,u(n);s_0) + (1-\tau_i)C(s_0) \end{aligned} \tag{E.8}$$

From (29), we know that $K(x,s)$ is independent of cash flow, but depends on initial value of debt. For now, let's assume that these values are known. Then, a particular solution will be:

$$D(x,i;s_0) = \bar{\zeta}^D_{n+k}(i;s_0). \tag{E.9}$$

Plug the particular solution and the expression of K into the ODE above,

$$\begin{aligned} r^n(i)\bar{\zeta}^D_{n+k}(i;s_0) &= \tilde{\lambda}_{i,u(1)}\frac{C(s_0)}{C(u(1))}D(X_0,u(1)) + \cdots + \tilde{\lambda}_{i,u(k)}\frac{C(s_0)}{C(u(k-1))}D(X_0,u(k-1)) \\ &\quad + \tilde{\lambda}_{i,u(k)}\bar{\zeta}^D_{n+k}(u(k);s_0) + \cdots + \tilde{\lambda}_{i,u(n)}\bar{\zeta}^D_{n+k}(u(n);s_0) + (1-\tau_i)C(s_0). \end{aligned}$$

The solutions can be written as

$$\bar{\zeta}^D_{n+k}(\mathcal{I}_{n+k};s_0) = C(s_0)\left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\left[(1-\tau_i)\,\mathbf{1}_{n-k+1} + \tilde{\mathbf{\Lambda}}_{[\mathcal{I}_{n+k},\mathcal{I}^c_{n+k}]}\left[\mathbf{D}(X_0) \oslash \mathbf{C}(X_0)\right]_{[\mathcal{I}^c_{n+k}]}\right], \tag{E.10}$$

where $\oslash$ denotes element-by-element division, $\mathbf{D}(X_0) = [D(X_0,1), \cdots, D(X_0,n)]'$.

The boundary conditions are as follows. First, as in the static model, there are n conditions specifying the value of debt at the n different default boundaries:

$$D\left(X^i_D, i; s_0\right) = H\left(X^i_D, i\right), \quad i = 1, \cdots, n. \tag{E.11}$$

Another n conditions specify the value of debt at the restructuring boundaries:

$$D\left(X^{u(i)}_U, u(i); s_0\right) = \frac{C(s_0)}{C(u(i))}D(X_0, u(i)), \quad i = 1, \cdots, n. \tag{E.12}$$

Moreover, we need to ensure that D is C^0 and C^1 at all the boundaries for which neither default or restructure has occurred.

$$\begin{aligned} \lim_{X \uparrow X^k_D} D(X,i;s_0) &= \lim_{X \downarrow X^k_D} D(X,i;s_0), \quad k = i+1, \cdots, n \\ \lim_{X \uparrow X^k_D} D_X(X,i;s_0) &= \lim_{X \downarrow X^k_D} D_X(X,i;s_0), \quad k = i+1, \cdots, n \end{aligned} \tag{E.13}$$

and

$$\begin{aligned} \lim_{X \uparrow X^{u(k)}_U} D(X,u(i);s_0) &= \lim_{X \downarrow X^{u(k)}_U} D(X,u(i);s_0), \quad k = 1, \cdots, i-1 \\ \lim_{X \uparrow X^{u(k)}_U} D_X(X,u(i);s_0) &= \lim_{X \downarrow X^{u(k)}_U} D_X(X,u(i);s_0), \quad k = 1, \cdots, i-1 \end{aligned} \tag{E.14}$$

Finally, we have one set of conditions that pin down the value of $\mathbf{D}(X_0)$:

$$D\left(X_0, s_0; s_0\right) = \sum_{j=1}^{2n} \bar{w}^D_{n,j} \mathbf{g}_{n,j}(s_0) X_0^{\beta_{n,j}} + \bar{\zeta}^D_n(s_0; s_0), \quad s_0 = 1, \cdots, n \tag{E.15}$$

There are $2n^2$ of unknown coefficients for $\left\{\bar{w}^D(s_0)\right\}$ for each s_0. In addition, there are n unknowns in $\mathbf{D}(X_0)$. Thanks to the scaling property, conditions in (E.13−E.25) are all linear in the unknowns, which gives us a system of $2n^3 + n$ linear equations. Solving the linear system gives $\left\{\bar{w}^D\right\}$ and $\mathbf{D}(X_0)$.

E.2 Equity

Let $E\left(x, s; s_0\right)$ be the value of equity after debt is issued, where s_0 again represents the dependence of equity value on coupon chosen at time 0. The dividend rate, default payment, and restructuring payment for equity are given in (23) and (30). When restructuring occurs in state s with cash flow X, applying the scaling property to the restructuring payment, we get:

$$K\left(X, s; s_0\right) = D\left(X_0, s; s\right)\left((1-q)\frac{X}{X_0} - \frac{C(s_0)}{C(s)}\right) + \frac{X}{X_0} E\left(X_0, s; s\right) \tag{E.16}$$

Since payoffs are linear in X in all regions, we can quickly write solutions for E as:

$$\mathbf{E}(X; s_0)_{[\mathcal{I}_k]} = \sum_{j=1}^{2k} \bar{w}^E_{k,j}(s_0)\mathbf{g}_{k,j} X^{\beta_{k,j}} + \bar{\xi}^E_k(\mathcal{I}_k; s_0) X + \bar{\zeta}^E_k(\mathcal{I}_k; s_0), \quad X \in \mathcal{D}_k, \quad k = 1, \cdots, n.$$

$$\mathbf{E}(X; s_0)_{\left[\mathcal{I}_{n+k}\right]} = \sum_{j=1}^{2(n-k+1)} \bar{w}^E_{n+k,j}(s_0)\mathbf{g}_{n+k,j} X^{\beta_{n+k,j}} + \bar{\xi}^E_{n+k}(\mathcal{I}_{n+k}; s_0) X + \bar{\zeta}^E_{n+k}(\mathcal{I}_{n+k}; s_0), \quad X \in \mathcal{U}_k, \quad k = 1, \cdots, n$$

When $X \in \mathcal{D}_k$ $(k = 1, \cdots, n)$, for $i \in \mathcal{I}_k$, the firm is alive but needs to issue equity to meet debt payments. The ODE is:

$$r^n(i) E\left(x, i; s_0\right) = \mathcal{A}^i E\left(x, i; s_0\right) + \tilde{\lambda}_{i,1} E\left(x, 1; s_0\right) + \cdots + \tilde{\lambda}_{i,k} E\left(x, k; s_0\right) + \frac{1-\tau_c^-}{1-e}\left(e^x - C(s_0)\right) \tag{E.17}$$

A particular solution is:

$$E\left(x, i; s_0\right) = \bar{\xi}^E_k\left(i; s_0\right) e^x + \bar{\zeta}^E_k\left(i; s_0\right), \tag{E.18}$$

and we can verify through the ODE system above that:

$$\begin{aligned} \bar{\xi}^E_k(\mathcal{I}_k; s_0) &= \frac{1-\tau_c^-}{1-e}\left(\mathbf{r}^n - \tilde{\theta}_X - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_k, \mathcal{I}_k]} \mathbf{1}_k \\ \bar{\zeta}^E_k(\mathcal{I}_k; s_0) &= -\frac{1-\tau_c^-}{1-e} C(s_0)\left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_k, \mathcal{I}_k]} \mathbf{1}_k \end{aligned} \tag{E.19}$$

When $X \in \mathcal{U}_1$, the firm has not restructured yet. It is easy to verify that a particular solution in this case is:

$$\begin{aligned} \bar{\xi}^E_{n+1}(\mathcal{I}_{n+1}; s_0) &= (1-\tau_d)(1-\tau_c^+)\left(\mathbf{r}^n - \tilde{\theta}_X - \tilde{\mathbf{\Lambda}}\right)^{-1}_{\left[\mathcal{I}_{n+1}, \mathcal{I}_{n+1}\right]} \mathbf{1}_n \\ \bar{\zeta}^E_{n+1}(\mathcal{I}_{n+1}; s_0) &= -(1-\tau_d)(1-\tau_c^+) C(s_0)\left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1}_{\left[\mathcal{I}_{n+1}, \mathcal{I}_{n+1}\right]} \mathbf{1}_n \end{aligned} \tag{E.20}$$

