Macroeconomic Risk and Debt Overhang[*]

Hui Chen[†]   Gustavo Manso[‡]

July 30, 2010

Abstract

Since debt is typically riskier in recessions, transfers from equity holders to debt holders associated with each investment also tend to concentrate in recessions. Such systematic risk exposure of debt overhang has important implications for the investment and financing decisions of firms and on the ex ante costs of debt overhang. Using a calibrated dynamic capital structure/real option model, we show that the costs of debt overhang become significantly higher in the presence of macroeconomic risk. We also provide several new predictions that relate the cyclicality of a firm's assets in place and growth options to its investment and capital structure decisions.

1 Introduction

A fundamental question in finance is what determines the optimal investment decisions for firms. A part of this problem is valuation — the classic rule of Net Present Value (NPV) prescribes that we value an investment opportunity by forecasting its future cash flows and discounting these cash flows at rates that appropriately reflect the risks embedded in them. The problem is greatly enriched by market frictions, especially agency problems and informational asymmetries, which not only can alter the levels of cash flows from investment, but also their risk exposure. To be able to properly assess the distortions these frictions can bring to corporate investment, we need to better understand how agents respond to these frictions in a dynamic economy, as well as the consequences of these actions for the systematic risk of investment.

In this paper, we focus on one specific type of frictions, *debt overhang*. Myers (1977) argues that, in the presence of risky debt, equity holders of a levered firm underinvest, because a fraction of the value generated by their new investment will accrue to the existing debt holders. Thus, investment decisions not only depend on the cash flows from investment, but also the transfers between different stake holders. We demonstrate how macroeconomic risk affects these transfers, which links the investment distortions to the cyclicality of assets in place and growth options. Moreover, we show that macroeconomic risk can substantially amplify the costs of debt overhang, which in turn affects firms' financing decisions ex ante.

Why is it important to take into account the effects of macroeconomic risk when analyzing the debt overhang problem? The distribution of agency costs across different macroeconomic states matters for their impact ex ante. Recessions are times of high marginal utilities, which means that the distortions caused by agency problems during such times will affect investors more than in booms. Thus, the agency cost will be amplified if agency conflicts are more severe in bad times, or reduced if agency conflicts are more severe in good times.

In the case of debt overhang, a key prerequisite for the agency conflict is debt being risky. It has been well documented empirically that credit spreads for an average investment-grade firm are strongly countercyclical, i.e., debt tends to become significantly more risky in aggregate bad times than in good times. Thus, controlling for the investment opportunity, transfers from equity holders to debt holders will tend to concentrate in bad times, which makes investment more risky for equity holders, causing them to become more reluctant to invest. This systematic risk component of debt

overhang is also important for measuring the agency costs. Moreover, the same intuition can be extended to the cross section, where firms' exposure to debt overhang will depend on the cyclicality of their assets in place and growth options.

To measure these effects, we need to take into account agents' ability to endogenously respond to changing macroeconomic conditions through their investment and financing decisions (e.g., delaying rather than deserting an investment; choosing a lower leverage). We build a dynamic capital structural model with investment decisions modeled as a real option. We incorporate macroeconomic risk into the model by imposing a stochastic discount factor that generates time variations in the riskfree rate and the risk prices for small shocks as well as large business cycle shocks. In addition, the cash flows from assets in place and growth options are allowed to have time-varying expected growth rates, conditional volatility, and jumps that coincide with changes in macroeconomic conditions. We then calibrate the stochastic discount factor to match the business cycle dynamics of asset prices, and examine the agency costs of debt for firms with different leverage, different present value of growth option (PVGO), as well as different systematic risk exposure for their assets in place and growth options.

Our model shows that debt overhang costs are substantially higher when macroeconomic risk is taken into account. For example, in our benchmark case, the debt overhang costs for a low leverage firm peak at 0.7% of the first-best firm value without macroeconomic risk, while these costs peak at 2.7% or 3.5% in booms and recessions respectively in the presence of macroeconomic risk. For a high leverage firm, the debt overhang costs peak at 4% without macroeconomic risk, while these costs peak at 7.2% or 8.6% in boom and recessions respectively with macroeconomic risk.

The impact of macroeconomic risk on debt overhang depends on the cyclicality of cash flows from assets in place and growth opportunities. More cyclical cash flows from assets in place increase the probability that firms will underinvest during recessions, when marginal utilities are higher, amplifying thus the impact of macroeconomic risk on agency cost of debt. The effect of more cyclical cash flows from growth opportunities is ambiguous. On one hand, more cyclical cash flows from growth opportunities increase the probability that firms will underinvest during recessions. On the other hand, the value lost from delaying investment in recessions is lower. In our calibrated dynamic capital structure model, we show that either of the two effects may dominate for reasonable set of parameters.

Another implication from the dynamic model is that debt overhang in bad times can also

significantly distort investment decisions in good times, which we refer to as the dynamic overhang effect. In anticipation of bad times arriving in the future, equity holders can become reluctant to invest, even though currently debt is relatively safe. Thus, as we increase the cyclical variation of the firm (by making the good state better and bad state worse), the conditional agency cost in the good state can rise rather than fall, which is in sharp contrast with the results in a static model. The more persistent the states are, the less the debt overhang problem in the bad states will propagate to the good states, hence the bigger the differences in the conditional agency costs between good and bad states.

The higher agency costs of debt due to macroeconomic risk will also affect firms' financing decisions. We compute the optimal leverage using the tradeoff between tax benefits and costs of debt overhang. For our benchmark parameters, the optimal interest coverage is 1.09 in a model without macroeconomic risk. After taking macroeconomic risk into account, it rises to 2.43 or 2.31 depending on whether the current state is boom or recession respectively. The optimal market leverage drops from 60% to 45% and 40% respectively.

Besides raising the costs of debt overhang and causing more delay in investment, we show that macroeconomic risk can lead to new distortions. Specifically, equity holders will want to reduce the transfer to debt holders by synchronizing the cash flows from investment with those from the assets in place. For example, if the assets in place are procyclical, equity holders might prefer to invest in procyclical projects, even if these projects have lower NPV. This result can be viewed as a general form of asset substitution in the presence of macroeconomic risk, whereby equity holders want to not just increase the volatility of the firm on average, but especially the volatility across different aggregate states. This result can explain why a highly levered firm (e.g., a large bank) might not have incentive to diversify its investments or hedge its market risk exposure, but would instead load on assets with high exposure to systematic risk. The result can also be applied to asset sales.[1]

In summary, our model produces the following testable predictions. First, the model predicts that underinvestment is more severe in recessions than in booms for firms with more cyclical assets in place or more cyclical growth options. Second, firms with more cyclical assets in place have higher agency costs of debt, and therefore should take on less debt. Third, firms with procyclical (countercyclical) assets in place have a bias to invest in procyclical (countercyclical) projects.

---

[1]Diamond and Rajan (2010) argue that debt overhang might make impaired banks reluctant to sell those bad assets with high systematic risk.

Our paper builds on a growing literature bringing macroeconomic risk into corporate finance. Almeida and Philippon (2007) use a reduced-form approach to measure the ex ante costs of financial distress. They show that the NPV of distress costs rises significantly after adjusting for the credit risk premium embedded in the losses. Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2009), and Chen (2009) use structural models to link capital structure decisions to macroeconomic conditions. A contemporaneous and independent paper by Arnold, Wagner, and Westermann (2010) extends the model of Hackbarth, Miao, and Morellec (2006) with real options to show that firms with growth options are more likely to default in recessions than those without growth options and thus should have higher credit spreads. They assume agents are risk neutral (no risk premium), and they do not measure the costs of debt overhang.