When $X \in \mathcal{U}_k$ $(k = 2, \cdots, n)$, for $i \in \mathcal{I}_{n+k}$, the firm has not restructured yet. The ODE is:

$$r^n(i)E\left(x,i;s0\right) = \mathcal{A}^i E\left(x,i;s0\right) + \tilde{\lambda}_{i,u(1)}K\left(x,u\left(1\right);s0\right) + \cdots + \tilde{\lambda}_{i,u(k-1)}K\left(x,u\left(k-1\right);s0\right)$$
$$+ \tilde{\lambda}_{i,u(k)}E\left(x,u\left(k\right);s0\right) + \cdots + \tilde{\lambda}_{i,u(n)}E\left(x,u\left(n\right);s0\right) + (1-\tau_d)(1-\tau_c^+)\left(e^x - C(s_0)\right)$$

Guess the particular solution is:

$$E\left(x,i;s_0\right) = \bar{\xi}^E_{n+k}\left(i;s_0\right)e^x + \bar{\zeta}^E_{n+k}\left(i;s_0\right). \tag{E.21}$$

Plug the particular solution and the expression of K in (E.16) into the ODE,

$$r^n(i)\left(\bar{\xi}^E_{n+k}\left(i;s_0\right)e^x + \bar{\zeta}^E_{n+k}\left(i;s_0\right)\right) = \bar{\xi}^E_{n+k}\left(i;s_0\right)\tilde{\theta}\left(i\right)e^x$$
$$+\sum_{j=1}^{k-1}\tilde{\lambda}_{i,u(j)}\left(D\left(X_0,u(j);u(j)\right)\left((1-q)\frac{e^x}{X_0} - \frac{C(s_0)}{C(u(j))}\right) + \frac{e^x}{X_0}E\left(X_0,u(j);u(j)\right)\right)$$
$$+\sum_{j=k}^{n}\tilde{\lambda}_{i,u(j)}\left(\bar{\xi}^E_{n+k}\left(u\left(j\right);s_0\right)e^x + \bar{\zeta}^E_{n+k}\left(u\left(j\right);s_0\right)\right) + (1-\tau_d)(1-\tau_c^+)\left(e^x - C(s_0)\right)$$

Collecting terms, we get:

$$r^n(i)\bar{\xi}^E_{n+k}\left(i;s_0\right) = \tilde{\theta}\left(i\right)\bar{\xi}^E_{n+k}\left(i;s_0\right) + \sum_{j=1}^{k-1}\tilde{\lambda}_{i,u(j)}\frac{1}{X_0}\left(D\left(X_0,u(j);u(j)\right)(1-q) + E\left(X_0,s;s\right)\right)$$
$$+\sum_{j=k}^{n}\tilde{\lambda}_{i,u(j)}\bar{\xi}^E_{n+k}\left(u\left(j\right);s_0\right) + (1-\tau_d)(1-\tau_c^+)$$
$$r^n(i)\bar{\zeta}^E_{n+k}\left(i;s_0\right) = -\sum_{j=1}^{k-1}\tilde{\lambda}_{i,u(j)}D\left(X_0,u(j);u(j)\right)\frac{C(s_0)}{C(u(j))} + \sum_{j=k}^{n}\tilde{\lambda}_{i,u(j)}\bar{\zeta}^E_{n+k}\left(u\left(j\right);s_0\right) - (1-\tau_d)(1-\tau_c^+)C(s_0)$$

which yields:

$$\begin{aligned}\bar{\xi}^E_{n+k}\left(\mathcal{I}_{n+k};s_0\right) &= \left(\mathbf{r}^n - \tilde{\theta}_X - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_{n+k},\mathcal{I}_{n+k}]}\begin{bmatrix} (1-\tau_d)(1-\tau_c^+)\mathbf{1}_{n-k+1} \\ +\frac{1}{X_0}\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_{n+k},\mathcal{I}^c_{n+k}]}\left((1-q)\,\mathbf{D}\left(X_0\right)_{[\mathcal{I}^c_{n+k}]} + \mathbf{E}\left(X_0\right)_{[\mathcal{I}^c_{n+k}]}\right)\end{bmatrix} \\ \bar{\zeta}^E_{n+k}\left(\mathcal{I}_{n+k};s_0\right) &= \left(\mathbf{r}^n - \tilde{\mathbf{\Lambda}}\right)^{-1}_{[\mathcal{I}_k,\mathcal{I}_k]}\left[-\tilde{\mathbf{\Lambda}}_{[\mathcal{I}_{n+k},\mathcal{I}^c_{n+k}]}\left[\mathbf{D}\left(X_0\right)\oslash\mathbf{C}\left(X_0\right)\right]_{[\mathcal{I}^c_{n+k}]} - (1-\tau_d)(1-\tau_c^+)\mathbf{1}_{n-k+1}\right]C(s_0)\end{aligned} \tag{E.22}$$

where the value of debt at time 0 has been solved in the previous section, and we are (temporarily) treating the value of equity at time 0 as known, $\mathbf{E}(X_0) = \left[E\left(X_0,1\right),\cdots,E\left(X_0,n\right)\right]'$.

The boundary conditions for E are similar to those for debt. First, as in the static model, there are n conditions specifying the value of debt at the n different default boundaries:

$$E\left(X^i_D,i;s_0\right) = 0, \quad i = 1,\cdots,n. \tag{E.23}$$

Another n conditions specify the value of debt at the restructuring boundaries, for $i = 1,\cdots,n$,

$$E\left(X^{u(i)}_U,u\left(i\right);s_0\right) = D\left(X_0,u(i);u(i)\right)\left((1-q)\frac{X^{u(i)}_U}{X_0} - \frac{C(s_0)}{C(u(i))}\right) + \frac{X^{u(i)}_U}{X_0}E\left(X_0,u(i);u(i)\right) \tag{E.24}$$

Moreover, we need to ensure that E is C^0 and C^1 , which lead to an identical set of conditions as for D.

Finally, we have one set of conditions that pin down the value of $\mathbf{E}(X_0)$:

$$E\left(X_0, s_0; s_0\right)=\sum_{j=1}^{2n} \bar{w}_{n+1,j}^E \mathbf{g}_{n+1,j}(s_0) X_0^{\beta_{n+1,j}}+\bar{\xi}_{n+1}^E(s_0; s_0) X_0+\bar{\zeta}_{n+1}^E(s_0; s_0), \quad s_0=1, \cdots, n \tag{E.25}$$

As in the case of debt, the boundary conditions and the value of equity at time 0 together form a system of linear equations $\left\{\bar{w}_{k,j}^E\right\}$, which is straightforward to solve.

F Calibrating the Continuous-time Markov Chain

The Markov chain for the expected growth rate and volatility of aggregate consumption is calibrated using a two-step procedure. Start with the discrete-time system of consumption and dividend dynamics of Bansal and Yaron (2004) (BY):

$$g_{t+1} = \mu_c + x_t + \sqrt{v_t}\eta_{t+1} \tag{F.1a}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \sigma_d\sqrt{v_t}u_{t+1} \tag{F.1b}$$

$$x_{t+1} = \kappa_x x_t + \sigma_x\sqrt{v_t}e_{t+1} \tag{F.1c}$$

$$v_{t+1} = \bar{v} + \kappa_v\left(v_t - \bar{v}\right) + \sigma_v w_{t+1} \tag{F.1d}$$

where g is log consumption growth, g_d is log dividend growth, and $\eta, u, e, w \sim i.i.d.N(0,1)$. I use the parameters from BY, which are at monthly frequency and calibrated to the annual consumption data from 1929 to 1998.

The restriction that shocks to consumption, η_{t+1}, and shocks to the conditional moments, e_{t+1}, w_{t+1}, are mutually independent, allows me to approximate the dynamics of (x, v) with a Markov chain. I first obtain a discrete-time Markov chain over a chosen horizon Δ, e.g. quarterly, using the quadrature method of Tauchen and Hussey (1991). For numerical reasons, I choose a small number of states ($n = 9$) for the Markov chain, with three different values for v, and three values for x for each v. Next, I convert the grid for (x, v) into a grid for (θ_m, σ_m) as in equation (4). Finally, I transform the discrete-time transition matrix $\mathbf{P} = [p_{ij}]$ into the generator $\mathbf{\Lambda} = [\lambda_{ij}]$ of a continuous-time Markov chain using the method of Jarrow, Lando, and Turnbull (1997) (an approximation based on the assumption that the probability of more than one change of state is close to zero within the period Δ). The details of the procedure are in Chen (2007).

< Figure 10 about here >

With just 9 states, the grid points are relatively far away from each other. I compute the discrete time Markov chain at the quarterly frequency so that the transition probabilities are not too small, and the assumption of no more than one jump within the period is reasonable. Under my calibration, the economy spends about 54% of the time in the "center" state with median expected growth rate and volatility (see Figure 10).

< Table VII about here >

Table VII Panel A shows the parameters for the discrete time consumption model of BY; Panel B compares the statistical properties of consumption growth rates in the data with those of the simulated data from the BY model and the Markov chain model. With just 9 states, the Markov chain approximation does a good in matching the mean, volatility, autocorrelation and variance ratio of consumption growth in the BY model. The noticeable differences are that the Markov chain appears to generate a distribution of volatility and variance ratios with lighter right tail, which is likely due to the non-extreme grid points.

References

Acharya, V. V., S. T. Bharath, and A. Srinivasan, 2007, "Does Industry-Wide Distress Affect Defaulted Firms? Evidence from Creditor Recoveries," *Journal of Financial Economics*, 85, 787–821.

Almeida, H., and T. Philippon, 2007, "The Risk-Adjusted Cost of Financial Distress," *Journal of Finance*, 62, 2557–2586.

Altinkihc, O., and R. S. Hansen, 2000, "Are There Economies of Scale in Underwriting Fees? Evidence of Rising External Financing Costs," *Review of Financial Studies*, 13, 191–218.

Altman, E., B. Brady, A. Resti, and A. Sironi, 2005, "The Link Between Default and Recovery Rates: Theory, Empirical Evidence and Implications," *Journal of Business*, 78, 2203–2227.

Altman, E., and B. Pasternack, 2006, "Defaults and Returns in the High Yield Bond Market: The Year 2005 in Review and Market Outlook," *Journal of Applied Research in Accounting and Finance*, 1, 3–29.

Andrade, G., and S. N. Kaplan, 1998, "How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions That Became Distressed," *Journal of Finance*, 53, 1443–1493.

Asquith, P., R. Gertner, and D. Scharfstein, 1994, "Anatomy of Financial Distress: An Examination of Junk-Bond Issuers," *Quarterly Journal of Economics*, 109, 625–658.

Bansal, R., and A. Yaron, 2004, "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481 – 1509.

Barlow, M. T., L. C. G. Rogers, and D. Williams, 1980, *Wiener-Hopf Factorization for Matrices, in Séminaire de Probabilités XIV, Lecture Notes in Math. 784* . pp. 324–331, Springer-Verlag, Berlin.

Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz, 2005, "Measuring Default Risk Premia from Default Swap Rates and EDFs," *Working paper.*

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev, 2007, "The Levered Equity Risk Premium and Credit Spreads: A Unified Framework," *Working paper.*

Brennan, M. J., and E. S. Schwartz, 1978, "Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure," *Journal of Business*, 51, 103–114.

Campbell, J. Y., and J. H. Cochrane, 1999, "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.

Chen, H., 2007, "Solving Asset Pricing Models with Recursive Preferences," *Working Paper, Massachusetts Institute of Technology.*

Chen, L., P. Collin-Dufresne, and R. S. Goldstein, 2006, "On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle," *Working Paper, Michigan State University, U.C. Berkeley, and University of Minnesota.*

Cochrane, J. H., 2006, "Financial Markets and the Real Economy," *Working Paper, University of Chicago.*

Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin, 2001, "The Determinants of Credit Spread Changes," *Journal of Finance*, 56, 2177–2207.

David, A., 2006, "Inflation Uncertainty, Asset Valuations, and the Credit Spreads Puzzle," *Working paper, University of Calgary.*

Davydenko, S. A., 2005, "When Do Firms Default? A Study of the Default Boundary," *Working Paper, University of Toronto.*

Driessen, J., 2005, "Is Default Event Risk Priced in Corporate Bonds?," *Review of Financial Studies*, 18, 165–195.

Duffee, G. R., 1998, "The Relation Between Treasury Yields and Corporate Bond Yield Spreads," *Journal of Finance*, 53, 2225–2241.

Duffie, D., 2001, *Dynamic Asset Pricing Theory*, Princeton University Press, Princeton, 3rd edn.

Duffie, D., and L. G. Epstein, 1992, "Stochastic Differential Utility," *Econometrica*, 60, 353–394.

Duffie, D., and D. Lando, 2001, “Term Structures of Credit Spreads with Incomplete Accounting Information,” *Econometrica*, 69, 633–664.

Duffie, D., and C. Skiadas, 1994, “Continuous-Time Security Pricing: A Utility Gradient Approach,” *Journal of Mathematical Economics*, 23, 107–132.

Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann, 2001, “Explaining the Rate Spread on Corporate Bonds,” *Journal of Finance*, 56, 247–277.

Epstein, L., and S. Zin, 1989, “Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework,” *Econometrica*, 57, 937–969.

Fischer, E. O., R. Heinkel, and J. Zechner, 1989, “Dynamic Capital Structure Choice: Theory and Tests,” *Journal of Finance*, 44, 19–40.

Geske, R., 1977, “The Valuation of Corporate Liabilities as Compound Options,” *Journal of Financial and Quantitative Analysis*, 12, 541–552.

Gilson, S. C., 1997, “Transactions Costs and Capital Structure Choice: Evidence from Financially Distressed Firms,” *Journal of Finance*, 52, 161–196.

Goldstein, R., N. Ju, and H. Leland, 2001, “An EBIT-Based Model of Dynamic Capital Structure,” *Journal of Business*, 74, 483–512.

Graham, J. R., 2000, “How Big Are the Tax Benefits of Debt?,” *Journal of Finance*, 55, 1901–1941.

Hackbarth, D., J. Miao, and E. Morellec, 2006, “Capital Structure, Credit Risk, and Macroeconomic Conditions,” *Journal of Financial Economics*, 82, 519–550.

Hansen, L. P., J. C. Heaton, and N. Li, 2005, “Consumption Strikes Back? Measuring Long Run Risk,” *Working Paper, University of Chicago.*

Hennessy, C. A., and T. M. Whited, 2005, “Debt Dynamics,” *Journal of Finance*, 60, 1129–1165.

Huang, J., and M. Huang, 2003, “How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?,” *Working paper, Penn State University and Stanford University.*

Jarrow, R. A., D. Lando, and S. M. Turnbull, 1997, "A Markov Model for the Term Structure of Credit Risk Spreads," *Review of Financial Studies*, 10, 481–523.

Jobert, A., and L. C. G. Rogers, 2006, "Option Pricing with Markov-Modulated Dynamics," *SIAM Journal on Control and Optimization*, 44, 2063–2078.

Kane, A., A. J. Marcus, and R. L. McDonald, 1985, "Debt Policy and the Rate of Return Premium to Leverage," *Journal of Financial and Quantitative Analysis*, 20, 479–499.

Karatzas, I., and S. E. Shreve, 1991, *Brownian Motion and Stochastic Calculus*, Springer-Verlag, New York, 2nd edn.

Kreps, D. M., and E. L. Porteus, 1978, "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46, 185–200.

Leary, M. T., and M. R. Roberts, 2005, "Do Firms Rebalance Their Capital Structures?," *Journal of Finance*, 60, 2575–2619.

Leland, H., 1994, "Corporate Debt Value, bond covenants, and Optimal Capital Structure," *Journal of Finance*, 49, 1213–1252.

Leland, H., 1998, "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53, 1213–1243.

Leland, H., and K. B. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987–1019.

Longstaff, F. A., S. Mithal, and E. Neis, 2005, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *Journal of Finance*, 60, 2213–2253.

Miller, M. H., 1977, "Debt and Taxes," *Journal of Finance*, 32, 261–275.