Lamont (1995) studies a static reduced-form model of debt overhang with macroeconomic conditions. The focus of the paper is on the multiplicity of equilibria that arises in a general equilibrium model in which firms make financing and investment decisions.

Our paper also contributes to the literature on dynamic investment and financing decisions of the firm. Mello and Parsons (1992), Mauer and Triantis (1994), Leland (1998), Mauer and Ott (2000), Décamps and Faure-Grimaud (2002), Hennessy (2004), Titman, Tompaidis, and Tsyplakov (2004), Childs, Mauer, and Ott (2005), Ju and Ou-Yang (2006), Moyen (2007), Sundaresan and Wang (2007), Lobanov and Strebulaev (2007), Manso (2008), Hackbarth and Mauer (2009), and Morellec and Schuerhoff (2010) are among the papers that develop dynamic models of investment to study distortions produced by debt financing. The bulk of these papers find that agency costs are typically below 1%. They do not consider, however, macroeconomic risk and its impact on the agency cost of debt.

The paper is also related to the real options literature which study dynamic investment decisions of the firm. McDonald and Siegel (1986), for example, study the timing of an irreversible investment decision. Dixit (1989) analyzes entry and exit decisions of a firm whose output price follows a geometric Brownian motion. Dixit and Pindyck (1994) provide a survey of this literature. Guo, Miao, and Morellec (2005) study a real options problem with regime shifts, but do not consider debt financing.
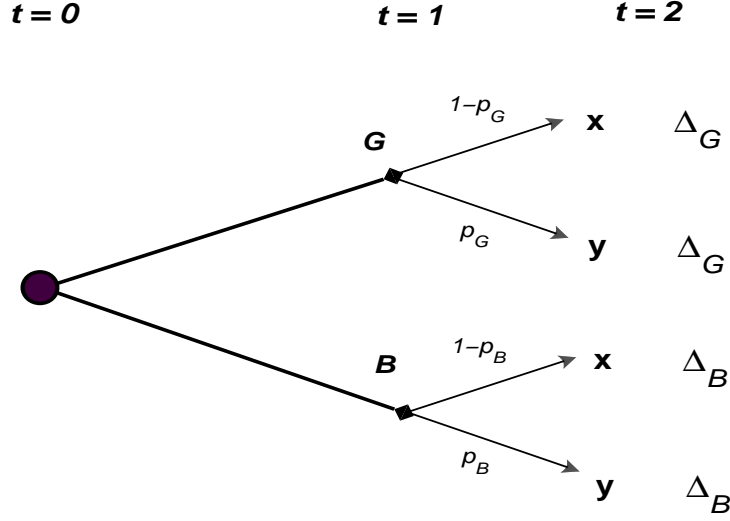
Figure 1: A Two-period Example.

2   Two-period example

We first study a simple two-period model that illustrates the interplay between macroeconomic conditions and debt overhang. This simple model will help with the intuition behind the results obtained in the dynamic model, which we develop in the next section.

The economy can be in one of two aggregate states $s \in \{G, B\}$ at $t = 1$. The time-0 price of a one-period Arrow-Debreu security that pays \$1 at $t = 1$ in state s is given by Q_s. Since the marginal utility in the bad state is higher than the marginal utility in the good state, agents will pay more for consumption in the bad state than in the good state: $Q_B > Q_G$. For simplicity, we assume that the risk-free interest rate is 0, so that $Q_G + Q_B = 1$.

At $t = 2$, the firm's assets in place produce cash flow x with probability $1 - p_s$ and y with probability p_s, where $x > y$, and the different realizations of cash flow in a given aggregate state are the result of firm-specific shocks in that state.

The firm has zero-coupon debt with face value F, $y < F \leq x$, which matures at time $t = 2$. Absolute priority is satisfied. As such, if the firm does not produce enough cash flow to pay back debt holders, then debt holders seize the realized cash flow of the firm (no bankruptcy costs). The fact that $y < F$ makes debt risky, without which there will be no debt overhang.

Let's first assume that the equity holders of the firm can choose whether or not to undertake

an investment I *after* learning the state s of the economy at $t = 1$. The investment produces an additional cash flow of $I + \Delta_s$ realized at the same time as the cash flows from assets in place. We assume that $\Delta_s > 0$ so that the investment opportunity has positive NPV.

We now derive conditions under which equity holders will undertake the available investment opportunity. The equity value of the firm when the manager makes the investment is

$$-I + (1 - p_s)(x + I + \Delta_s - F) + p_s(y + I + \Delta_s - F) \tag{1}$$

if $y + I + \Delta_s \geq F$, and

$$-I + (1 - p_s)(x + I + \Delta_s - F) \tag{2}$$

if $y + I + \Delta_s < F$. The equity value of the firm when equity holders choose not to make the investment is

$$(1 - p_s)(x - F). \tag{3}$$

It follows that equity holders will make the investment if

$$p_s \times \min(F - y, I + \Delta_s) < \Delta_s. \tag{4}$$

The left-hand side of the inequality gives the expected value of the transfer from equity holders to existing debt holders after the investment is made. Thus, equity holders will only make the investment if the expected transfer is less than the NPV of the investment, so that the "overhang-adjusted NPV" is positive. It is easy to see that a higher leverage (larger F) will tend to increase the transfer, making the above condition harder to satisfy.

We define the indicator function Ω_s as

$$\Omega_s \equiv \begin{cases} 0 & \text{if } p_s \times \min(F - y, I + \Delta_s) < \Delta_s \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

The function is equal to 1 if the equity holders do not undertake the investment opportunity, and 0 otherwise.

We next turn to the valuation of the securities of the firm and to the measurement of the agency cost of debt. To provide a benchmark, we first calculate the value V of the unlevered firm at time

$t = 0$ (for which $F = 0$). If the firm is unlevered, equity holders will always make the investment and therefore

$$V = \sum_{s \in \{G,B\}} Q_s((1 - p_s)x + p_s y + \Delta_s).$$ (6)

With $F > 0$, the value of debt at the initial date is

$$D = \sum_{s \in \{G,B\}} Q_s \{(1 - p_s)F + p_s((1 - \Omega_s) \min(F, y + I + \Delta_s) + \Omega_s y)\}.$$ (7)

The value of equity at the initial date is:

$$E = \sum_{s \in \{G,B\}} Q_s \{(1 - p_s)(x + (1 - \Omega_s)(I + \Delta_s) - F)$$

$$+ p_s((1 - \Omega_s) \max(0, y + I + \Delta_s - F)) - (1 - \Omega_s)I\}.$$ (8)

The total value of the firm is thus

$$E + D = \sum_{s \in \{G,B\}} Q_s \{(1 - p_s)x + p_s y + (1 - \Omega_s)\Delta_s\}.$$ (9)

For the purposes of this example, we define the agency cost of debt as

$$A = V - (E + D),$$ (10)

the value of the unlevered firm minus the value of the levered firm.[2] Using equations (6) and (9) we obtain that

$$A = Q_G \Omega_G \Delta_G + Q_B \Omega_B \Delta_B.$$ (11)

The agency cost of debt is equal to the sum over the two states of the product of the value Q_s of 1 dollar in state s, the indicator function Ω_s which is equal to 1 when underinvestment takes place, and the losses Δ_s from underinvestment.