Pástor, L., and P. Veronesi, 2005, "Rational IPO Waves," *Journal of Finance*, 60, 1713–1757.

Piazzesi, M., and M. Schneider, 2006, "Equilibrium Yield Curves," *NBER Macroeconomics Annual.*

Shleifer, A., and R. W. Vishny, 1992, "Liquidation Values and Debt Capacity: A Market Equilibrium Approach," *Journal of Finance*, 47, 1343–1366.

Strebulaev, I. A., 2006, “Do Tests of Capital Structure Theory Mean What They Say?,” *forthcoming, Journal of Finance.*

Tauchen, G., and R. Hussey, 1991, “Quadrature Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models.,” *Econometrica*, 59, 371–396.

Titman, S., and S. Tsyplakov, 2005, “A Dynamic Model of Optimal Captial Structure,” *Working paper.*

Weil, P., 1990, “Non-Expected Utility in Macroeconomics,” *Quarterly Journal of Economics*, 105, 29–42.

Welch, I., 2004, “Capital Structure and Stock Returns,” *Journal of Political Economy*, 112, 106–131.

Notes

[1] The correlation between default rates and annual averages of monthly spreads is 0.65.

[2] Moody's calculate recovery rates as the weighted average of all corporate bond defaults, using closing bid prices on defaulted bonds observed roughly 30 days after the default date. For robustness, I also plot the value-weighted recovery rates from Altman and Pasternack (2006), who use the Altman Defaulted Bonds Data Set and measure recovery rates using closing bid prices as close to default date as possible. Results from the two samples are similar.

[3] Suppose the true 10-year default probability for a firm is 0.5%. Assuming risk neutrality, if the estimated default probability is 1% higher than the true value, it will result in a 200% increase in the predicted spread.

[4] This is a standard assumption in dynamic capital structure models.

[5] A contemporaneous and independent paper by Bhamra, Kuehn, and Strebulaev (2007) uses a theoretical framework similar to this paper. Their model only considers static capital structure decisions, and their main goal is to identify credit risk premium as an important common component in the equity premium and credit spreads. In contrast, I build a dynamic capital structure model directly on the long-run risk framework, and focus on identifying the common causes of high credit spreads and low leverage ratio.

[6] See Leland (1994, 98), Leland and Toft (1996), Goldstein, Ju, and Leland (2001), Titman and Tsyplakov (2005), Hennessy and Whited (2005), Hackbarth, Miao, and Morellec (2006), and earlier work of Brennan and Schwartz (1978), Kane et al. (1985), Fischer et al. (1989).

[7] Welch (2004) documents that firms do not actively adjust their debt levels in response to changes in the market value of equity. Leary and Roberts (2005) provide empirical evidence that such behavior is likely due to adjustment costs, and Strebulaev (2006) shows that a trade-off model with lumpy adjustment costs can account for such effects.

[8] To avoid the fix-point problem, I assume debt-holders cannot lever up again after taking over the assets.

[9] A formal proof is available from the author upon request.

[10] Duffee's estimates are based on corporate bonds without option-like features. His sample covers the period 1985-1995, a period when the Baa-Aaa spread is relatively low and smooth. Huang and Huang (2003) estimate credit spreads over the sample period 1973-1993. Their estimates are higher (194 bp for Baa, 131 bp for Baa-Aaa) because of the embedded call options and the inclusion of two recessions with high spreads.

[11] Approximating the BY model with a 2-state Markov chain will require the states to be far apart and much more persistent than the business cycles.

[12] David (2006) argues that the time-varying leverage ratios can also lead to higher average credit spreads over time, because credit spreads are convex functions of the solvency ratio (inverse of leverage ratio). However, CCDG (2006) show that the bias due to convexity is small once the model is calibrated to match historical default rates, recovery rates, and Sharpe ratios.

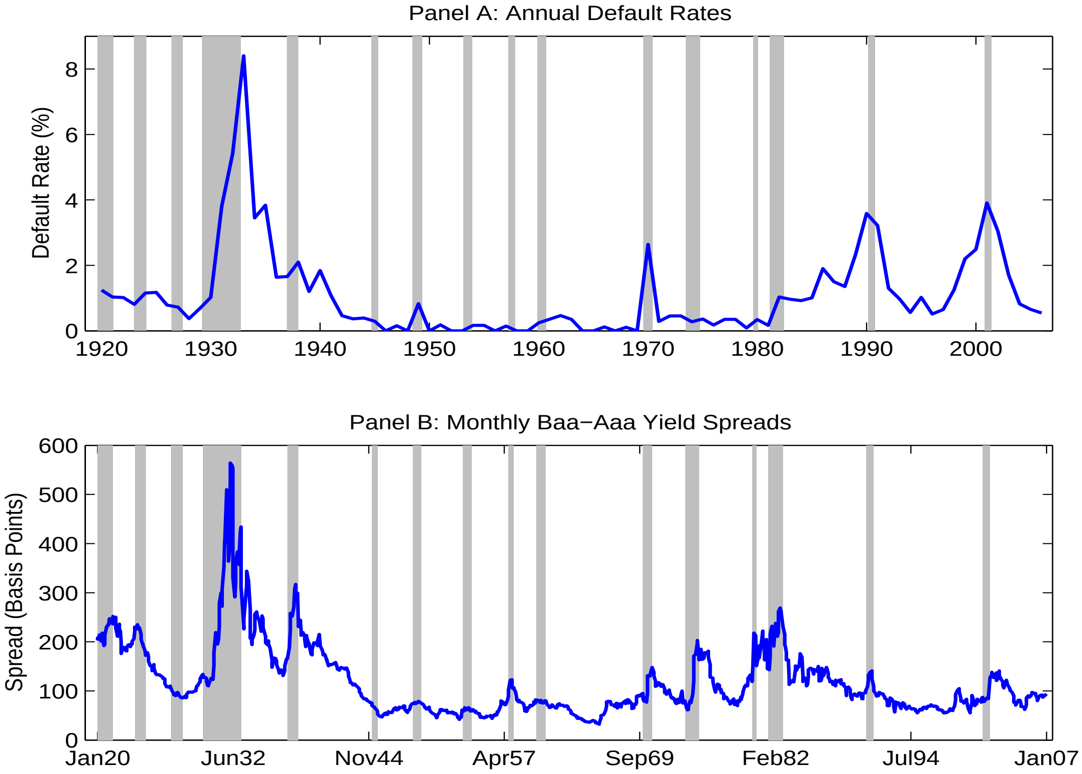

Figure 1: Annual Global Corporate Default Rates and Monthly Baa-Aaa Credit Spreads, 1920-2006. Shaded areas are NBER-dated recessions. For annual data, any calendar year with at least 5 months being in a recession as defined by NBER is treated as a recession year. Data source: Moody's.

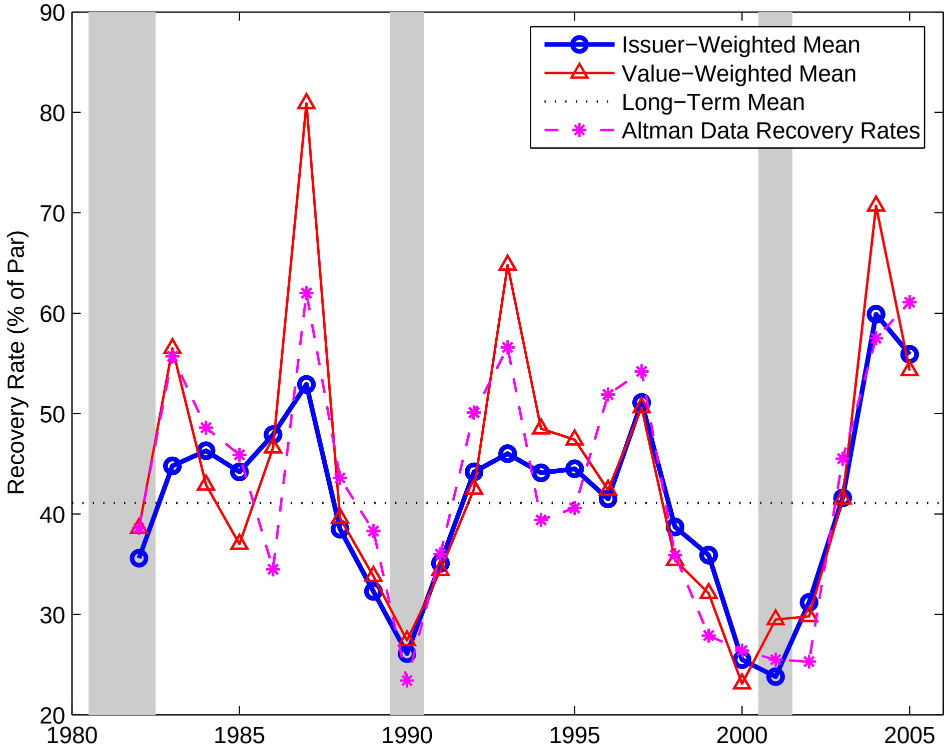

Figure 2: Annual Average Recovery Rates, 1982-2005. Issuer-weighted and value-weighted mean recovery rates are from Moody's. "Altman Data Recovery Rates" are from Altman and Pasternack (2006). Shaded areas are NBER-dated recessions.

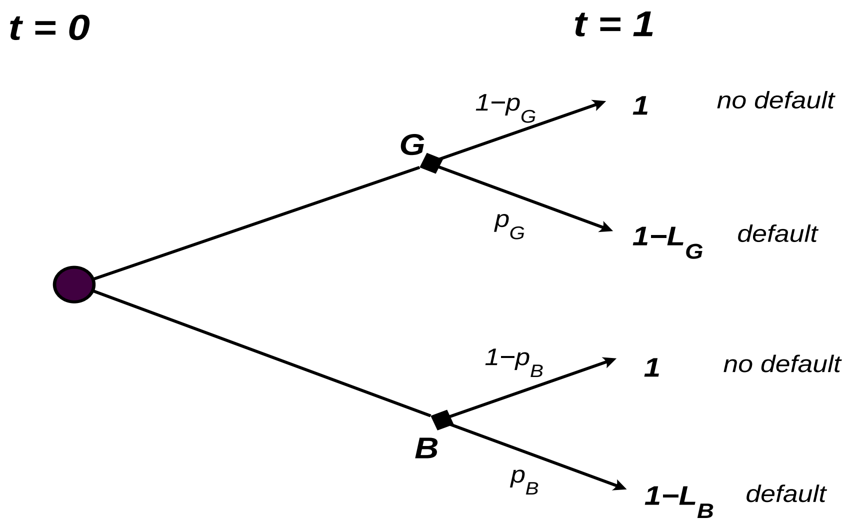

Figure 3: Payoff Diagram of a Defaultable Zero Coupon Bond in a Two-period Example.

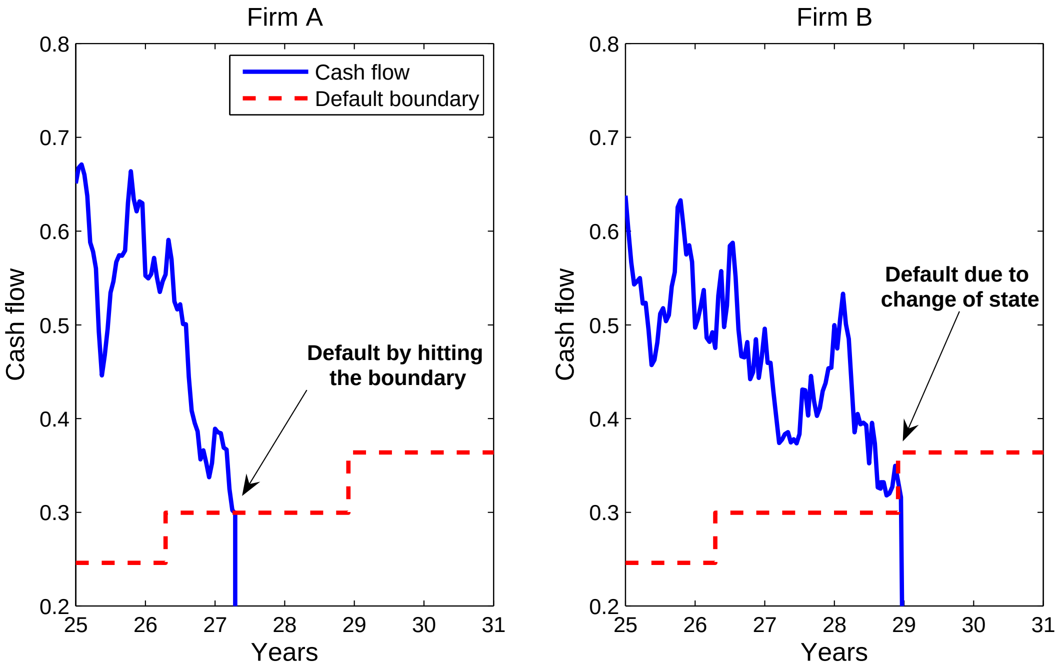

Figure 4: Illustration of Two Types of Defaults. In the left panel, default occurs when cash flow drops below a default boundary; in the right panel, default occurs when the default boundary jumps up, which is triggered by a change of the aggregate state.

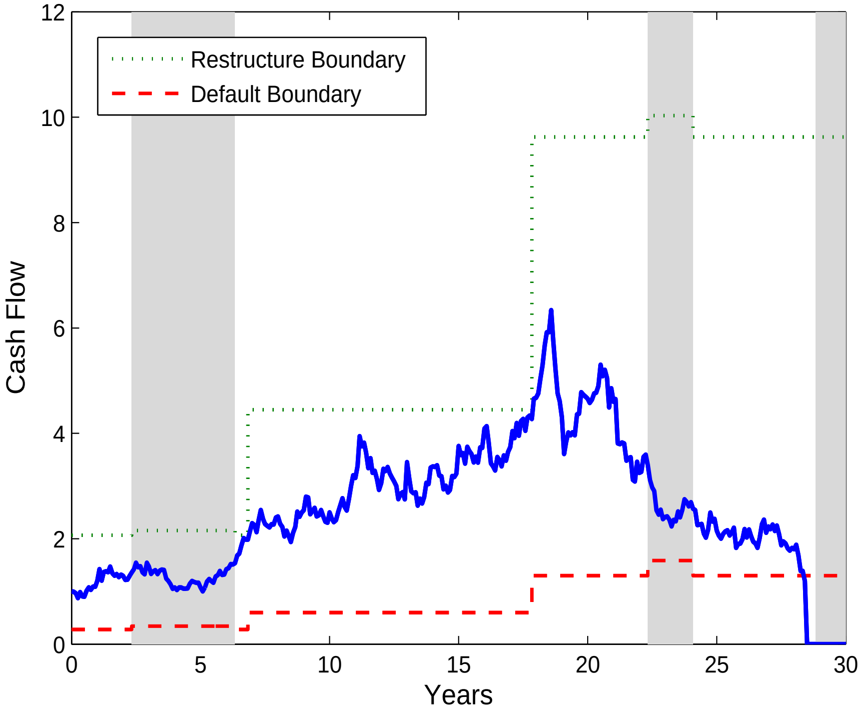

Figure 5: Illustration of the Scaling Property. The blue line is a cash flow sample path for a firm in the dynamic model. Green and red lines are restructuring and default boundaries. Shaded areas denote bad states.