To asses the impact of variations in state prices on the agency cost of debt, we subtract the

---

[2]In the dynamic model of state contingent agency costs of the next section, we will extend this definition to a setting with bankruptcy costs and tax benefits of debt.

agency cost of debt when $Q_G = Q_B$ from (11) to obtain:

$$\left(\frac{1}{2} - Q_G\right)(\Omega_B\Delta_B - \Omega_G\Delta_G). \tag{12}$$

Since $Q_G < \frac{1}{2}$, variations in state prices exacerbate the agency cost of debt if $\Omega_B\Delta_B > \Omega_G\Delta_G$.

In the following discussions, we say that the assets in place are *procyclical* if $p_G < p_B$. We say that the growth option is *procyclical* if $\Delta_G > \Delta_B$.

Keeping all else constant, more cyclical cash flows from assets in place, i.e., lower p_G and higher p_B, makes the condition for investment (4) easier to satisfy in state G but harder in state B. As a result, underinvestment becomes more concentrated in the bad state, exacerbating the costs of debt overhang when macroeconomic risk is taken into account.

Next, keeping all else constant, more cyclical cash flows $I + \Delta_s$ from the investment also make the condition for investment (4) easier to satisfy in state G but harder in state B. However, it also has the additional effect of reducing the potential loss if the investment is not made in state B. Therefore, the effect of stronger cyclicality of the growth option on the costs of debt overhang is ambiguous.

So far the investment we consider is riskless – its cash flow is constant after investment is made. We now consider a risky investment opportunity that is only exposed to aggregate shocks. This is accomplished by assuming that the investment I is made at $t = 0$ as opposed to $t = 1$, while the cash flows from investment at $t = 2$ remain the same. When would equity holders make the investment? The condition is

$$Q_G p_G \min(F - y, I + \Delta_G) + Q_B p_B \min(F - y, I + \Delta_B) < Q_G\Delta_G + Q_B\Delta_B. \tag{13}$$

The right-hand side of the inequality gives the NPV of the investment, while the left-hand side again gives the expected transfer from equity holders to debt holders. In the case where the cash flow from new investment is sufficiently high to make the existing debt riskfree in both states, the inequality (13) simplifies to

$$Q_G p_G (F - y) + Q_B p_B (F - y) < Q_G\Delta_G + Q_B\Delta_B.$$

In this case, the cyclicality of the growth option does not matter for the investment decision (only

the NPV matters). The cyclicality of assets in place does matter for investment, as higher p_B and lower p_G will raise the total value of transfer.

However, if the cash flow from new investment is not enough to pay off the debt holders in the states with low cash flows from assets in place, then the condition becomes

$$Q_G p_G (I + \Delta_G) + Q_B p_B (I + \Delta_B) < Q_G \Delta_G + Q_B \Delta_B.$$

Holding the NPV constant, making the investment opportunity more procyclical means raising Δ_G while lowering Δ_B so that $Q_G \Delta_G + Q_B \Delta_B$ is unchanged. If $Q_G p_G < Q_B p_B$ (e.g., when the assets in place are procyclical), then a more procyclical investment can lower the expected transfer from equity holders to debt holders, making equity holders more willing to make such an investment. In fact, the stronger the cyclicality of the investment, the better off the equity holders. Finally, it is also easy to check that when the assets in place are countercyclical, equity holders would prefer to invest in countercyclical growth options.

To summarize, our two-period model provides the following predictions:

- More cyclical assets in place make underinvestment more likely in bad times, exacerbating the costs of debt overhang when macroeconomic risk is taken into account.

- More cyclical investment opportunities also make underinvestment more likely in bad times. The overall effect on the costs of debt overhang when macroeconomic risk is taken into account is ambiguous.

- Among the growth options that are not too profitable (so that debt is still risky), equity holders would prefer to invest in ones that have the same cyclicality as their assets in place.

3 A Dynamic Model of Debt Overhang

In this section, we set up a dynamic real option/captial structure model to assess the quantitative impact of macroeconomic risk on investments and the costs of debt overhang. While earlier studies have examined the impact of macroeconomic risk on investment (e.g. Guo, Miao, and Morellec 2005) and financing (e.g. Hackbarth, Miao, and Morellec 2006, Chen 2009) separately, we emphasize the interactions between investment and financing in the presence of business cycle fluctuations in cash flows and risk prices.

3.1 Model Setup

The Economy   We consider an economy with business cycle fluctuations in the levels of cash flow, expected growth rates, economic uncertainty, and risk prices. For simplicity, we assume the economy has two aggregate states, $s_t = \{G, B\}$ (boom and recession).[3] The state s_t follows a continuous-time Markov chain, where the probability of the economy switching from state G (boom) to state B (recession) within a small period Δ is approximately equal to $\lambda(G)\Delta$, while the probability of switching from state B to G is approximately $\lambda(B)\Delta$. The long-run probability of the economy being in state G is $\lambda(B)/(\lambda(G) + \lambda(B))$.

We specify an exogenous stochastic discount factor (SDF),[4] which captures business cycle fluctuations in the risk free rate and the risk prices for small and large shocks in the economy:

$$\frac{dm_t}{m_t} = -r\left(s_t\right)dt - \eta\left(s_t\right)dW_t^m + \delta_G\left(s_t\right)\left(e^\kappa - 1\right)dM_t^G + \delta_B\left(s_t\right)\left(e^{-\kappa} - 1\right)dM_t^B, \qquad (14)$$

with

$$\delta_G\left(G\right) = \delta_B\left(B\right) = 1, \quad \delta_G\left(B\right) = \delta_B\left(G\right) = 0,$$

where W_t^m is a standard Brownian motion that generates systematic small shocks, and $\{M_t^G, M_t^B\}$ are compensated Poisson processes with intensity $\{\lambda(G), \lambda(B)\}$ respectively, which provide large shocks in the economy.

The first two terms in the stochastic discount factor process are standard. The instantaneous risk-free rate is $r(s_t)$, and the risk price for Brownian shocks is $\eta(s_t)$, both of which will change value when the state of the economy changes. The last two terms in (14) introduce jumps in the SDF that coincide with a change of state in the Markov chain specified earlier. For example, if the current state is G, a positive relative jump size ($\kappa > 0$) will imply that the SDF jumps up when the economy moves from a boom into a recession. The value κ determines the risk price for the large shocks in the economy.

---

[3]It is straightforward to extend the model to allow for more aggregate states, which does not change the main insight of the paper.

[4]Chen (2009) (Proposition 1) shows that such a stochastic discount factor can be generated in a consumption-based model when the expected growth rate and volatility of aggregate consumption follow a discrete-state Markov chain, and the representative agent has recursive preferences. His calibration is based on the long-run risk model of Bansal and Yaron (2004).

The Firm   A firm has assets in place that generate cash flow stream $a_t x_t + f_t^a$, where a_t and f_t^a take two possible values $\{a(G), a(B)\}$ and $\{f^a(G), f^a(B)\}$ in booms and recessions respectively, and x_t follows a Markov-modulated diffusion process:

$$dx_t = \mu(s_t)x_t dt + \sigma_m(s_t) x_t dW_t^m + \sigma_f x_t dW_t^f, \tag{15}$$

where W_t^f is a standard Brownian motion independent of W_t^m; $\mu(s_t)$ and $\sigma_m(s_t)$ are the expected growth rate and systematic volatility of cash flow, both of which can change with the aggregate state; σ_f is the idiosyncratic volatility, which is constant over time.