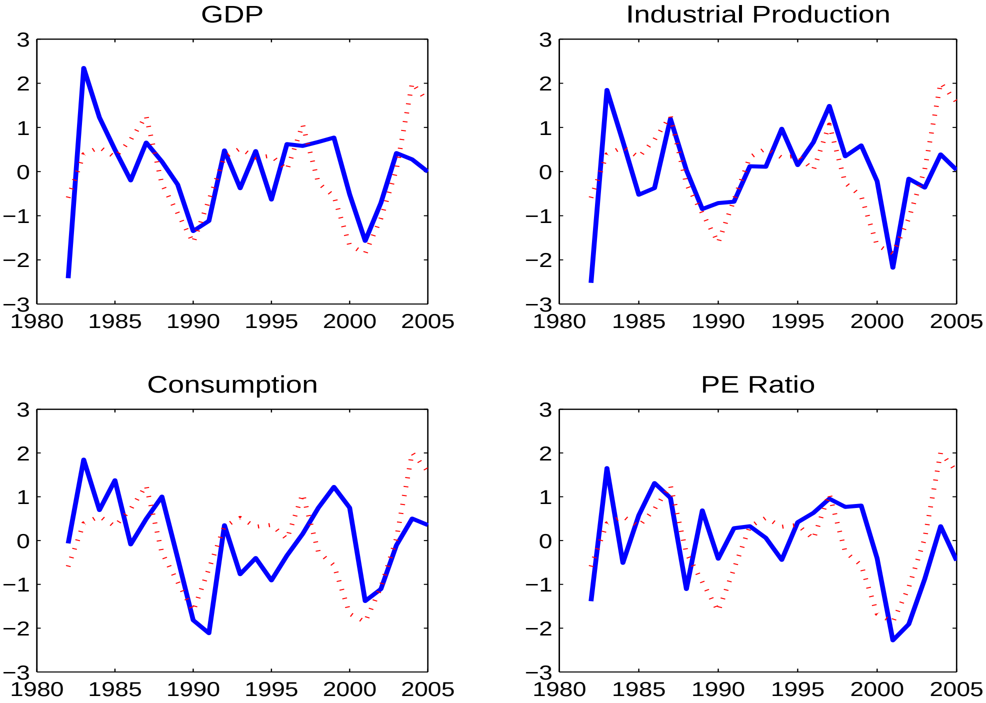

Figure 6: Recovery Rates and Macroeconomic Variables, 1982-2005. All the series are normalized to have mean 0 and standard deviation 1. The dotted line is the normalized recovery rate. GDP, IP and consumption data are from NIPA. Consumption is the sum of nondurables and services deflated with a chain-weighted price indice. Price-Earnings ratios are from Robert Shiller's web site. All macro variables are annual growth rates.

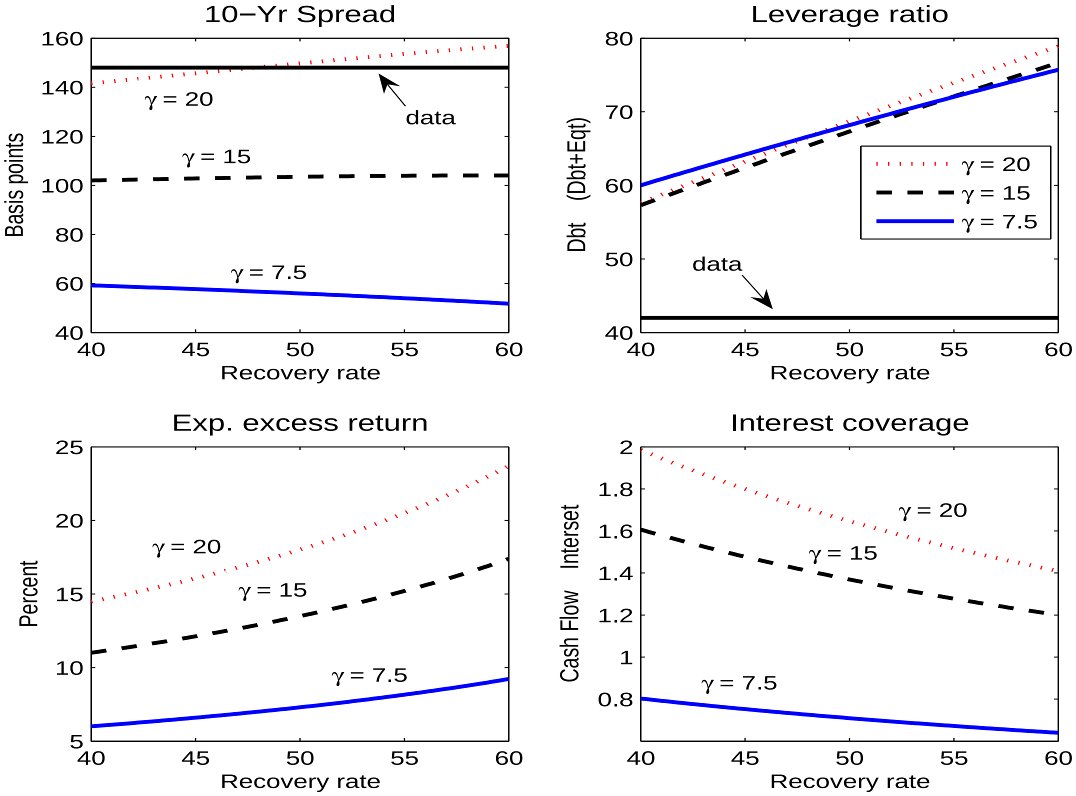

Figure 7: Results of Benchmark Case: No Variation in Macroeconomic Conditions. All variables are set to their unconditional averages, except for α and σ, which are calibrated to match the recovery rate and 10-year default probability for Baa-rated firms.

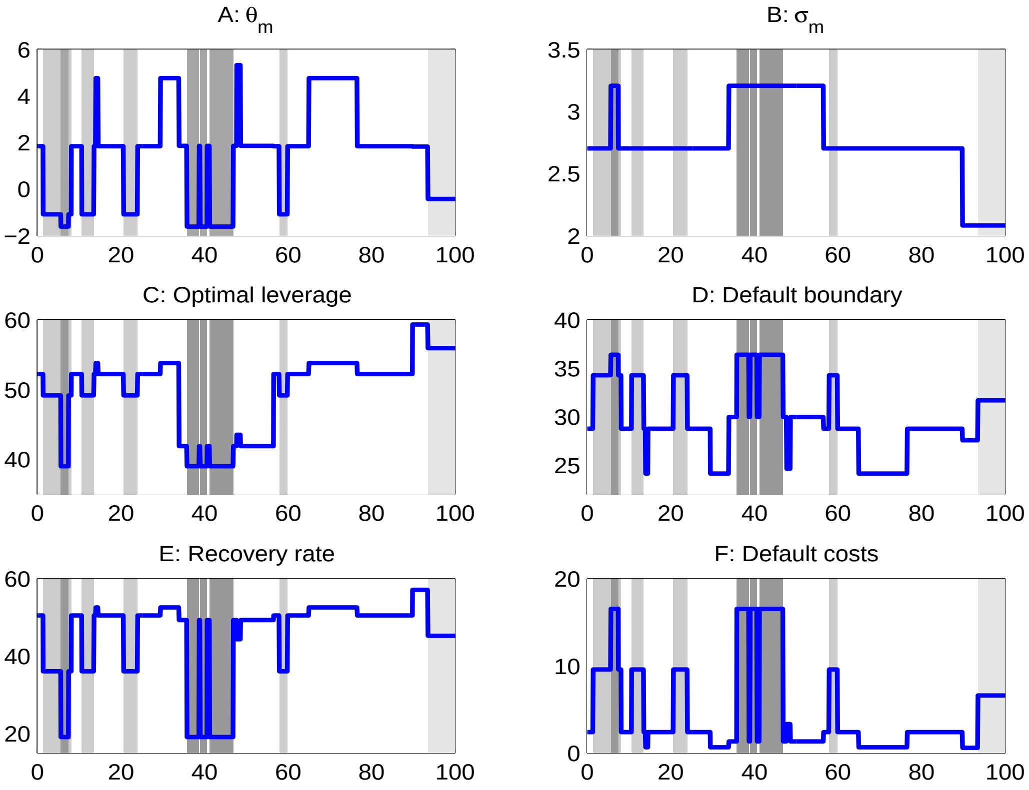

Figure 8: Dynamics of Consumption and Capital Structure in Simulation. This figure plots the conditional moments of aggregate consumption, optimal leverage ratios, default boundaries, recovery rates and default losses in a simulation. Default boundaries are relative to initial cash flow level. Default losses are relative to *pre*-distress firm value. All variables are in percentages.