This affine functional form for cash flow captures the impact of business cycles in several dimensions. Let's assume that $f_t^a = 0$. First, holding a_t fixed, when the state of the economy changes, the expected growth rate $\mu(s_t)$ and the systematic volatility $\sigma_m(s_t)$ of cash flow can both change. These shocks on the conditional moments have permanent effects on cash flow. Second, when the economy enters into a recession, the level of cash flow jumps by a factor of $a(B)/a(G)$, which could be due to a significant change in productivity or adjustment in the amount of productive assets. For a firm with procyclical assets in place, $a(G) > a(B)$, and increasing the spread between $a(G)$ and $a(B)$ makes assets in place more procyclical. The effects of these shocks on cash flow are temporary, as they are reversed when the economy moves out of the recession.[5] Third, if $a_t = 0$ and f_t^a is constant, then the cash flow from assets in place becomes riskless. Fourth, by changing the relative composition of a_t and f_t^a, we can change the degree to which cash flow from assets in place is correlated with the market.

Next, the firm faces an investment opportunity. The investment requires a one-time lump-sum cost ϕ, and generates a cash flow stream that takes a similar form to that of assets in place, $g_t x_t + f_t^g$. Again, this cash flow process captures the cyclicality of growth option in a variety of ways. We will investigate how these different aspects of cyclicality of cash flows from assets in place and growth option affect investment and the agency costs of debt. We assume that investment is irreversible.[6]

The firm has debt in the form of a consol with coupon c. We first take the firm's debt level c as given and focus on the effects of existing debt on investments. Then, in Section 5, we compute the

---

[5]Hackbarth, Miao, and Morellec (2006) and Gorbenko and Strebulaev (2010) have studied the effects of temporary jumps in cash flows on the capital structure.

[6]Manso (2008) shows that if investment is perfectly reversible then there is no agency cost of debt. The bulk of the previous literature that study debt overhang assumes irreversible investment, which produces higher agency cost of debt.

optimal capital structure using the tradeoff between tax benefits and costs of debt overhang. There are two reasons for not restricting our analysis of debt overhang exclusively to the case of optimal leverage. First, in practice it is costly for firms to readjust their leverage, which often results in leverage ratios that are far from optimal levels. Second, other factors beyond tax benefits and costs of debt overhang (such as bankruptcy costs, asymmetric information, diversification benefits) could also be important determinants of the optimal leverage, which are outside of our model.

We assume that at each point in time the firm first makes the coupon payment c, then pays taxes at rate τ, and distributes all the remaining profit to its equity holders (no cash holdings). At the time of default, we assume that the absolute priority rule applies. The value of equity will be zero. Debt holders take over the firm and implement the first-best policies for the all-equity firm, but loses a fraction $1 - \alpha(s_t)$ of the value due to financial distress.[7]

The agency problem stems from the assumption that the firm acts in the interest of its equity holders. It chooses the optimal timing of default and investment to maximize the value of equity. We also assume that the investment is entirely financed by equity holders, and there are no ex post renegotiations between debt holders and equity holders. In particular, we rule out the possibility of financing the investment with new senior debt (likely restricted by covenants in practice).[8] Ex post renegotiations can be quite costly due to the free-rider problem among debt holders and the lack of commitment by equity holders.

3.2 Model Solution

We first introduce some notations. The value of equity before investment is $e_s(x)$ in state s. The value of equity after investment is $E_s(x)$. Similarly, the value of debt before and after investment is $d_s(x)$ and $D_s(x)$, respectively.

The optimal investment policy is summarized by a pair of investment boundaries $\{x_u(G), x_u(B)\}$. The firm invests when x_t is above $x_u(G)$ $(x_u(B))$ while the economy is in state G (B). The default policy is summarized by two pairs of default boundaries: $\{x_d(G), x_d(B)\}$ applies before investment is made, while $\{x_D(G), x_D(B)\}$ applies after investment. We first derive the value of equity and

---

[7]Alternatively, one can assume that debt holders lose the growth option in bankruptcy, and only recover a fraction of the value from assets in place. This assumption does not affect the investment policy equity holders choose, but does change the costs of bankruptcy.

[8]Hackbarth and Mauer (2010) argue that it could be in the interest of existing debt holders to allow for issuance of new senior debt to finance investment. However, such priority structures could become harder to implement when there is uncertainty about the quality of investment.

debt for given investment and default policies, and then search for the optimal policies.

While the ordering of the default and investment boundaries is endogenous, we assume the following ordering is true when presenting the model solution:

$$x_D(G) < x_D(B) < x_d(G) < x_d(B) < x_u(G) < x_u(B).$$

This ordering is satisfied when leverage is not too high, and the cash flows from the firm's assets in place and growth option are sufficiently procyclical. It has the intuitive implication that the firm defaults earlier and invests later in bad times. The ordering is satisfied by most of the parameter regions we consider in this paper. The solution can be easily adjusted for those cases with different orderings of the boundaries.

We value debt and equity under the risk-neutral probability measure $\mathcal{Q}$, as implied by the stochastic discount factor (14). Under $\mathcal{Q}$, the process for x_t becomes

$$dx_t = \widetilde{\mu}(s_t)x_t dt + \sigma(s_t)x_t d\widetilde{W}_t, \tag{16}$$

where

$$\widetilde{\mu}(s_t) = \mu(s_t) - \eta(s_t)\sigma_m, \tag{17}$$

$$\sigma(s_t) = \sqrt{\sigma_m^2(s_t) + \sigma_f^2}, \tag{18}$$

and $\widetilde{W}_t$ is a standard Brownian motion under $\mathcal{Q}$. In addition, the transition intensities of the Markov chain under $\mathcal{Q}$ become

$$\widetilde{\lambda}(G) = \lambda(G)e^{\kappa}, \quad \widetilde{\lambda}(B) = \lambda(B)e^{-\kappa}. \tag{19}$$

Thus, if the stochastic discount factor m_t jumps up when the economy changes from state G to B ($\kappa > 0$), then $\widetilde{\lambda}(G) > \lambda(G)$, while $\widetilde{\lambda}(B) < \lambda(B)$. Intuitively, the jump risk premium in the model makes the duration of the good state shorter and bad state longer under the risk neutral measure.

3.2.1 Value of Equity

After Investment   After the firm exercises the investment option, the problem becomes the same as the static capital structure model with two aggregate states. As discussed earlier, we conjecture that the default boundaries satisfy $x_D(G) < x_D(B)$. Then, taking $x_D(G)$ and $x_D(B)$ as given, the value of equity can be solved in two regions: $J_1 = [x_D(G), x_D(B))$ and $J_2 = [x_D(B), \infty)$.

For $x \in J_1$, the firm has not defaulted yet in state G, but has already defaulted in state B. Thus, $E_B(x) = 0$ in this region. The Feynman-Kac formula implies that $E_G(x)$ satisfies:

$$(r(G) + \widetilde{\lambda}(G))E_G = (1-\tau)\left((a(G) + g(G))x + f^a(G) + f^g(G) - c\right) + \widetilde{\mu}(G)xE_G' + \frac{1}{2}\sigma^2(G)x^2E_G''. \quad (20)$$

In Appendix A, we show that

$$E_G(x) = w_{1,1}^E x^{\alpha_1} + w_{1,2}^E x^{\alpha_2} + h_1^E(G)x + k_1^E(G), \quad (21)$$

where the values of α, $h_1^E(G)$, and $k_1^E(G)$ are given in the appendix.

Next, for $x \in J_2$, the firm is not in default yet in either state, and $E_G(x)$ and $E_B(x)$ satisfy a system of ODEs:

$$(r(G) + \widetilde{\lambda}(G))E_G = (1-\tau)\left((a(G) + g(G))x + f^a(G) + f^g(G) - c\right) + \widetilde{\mu}(G)xE_G' + \frac{1}{2}\sigma^2(G)x^2E_G'' + \widetilde{\lambda}(G)E_B, \quad (22a)$$

$$(r(B) + \widetilde{\lambda}(B))E_B = (1-\tau)\left((a(B) + g(B))x + f^a(B) + f^g(B) - c\right) + \widetilde{\mu}(B)xE_B' + \frac{1}{2}\sigma^2(B)x^2E_B'' + \widetilde{\lambda}(B)E_G. \quad (22b)$$

The solution is

$$E_s(x) = \sum_{j=1}^{4} w_{2,j}^E \theta_j(s)x^{\beta_j} + h_2^E(s)x + k_2^E(s). \quad (23)$$

The values of β, θ, $\mathbf{h}_2^E$, $\mathbf{k}_2^E$ are given in Appendix A.

In addition, we have the following boundary conditions that help pin down the values of the coefficients $\mathbf{w}^E$. First, the absolute priority rule implies that the value of equity at default is zero,

$$\lim_{x \downarrow x_D(G)} E_G(x) = 0, \quad (24)$$

$$\lim_{x \downarrow x_D(B)} E_B(x) = 0. \quad (25)$$

Next, the value of $E_G(x)$ must be continuous and smooth at the boundary of regions J_1 and J_2

14

(see Karatzas and Shreve (1991)), which implies

$$\lim_{x \uparrow x_D(B)} E_G(x) = \lim_{x \downarrow x_D(B)} E_G(x), \tag{26}$$

$$\lim_{x \uparrow x_D(B)} E'_G(x) = \lim_{x \downarrow x_D(B)} E'_G(x). \tag{27}$$

Finally, to rule out bubbles, we also impose the following conditions:

$$\lim_{x \uparrow +\infty} \frac{E_G(x)}{x} < \infty, \tag{28}$$

$$\lim_{x \uparrow +\infty} \frac{E_B(x)}{x} < \infty. \tag{29}$$

As the Appendix shows, these boundary conditions lead to a system of linear equations for $\mathbf{w}^E$, which can be solved in closed form.

Before Investment   Before the investment is made, we have conjectured that $x_d(G) < x_d(B) < x_u(G) < x_u(B)$, which gives 3 relevant regions for cash flow x_t: $I_1 = [x_d(G), x_d(B))$, $I_2 = [x_d(B), x_u(G))$, and $I_3 = [x_u(G), x_u(B))$. Again, we can solve for $e_G(x)$ and $e_B(x)$ analytically when taking $x_d(G), x_d(B), x_u(G), x_u(B)$ as given.

In region I_1, the firm has already defaulted in state B. Thus, $e_B(x) = 0$ in this region. In state G, $e_G(x)$ satisfies the same ODE as (20), except that before investment, the firm's cash flow at time t becomes $a(G)x_t + f^a(G)$ instead of $(a(G) + g(G))x_t + f^a(G) + f^g(G)$. The solution is

$$e_G(x) = w_{1,1}^e x^{\alpha_1} + w_{1,2}^e x^{\alpha_2} + h_1^e(G)x + k_1^e(G), \tag{30}$$

where α is the same as in the post-investment case; $h_1^e(G)$ and $k_1^e(G)$ are given in Appendix A.

In region I_2, the firm has not defaulted or made investment in either state, and $e_G(x)$ and $e_B(x)$ satisfy the same ODE system as (22a-22b), again with instantaneous profit $(a_t + g_t)x_t + f_t^a + f_t^g$ replaced by $a_t x_t + f_t^a$. The solution is

$$e_s(x) = \sum_{j=1}^{4} w_{2,j}^e \theta_j(s) x^{\beta_j} + h_2^e(s)x + k_2^e(s), \tag{31}$$

where the values of β and θ are the same as in the post-investment case; $\mathbf{h}_2^e$ and $\mathbf{k}_2^e$ are given in Appendix A.

15

In region I_3, the firm will have already made the investment in state G. In state B, $e_B(x)$ satisfies:

$$(r(B) + \widetilde{\lambda}(B))e_B = (1 - \tau)(a(B)x + f^a(B) - c) + \widetilde{\mu}(B)xe'_B + \frac{1}{2}\sigma^2(B)x^2e''_B + \widetilde{\lambda}(B)(E_G - \phi). \quad (32)$$

The last term captures the fact that the firm will invest immediately if the state changes from B to G. The solution is

$$e_B(x) = w^e_{3,1}x^{\gamma_1} + w^e_{3,2}x^{\gamma_2} + h^e_3(B)x + k^e_3(B) + \sum_{j=1}^{4}\omega^e_{3,j}x^{\beta_j}, \quad (33)$$

where the values of γ, $h^e_3(B)$, $k^e_3(B)$, and ω^e_3 are given in Appendix A.

The values of the coefficients $\mathbf{w}^e$ are determined by the following boundary conditions. First, the value of equity is 0 at default:

$$\lim_{x \downarrow x_d(G)} e_G(x) = 0, \quad (34)$$

$$\lim_{x \downarrow x_d(B)} e_B(x) = 0. \quad (35)$$

Next, $e_G(x)$ and $e_B(x)$ must be piecewise $\mathcal{C}^2$,

$$\lim_{x \uparrow x_d(B)} e_G(x) = \lim_{x \downarrow x_d(B)} e_G(x), \quad (36)$$

$$\lim_{x \uparrow x_d(B)} e'_G(x) = \lim_{x \downarrow x_d(B)} e'_G(x), \quad (37)$$

$$\lim_{x \uparrow x_u(G)} e_B(x) = \lim_{x \downarrow x_u(G)} e_B(x), \quad (38)$$

$$\lim_{x \uparrow x_u(G)} e'_B(x) = \lim_{x \downarrow x_u(G)} e'_B(x). \quad (39)$$

Finally, at the two investment boundaries $x_u(G)$ and $x_u(B)$, the value-matching conditions imply

$$\lim_{x \uparrow x_u(G)} e_G(x) = \lim_{x \downarrow x_u(G)} E_G(x) - \phi, \quad (40)$$

$$\lim_{x \uparrow x_u(B)} e_B(x) = \lim_{x \downarrow x_u(B)} E_B(x) - \phi. \quad (41)$$

Again, the boundary conditions are all linear in the coefficients $\{\mathbf{w}^e\}$, so we can solve for them analytically from a system of linear equations.

For a given coupon and the default and investment boundaries, we can also price the defaultable debt ($d_s(x)$ and $D_s(x)$) in closed form. Similarly, the value of an all-equity firm can be computed analytically for given investment boundaries. Appendix B provides the details.

3.2.2 Optimal Default and Investment, Agency Costs, and PVGO

Next, we discuss the conditions that determine the optimal default and investment boundaries. Whenever the optimal default boundaries post investment $\{x_D(G), x_D(B)\}$ are in the interior region (above 0), they satisfy the smooth-pasting conditions:

$$\lim_{x \downarrow x_D(G)} E'_G(x) = 0, \tag{42}$$

$$\lim_{x \downarrow x_D(B)} E'_B(x) = 0. \tag{43}$$

Since E_G and E_B are given in closed form, these smooth-pasting conditions render two nonlinear equations for $x_D(G)$ and $x_D(B)$ that can be solved numerically.