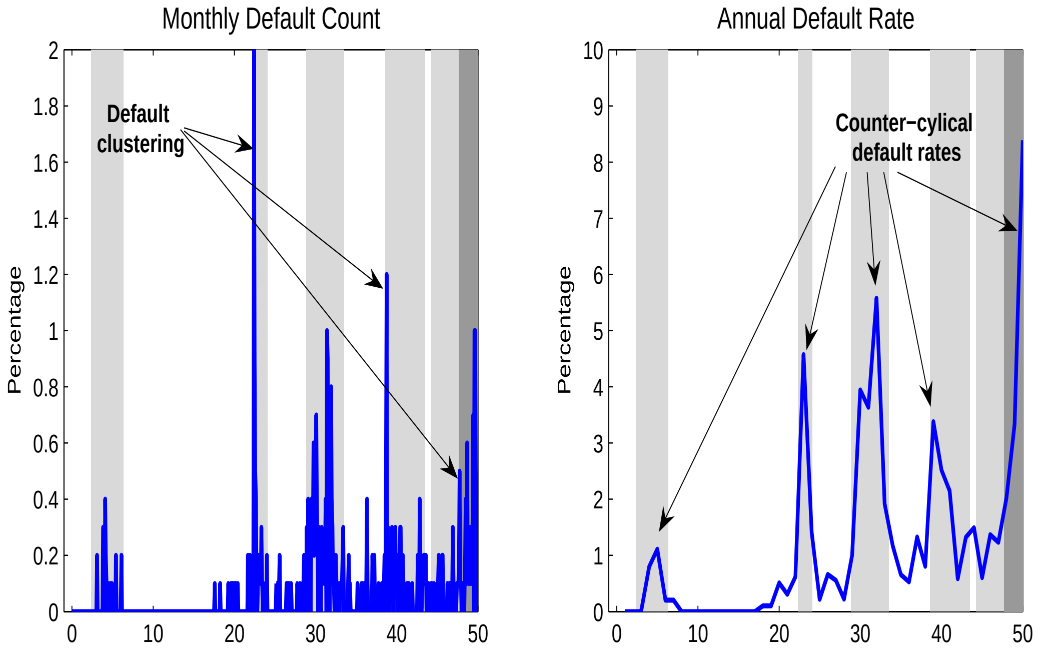

Figure 9: Simulated Default and the Annualized Default Rates. This figure plots the monthly default counts and annual default rates from a simulation of 1000 firms over 50 years. Areas with no shades are periods where the economy is in the state of high growth and median uncertainty. Light shades denote the state of low growth and median uncertainty. Dark shades denote the state of low growth and high uncertainty.

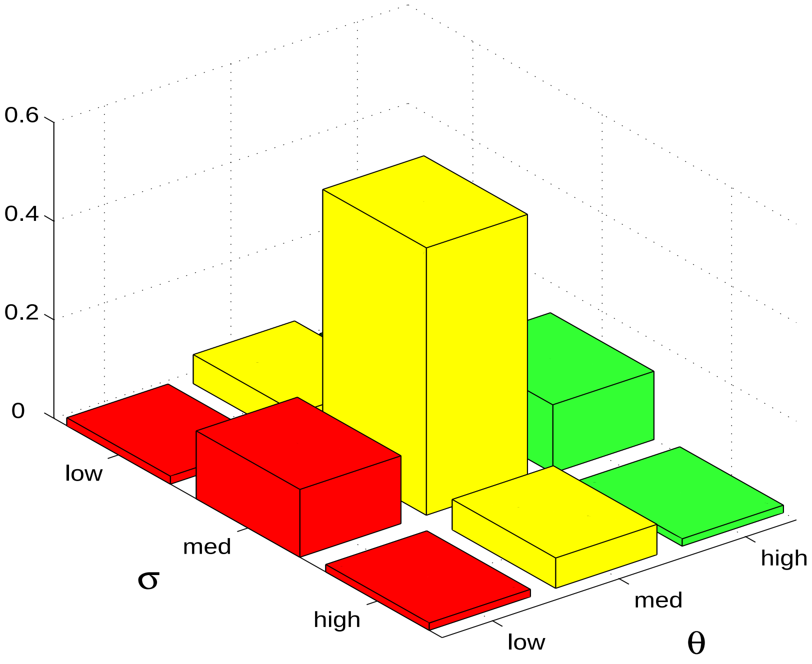

Figure 10: Stationary Distribution of the Markov Chain. This figure plots the stationary distribution of the conditional moments of consumption growth according to the 9-state Markov chain.

Table I: ASSET PRICING IMPLICATIONS OF THE MARKOV CHAIN MODEL

The table compares the model-generated moments of the equity market with the data. The statistics of the data are from BY (2004) (Table IV). The variables r_m and r_f are returns on the market portfolio and risk-free rate; SR is the Sharpe ratio; P/D is the price-dividend ratio for the market portfolio. Two additional preference parameters are $\psi = 1.5$, and $\rho = 0.015$. All values are annualized when applicable.

	Data		Model	
Variable	Estimate	SE	$\gamma = 7.5$	$\gamma = 10$
$E(r_m - r_f)$	6.33	(2.15)	6.71	7.54
$E(r_f)$	0.86	(0.42)	1.37	0.92
$\sigma(r_m)$	19.42	(3.07)	16.45	14.95
$\sigma(r_f)$	0.97	(0.28)	1.20	1.05
$E(SR)$	0.33		0.41	0.51
$E(P/D)$	26.56	(2.53)	21.54	18.80
$\sigma(log(P/D))$	0.29	(0.04)	0.23	0.18

Table II: PARAMETERS OF THE MODEL

The table reports the calibrated parameters of the model. All variables are annualized, when applicable. Inflation data are from NIPA. Tax rates (except τ_c^-) are from Graham (2000). Issuance costs are from Altinkihc and Hansen (2000).

Inflation, Taxes, and Issuance Costs

π	σ_P	$\rho_{P,m}$	τ_c^+	τ_c^-	τ_d	τ_i	q	e
0.036	0.014	-0.12	0.35	0.20	0.12	0.296	0.01	0.05

Cash Flow Process

	$\overline{\theta}_m^i$	$\overline{\sigma}_m^i$	σ_f^i	a_i	b_i
Baa	0.018	0.141	0.206	3.0	4.5
Aaa	0.018	0.093	0.137	2.0	3.0

Table III: Explaining Aggregate Default Rates and Recovery Rates

This table reports results from regressions of aggregate default rates and recovery rates on macro variables. ΔIP - real industrial production growth, ΔGDP - real GDP growth, ΔPE - growth rate of Price/Earnings ratio, g - real consumption growth, r_f - real riskfree rate. Numbers in brackets are Newey-West standard errors with lag 3. All variables are annualized, from 1982 to 2005. GDP, IP, consumption and CPI series are from NIPA. PE ratios are from Robert Shiller's web site. Riskfree rates are the 1-month T-bill rates. Default rates and recovery rates are from Moody's.

Panel A: Dependent Variable – Default Rate (DR)

Intercept	2.01	2.50	1.74	2.75	3.62	3.59	3.74
	(0.38)	(0.64)	(0.27)	(0.54)	(0.40)	(0.31)	(0.33)
ΔIP	-0.14					-0.09	-0.10
	(0.07)					(0.06)	(0.05)
ΔGDP		-0.28					
		(0.15)					
ΔPE			-0.03				
			(0.01)				
g				-0.55	-1.82	-1.70	-1.77
				(0.17)	(0.38)	(0.28)	(0.31)
g^2					0.33	0.34	0.37
					(0.12)	(0.09)	(0.10)
r_f							-0.24
							(0.23)
R^2	0.28	0.29	0.15	0.32	0.50	0.60	0.61
Adj R^2	0.25	0.26	0.11	0.29	0.45	0.54	0.53

Panel B: Dependent Variable – Recovery Rate (RR)

Intercept	52.96	37.05	32.92	39.60	33.58	28.03	31.68	31.79
	(2.69)	(2.61)	(4.09)	(2.27)	(3.47)	(2.47)	(2.90)	(2.37)
DR	-7.36							-6.95
	(1.14)							(1.83)
ΔIP		1.43						
		(0.44)						
ΔGDP			2.55					
			(0.95)					
ΔPE				0.33		0.31	0.35	0.35
				(0.07)		(0.08)	(0.09)	(0.07)
g					3.63	12.86	11.04	10.99
					(1.42)	(3.36)	(2.52)	(2.83)
g^2						-2.83	-2.29	-2.28
						(0.85)	(0.67)	(0.74)
r_f							-5.39	-5.55
							(3.54)	(2.38)
R^2	0.60	0.32	0.27	0.23	0.16	0.42	0.50	0.74
Adj R^2	0.58	0.29	0.24	0.20	0.12	0.33	0.40	0.67

Table IV: ESTIMATING DEFAULT LOSSES

The table reports the moments used in the simulated method of moments estimation of state-dependent default losses, and the estimation results for Baa and Aaa firms. I model default losses as:

$$\alpha(s) = a_0 + a_1\theta_m(s) + a_2\theta_m^2(s) + a_3\sigma_m(s),$$

where θ_m and σ_m are the conditional mean and volatility of consumption growth. All variables are annualized, when applicable.