Similarly, the optimal investment and default boundaries $\{x_d(G),\ x_d(B),\ x_u(G),\ x_u(B)\}$ satisfy 4 smooth-pasting conditions:

$$\lim_{x \downarrow x_d(G)} e'_G(x) = 0, \tag{44}$$

$$\lim_{x \downarrow x_d(B)} e'_B(x) = 0, \tag{45}$$

$$\lim_{x \uparrow x_u(G)} e'_G(x) = \lim_{x \downarrow x_u(G)} E'_G(x), \tag{46}$$

$$\lim_{x \uparrow x_u(B)} e'_B(x) = \lim_{x \downarrow x_u(B)} E'_B(x), \tag{47}$$

which again translate into a system of nonlinear equations in $\{x_d(G),\ x_d(B),\ x_u(G),\ x_u(B)\}$.

The first-best investment policy is achieved when the firm has no debt, i.e., $c = 0$. We denote the optimal investment boundaries in this case as $\{x_u^*(G), x_u^*(B)\}$. The existence of risky debt makes equity holders raise the investment thresholds, so that $x_u(G) > x_u^*(G)$ and $x_u(B) > x_u^*(B)$. To define the agency costs of debt, we need a few more notations. Let $v_s^{AE}(x; \widehat{x}_u(G), \widehat{x}_u(B))$ be the value of an all-equity firm (before investment) in state s with current cash flow x and investment thresholds $\{\widehat{x}_u(G), \widehat{x}_u(B)\}$. Let $v_s^{FB}(x)$ be the value of the first-best levered firm (which maximizes the value the firm), and let $v_s^{SB}(x)$ be the value of the second-best levered firm (which maximizes

17

the value equity).

A common measure of the agency costs of debt is the difference between the value of the firm under the first best and that under the second best (see, e.g., Hackbarth and Mauer 2010). Following this definition, we can define the state-dependent agency cost in our model as

$$\widetilde{ac}_s(x_0) = \frac{v_s^{FB}(x_0) - v_s^{SB}(x_0)}{v_s^{FB}(x_0)}, \quad s = G, B. \tag{48}$$

However, this measure of agency cost includes the costs of debt overhang, the costs of bankruptcy, and the tax benefit of debt. To isolate the investment distortions due to debt overhang, we can instead compute the agency costs as the difference in the value of an otherwise identical all-equity firm under the first and second best investment policy:

$$ac_s(x_0) = \frac{v_s^{AE}(x_0; x_u^*(G), x_u^*(B)) - v_s^{AE}(x_0; x_u(G), x_u(B))}{v_s^{AE}(x_0; x_u^*(G), x_u^*(B))}, \quad s = G, B. \tag{49}$$

It is possible that current cash flow x_0 is higher than some of the investment thresholds under the first or second best. In that case, the firm will invest immediately, and the value of the firm before investment will be equal to the value of the firm after investment minus the fixed costs of investment ϕ. If we set the tax rate $\tau = 0$ and the recovery rate $\alpha(G) = \alpha(B) = 1$, then there will be neither tax benefit nor bankruptcy costs. In this case, the agency cost $ac_s(x)$ as defined in (49) are the same as $\widetilde{ac}_s(x)$ in equation (48). Our measure has the benefit of being independent of the assumptions on tax rate and bankruptcy costs.

Finally, the size of agency costs will depend on how valuable the growth option is relative to the firm's assets in place. In the extreme case, if the growth option is worthless, there will be no costs of debt overhang. Thus, we also define a measure of the growth option using PVGO (present value of growth option), which is equal to the present value of the cash flows from investment normalized by the first-best firm value.

Having described the model and its solution, next we examine its quantitative implications.

4 Quantitative Analysis

In this section, we first discuss the calibration strategy, and then analyze the quantitative effects of macroeconomic risk on the costs of debt overhang.

We calibrate the transition intensities ($\lambda(G)$ and $\lambda(B)$) of the Markov chain by matching the average duration of NBER-dated expansions and recessions. Historically, the average length of an expansion is 38 months, while the average length of a recession is 17 months, which yields $\lambda(G) = 0.32$ and $\lambda(B) = 0.71$. As a result, the unconditional probability of being in an expansion and a recession state are 0.69 and 0.31, respectively. We then calibrate the real expected growth rate and systematic conditional volatility of cashflow in the two states to match the first and second moment of the conditional expected growth rate and volatility of real aggregate corporate profits. The nominal expected growth rate is obtained by assuming a constant annual inflation rate $\pi = 3\%$.

Next, we calibrate the real interest rate in the two states to match the mean and standard deviation of the real riskfree rate in the data, which are again converted to nominal rates using the constant inflation rate $\pi = 3\%$. Then we set $\kappa = \ln(2.5)$, which implies the risk-neutral probability of a jump from state G to B is 2.5 times as high as the physical probability.[9] The remaining parameters of the stochastic discount factor, the prices of Brownian shocks $\eta(s_t)$, are calibrated to match the average equity premium and the Sharpe ratio of the unlevered firm with those of the market portfolio.

The resulting parameter values are reported in Panel A of Table 1, where the means and standard deviations are computed using the stationary distribution of the Markov chain. The asset pricing implications of the stochastic discount factor are in Panel B, where the dividend process of the market portfolio is assumed to be the same as x_t in equation (15), with the idiosyncratic volatility σ_f calibrated to give an average correlation between the market and the SDF of 0.7.

Chen, Collin-Dufresne, and Goldstein (2009) and Chen (2009) show that the amount of systematic risk in a firm can significantly affect the pricing of corporate claims. They use the Sharpe ratio of equity as a key statistic to gauge whether the systematic risk exposure in a firm is reasonable. For this reason, when comparing models with and without macroeconomic risk, we always match the average Sharpe ratio of the market portfolio as well as the Sharpe ratio of equity for the firm. More specifically, we recalibrate the idiosyncratic volatility of cashflow σ_f for the levered firms to fix the Sharpe ratio of equity at 0.25, which is roughly the median firm-level Sharpe ratio in the data.

Finally, the assumption on tax rate and bankruptcy recovery rate does not affect the investment

---

[9]This jump-risk premium is consistent with the calibration adopted in Chen (2009). Later on we examine how different values of κ affect the results.

Table 1: Calibration Of The Markov Chain Model

The table reports the calibrated parameters and the model-generated moments of the equity market. The expression $E(r_m - r_f)$ is the annualized equity premium. The expression $\sigma(r_m - r_f)$ is the annualized volatility of the market excess return.

Variable	G	B	mean	std
A. Calibrated Parameters				
$\lambda(s_t)$	0.32	0.71	-	-
$r_f(s_t)$	4.51	2.41	3.86	0.97
$\eta(s_t)$	0.17	0.43	0.25	0.12
$\mu(s_t)$	5.97	2.18	4.80	1.75
$\sigma_m(s_t)$	9.82	17.39	12.16	3.50
B. Asset Pricing Implications				
$E(r_m - r_f)$	4.75	10.42	6.51	2.62
$\sigma(r_m - r_f)$	16.05	22.02	17.89	2.76
$E(r_m - r_f)/\sigma(r_m - r_f)$	0.30	0.47	0.35	0.08

and default decisions for equity holders as long as the after-tax fixed cost of investment ϕ is unaffected. Thus, we set $\tau = 0$ and $\alpha = 1$ in this section, so that our measure of agency cost is consistent with that in the literature. In Section 5 where we study the effect of debt overhang on optimal leverage, we will adopt a more realistic tax rate.

4.1 Static Investment Model

We first consider a simple case, where investment is assumed to be a static "take-it-or-leave-it" decision. In this case, the firm does not have the option to choose when to invest, but would have to decide whether to immediately invest in a given project. This exercise serves two purposes. First, the effects of macroeconomic risk on debt overhang are more transparent and easier to quantify in this case. Second, we use this example to highlight a new and important aspect of asset substitution in the presence of macroeconomic risk.

The optimal investment rule under the first best (without leverage) is the NPV rule, which prescribes making an investment whenever the net present value of cash flows from investment exceeds the cost. When the firm has risky debt in place, the value of investment accrued to equity holders would be equal to the NPV of investment minus the transfer from equity holders to debt

holders. As a result, debt overhang causes equity holders to value the investment with a discount. Naturally, this discount is larger when firm leverage is higher. We will show that the investment discount also varies significantly with the cyclicality of assets in place and growth option, which generates predictions on what types of projects equity holders would prefer to invest in.

Specifically, we first compute the initial value of equity assuming that the firm does not have any investment opportunities. This value is $e_s^n(x_0)$, where the superscript n (stands for "no investment option") distinguishes the variable from $e_s(x_0)$, which is the initial value of equity before investment is made. While there is no investment decision, the firm still needs to make optimal default decisions, which are characterized by the default thresholds $x_d^n(G)$ and $x_d^n(B)$. Next, assuming the firm accepts the project, we can compute the value of equity immediately after investment. Since the firm's problem after investment is identical to the case with investment option, the value of equity post investment will be $E_s(x_0)$. Then, the value of the investment to equity holders will be $E_s(x_0) - e_s^n(x_0)$. Denote the NPV of the investment in state s as $NPV_s(x_0)$, then the investment distortion relative to the first best can be measured by the investment discount:

$$ID_s(x_0) = 1 - \frac{E_s(x_0) - e_s^n(x_0)}{NPV_s(x_0)}, \tag{50}$$

which can be computed in closed form.

As a benchmark, we assume that $a(G) = a(B) = 1$, $f^a(G) = f^a(B) = 0$, $g(G) = g(B) = 0.4$, and $f^g(G) = f^g(B) = 0$. Thus, the investment will increase the firm's cash flows by 40%. Figure 2 reports the investment discount for the firm as we vary the cyclicality of assets in place and growth option. We focus on the case where the initial state is the good state, which is when firms are more likely to be making investment decisions in practice. In the left panels, the leverage is lower, with coupon of the consol fixed at $c = 0.4$, which corresponds to initial market leverage in the range of 42% to 44%. In the right panels, the coupon is fixed at $c = 1.0$, which corresponds to leverage in the range of 75% to 80%.

We first examine how the investment discount changes with the cyclicality of assets in place and growth option via the transitory business cycle shocks $a(s)$ and $g(s)$. Specifically, while *holding the NPV of cash flows fixed*, we can increase the spread between $a(G)$ and $a(B)$ ($g(G)$ and $g(B)$) to make the assets in place (growth option) more cyclical. Thus, the closer $a(B)$ (or $g(B)$) is to 0, the more procyclical the assets in place (or growth option) becomes.
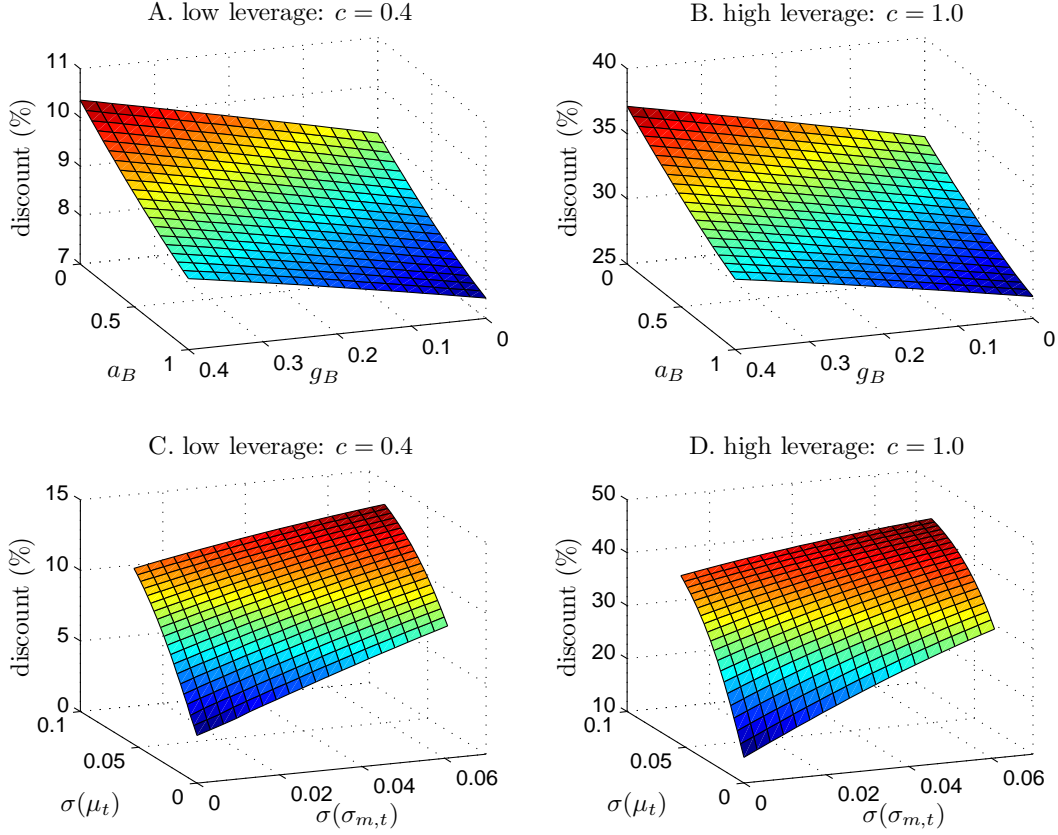
Figure 2: **Macroeconomic Risk and Deviation from the NPV Rule.** This figure plots the discount at which a levered firm values static investment opportunities (relative to the first best). The top panels show how the investment discount changes with the cyclicality of the assets in place and growth option (through $a(s)$ and $g(s)$). The bottom panels show how the discount changes with the business cycle variations in the conditional moments of cash flows ($\mu(s)$ and $\sigma_m(s)$).

In Panels A and B, we see that the investment discount rises as the firm's assets in place become more procyclical, but the discount decreases as the growth option becomes more procyclical. When leverage is low, the investment discount is relatively small, ranging from 11% of the NPV when assets in place are highly procyclical while growth option is highly countercyclical, to about 7% when assets in place are highly countercyclical while growth option is highly procyclical. With high leverage, not only is the average level of investment discount significantly higher, but it also becomes more sensitive to changes in the cyclicality of cash flows.

Intuitively, whenever cash flow from assets in place falls short of the coupon payment, part of the cash flow from investment will be paid to debt holders. Holding the growth option fixed, making assets in place more cyclical increases the probability of such transfers in the bad state, while

lowering their probability in the good state. The net effect is higher expected total transfer because of the higher systematic risk associated with the bad state. Put differently, due to debt overhang, stronger cyclicality of assets in place makes the part of cash flows equity holders receive from the investment more risky, even though the total cash flow from investment remains unchanged.