Panel A: Moments for Recovery Rates

Mean:	48%
Volatility:	7%
Correlation with default rates:	-0.77
Correlation with consumption growth:	0.40
Correlation with changes in price-earnings ratio:	0.48

Panel B: SMM Estimates

Baa:	$\alpha(s) = -0.04 - 12.88 \times \theta_m(s) + 209.02 \times \theta_m^2(s) + 10.29 \times \sigma_m(s)$
Aaa:	$\alpha(s) = -0.15 - 9.61 \times \theta_m(s) + 109.26 \times \theta_m^2(s) + 19.80 \times \sigma_m(s)$

Table V: Results For The Static Model

The table reports results of the static capital structure model, including the benchmark case (no business cycle variations), and the case with business cycle variations. It also reports 4 sets of comparative statics results: the first case lowers marginal tax rate when earnings are negative; the second adds equity issuance costs; the third combines the first two (the full model); the fourth case sets default loss α to a constant. Def10 - 10-year cumulative default probability; Rec - average recovery rate for firm's debt; VolRec - volatility of recovery rates; Spr10 - average credit spread for a 10-year coupon bond; Lev - market leverage; IntCov - Interest Coverage (Cash Flow/Coupon); TaxBen - Net tax benefits as measured by percentage increases in firm value; sprd - average credit spread of consol bond; ERx - exp. excess return on equity.

Panel A: Benchmark Case

	Def10	Rec	VolRec	Spr10	Lev	IntCov	TaxBen	Sprd	ERx
Baa	4.9%	48.0%	-	56.5	66.7%	0.7	10.8%	79.1	7.0%
Aaa	0.6%	48.0%	-	7.1	91.2%	0.1	20.0%	4.2	8.3%

Panel B: Model with Business-cycle Variation

	Def10	Rec	VolRec	Spr10	Lev	IntCov	TaxBen	Sprd	ERx
Baa	4.9%	45.3%	6.9%	141.3	50.4%	1.7	5.3%	262.8	9.3%
	(1.2%)	(1.8%)	(0.3%)	(9.7)	(3.5%)	(0.4)	(0.4%)	(50.5)	(1.6%)
Aaa	0.6%	46.5%	7.0%	43.4	52.2%	1.3	6.9%	81.4	6.6%
	(0.1%)	(1.6%)	(0.2%)	(1.1)	(1.8%)	(0.2)	(0.3%)	(17.1)	(1.2%)

Panel C: Comparative Statics for Baa Firms

	Def10	Rec	Spr10	Lev	IntCov	TaxBen	Sprd	ERx
Risky Tax	3.9%	48.8%	122.4	45.9%	1.96	4.7%	238.1	8.8%
Benefits	(0.8%)	(1.6%)	(6.2)	(2.9%)	(0.41)	(0.3%)	(42.0)	(1.6%)
Costly Eqt	4.7%	47.7%	136.5	49.1%	1.79	5.1%	255.9	9.2%
Issuance	(1.1%)	(1.6%)	(8.6)	(3.3%)	(0.38)	(0.4%)	(47.8)	(1.6%)
Combine	3.8%	49.2%	119.5	44.9%	2.02	4.6%	233.1	8.7%
Above Two	(0.8%)	(1.6%)	(5.7)	(2.8%)	(0.42)	(0.3%)	(40.5)	(1.6%)
Constant	13.4%	50.6%	281.8	65.6%	1.19	7.9%	321.9	12.1%
Deflt Losses	(0.7%)	(1.1%)	(28.0)	(0.5%)	(0.16)	(0.2%)	(42.6)	(2.4%)

Table VI: Results Of The Dynamic Model

The table reports results of the dynamic capital structure model, including the full model, and 2 sets of comparative statics: the first case sets default loss α to a constant; the second case sets equity issuance costs to zero. Def10 - 10-year cumulative default probability; Rec - average recovery rate for firm's debt; VolRec - volatility of recovery rates; Spr10 - average credit spread for a 10-year coupon bond; Lev - market leverage; IntCov - Interest Coverage (Cash Flow/Coupon); TaxBen - Net tax benefits as measured by percentage increases in firm value; sprd - average credit spread of consol bond; ERx - exp. excess return on equity.

Panel A: Full Model

	Def10	Rec	Spr10	Lev	IntCov	TaxBen	Sprd	ERx
Baa	4.9%	48.5%	135.0	37.2%	2.5	6.0%	232.0	7.2%
	(0.8%)	(1.3%)	(7.5)	(1.9%)	(0.4)	(0.3%)	(40.4)	(1.4%)
Aaa	0.6%	48.0%	42.4	45.5%	1.4	7.6%	78.8	5.9%
	(0.1%)	(1.2%)	(0.9)	(1.6%)	(0.1)	(0.3%)	(14.9)	(1.1%)

Panel B: Comparative Statics for Baa Firm

	Def10	Rec	Spr10	Lev	IntCov	TaxBen	Sprd	ERx
Constant	11.39%	50.0%	295.3	57.4%	1.37	9.5%	318.1	10.6%
Deflt Losses	(0.58%)	(0.9%)	(29.5)	(0.5%)	(0.14)	(0.2%)	(18.8)	(2.1%)
Zero Eqt	5.1%	45.0%	139.8	41.8%	2.1	6.9%	261.5	7.7%
Issue Costs	(0.9%)	(1.5%)	(8.5)	(2.4%)	(0.3)	(0.4%)	(48.0)	(1.5%)

Table VII: Markov Chain Approximation of the BY Model

The table compares the moments of consumption from the data, the model of Bansal and Yaron (2004), and the Markov chain model in this paper. Parameters in Panel A are from the discrete time model of BY (Table IV). In Panel B, the statistics of the data are from BY (2004) (Table I), based on annual observations from 1929 to 1998. The statistics for the two models are based on 5,000 simulations, each with 70 years of data. The simulations are done at high frequency and then aggregated to get annual growth rates. The symbols $\mu(g)$ and $\sigma(g)$ are mean and standard deviation of growth rates; $AC(j)$ is the jth autocorrelation; $VR(j)$ is the jth variance ratio.

Panel A: Paramters for the BY Model

μ_c	μ_d	ϕ	σ_d	κ_x	σ_x	κ_v	$\bar{v}$	σ_v
0.0015	0.0015	3	4.5	0.979	0.044	0.987	6.08×10^{-5}	0.23×10^{-5}

Panel B: Properties of Annualized Time-Averaged Growth Rates

	Data		BY			Markov Chain		
Variable	Estimate	SE	5%	50%	95%	5%	50%	95%
$\mu(g)$	1.80	-	0.59	1.79	2.99	0.74	1.81	2.90
$\sigma(g)$	2.93	(0.69)	2.26	2.79	3.44	2.19	2.64	3.12
AC(1)	0.49	(0.14)	0.25	0.46	0.63	0.23	0.42	0.58
AC(2)	0.15	(0.22)	-0.04	0.22	0.45	-0.05	0.18	0.39
AC(5)	-0.08	(0.10)	-0.17	0.06	0.30	-0.17	0.05	0.28
AC(10)	0.05	(0.09)	-0.24	-0.03	0.20	-0.24	-0.02	0.19
VR(2)	1.61	(0.34)	1.25	1.46	1.63	1.23	1.42	1.58
VR(5)	2.01	(1.23)	1.36	2.13	2.91	1.33	2.01	2.69
VR(10)	1.57	(2.07)	1.20	2.47	4.21	1.15	2.33	3.85