The effects of a more procyclical growth option depend on the cyclicality of assets in place. Since the firm's assets in place are procyclical, debt is more risky in the bad state. In this case, having a more procyclical growth option reduces the transfer to debt in the bad state, hence lowering the investment discount. However, if the firm's assets in place are countercyclical instead, then debt will be more risky in the good sate. In that case, having a more procyclical growth option will raise the investment discount.

The interactions between the cyclicality of assets in place and growth option bring us new insights on asset substitution in the presence of macroeconomic risk. In a risk-neutral world, the standard asset substitution argument (Jensen and Meckling (1976)) implies that equity holders of a levered firm will prefer to invest in projects with cash flows that are more correlated with assets in place. Higher correlation raises the volatility of the firm overall, and reduces the amount of transfer to debt holders. With macroeconomic risk, equity holders will not only care about the average correlation, but especially want to line up the cyclicality of the investment with that of assets in place. For example, a highly-levered procyclical firm, such as large banks, will have strong incentive to invest in assets with high systematic risk exposure, even if these assets have lower NPV, because such assets will give equity holders more upside in good times while providing limited transfer to debt holders in bad times. Such incentives can lead to severe negative externality for the economy, as highlighted by the recent financial crisis.

Next, we change the cyclicality of the firm by changing the amount of business cycle variations in the conditional moments of cash flow growth rates. Both $a(s)$ and $g(s)$ are assumed to be constant again. As reported in Table 1, for the benchmark firm, the volatility of the conditional expected growth rate is $\sigma(\mu_t) = 1.75\%$, while the volatility of the systematic volatility of cash flows is $\sigma(\sigma_{m,t}) = 3.5\%$. In Panels C and D of Figure 2, we plot investment discount as a function of the volatilities of the conditional moments while holding the means of the conditional moments fixed. The lowest investment discount occurs when both $\sigma(\mu_t)$ and $\sigma(\sigma_{m,t})$ are 0 (as in the case without macroeconomic risk), so that there is no business cycle variation in the conditional moments of cash flows. When we increase $\sigma(\mu_t)$ and $\sigma(\sigma_{m,t})$, the investment discount rises. In the case of low

leverage, the discount rises from 3% of NPV to 15%. With high leverage the discount can rise from 15% to 48%.

These results demonstrate how macroeconomic risk can significantly amplify the effect of debt overhang. Making those shocks with permanent effect (shocks on the conditional moments of growth rates) more cyclical can have particularly strong impact on the investment distortions.

4.2 Dynamic Debt Overhang

While the case of take-it-or-leave-it investment opportunity allows for easy comparison with the NPV rule, in practice firms usually have the ability to choose when to invest. An investment opportunity that will be rejected by equity holders under current market conditions could become attractive again in the future, for example, when debt becomes less risky. Thus, it is important to take into account the option to wait when measuring the costs of debt overhang.

As mentioned when we define the measure of agency cost in Section 3, the costs of debt overhang depend on the value of the growth option. If the growth option is too far out of the money, the firm is unlikely to invest soon regardless of whether it has debt in place or not. In this case, the agency costs as defined in equation (49) will be (essentially) zero. As the value of the growth option increases, the investment thresholds are likely to drop. If the growth option is sufficiently in the money, the optimal investment thresholds can be below x_0 both under the first best (no debt) and the second best ($c > 0$). In this case, the firm invests immediately, and there will be no difference in the actual investment thresholds under the first and second best. Then, the agency costs will again be zero.

In Figure 3, we plot the agency cost for growth options with PVGO ranging from 0 to 50% of the first best firm value. For assets in place, we assume $f^a(G) = f^a(B) = 0$, $a(G) = 1.1$, $a(B) = 0.77$, so that the unconditional average of a_t is still 1, and cash flow falls by 30% going into a recession. For growth option, we assume $g(G) = g(B) = 1$, and $f^g(G) = f^g(B) = 0.14$. It is unrealistic to have cash flows from the growth option perfectly correlated with the assets in place. Adding a component $f^g(s) > 0$ (which is independent of x_t) is one way to reduce this correlation. The NPV of this riskless component will be on average 20% of the total growth option. We then vary the PVGO by changing the fixed cost ϕ, but recalibrating σ_f each time to fix the Sharpe ratio of equity at 0.25. Finally, to turn off macroeconomic risk (for comparison), we fix μ_t, $\sigma_{m,t}$, a_t, g_t, f_t^a, and f_t^g all at their unconditional means for the benchmark firm. As for the stochastic discount
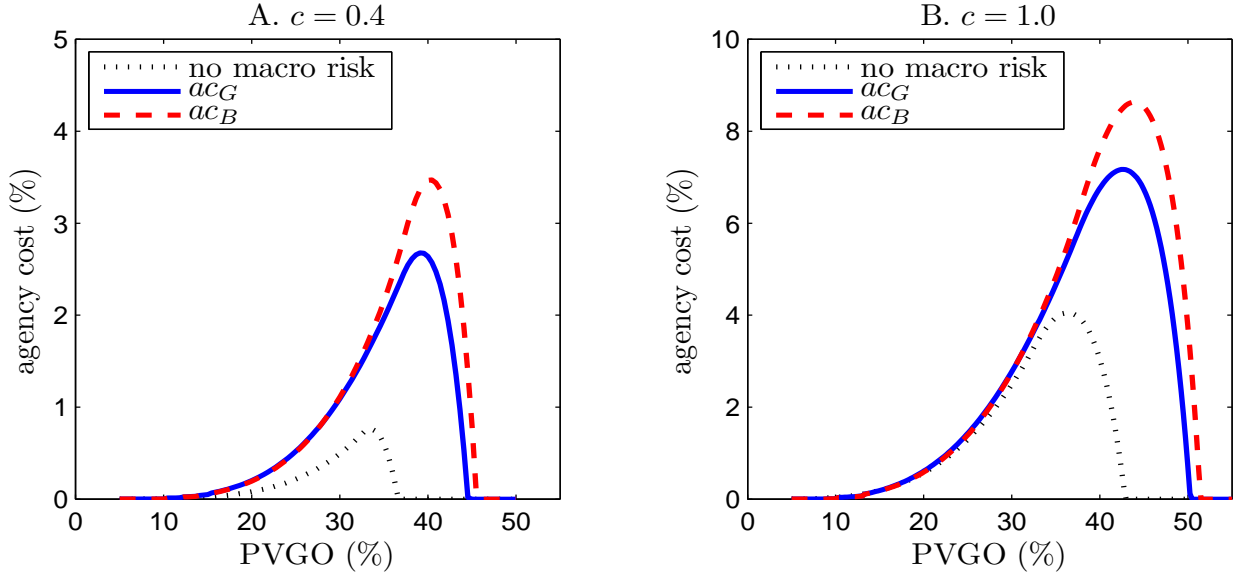
24

Figure 3: **Costs of Debt Overhang.** This figure plots the costs of debt overhang (in percentage of first best firm value) for investments with different PVGO. ac_G and ac_B are the conditional costs of debt overhang in good and bad state. The market Sharpe ratio in the "no macro" case is matched to the average market Sharpe ratio in the case with macro risk. The equity Sharpe ratio is always fixed at 0.25 through recalibration of σ_f.

factor, we remove the jumps ($\kappa = 0$) from Equation (14), fix r at its unconditional mean, and set $\eta = 0.49$ to match the average Sharpe ratio of the market portfolio in Table 1. The fixed cost ϕ and idiosyncratic volatility σ_f are then calibrated to generate different levels of PVGO while keeping the equity Sharpe ratio at 0.25.

Panel A of Figure 3 plots the agency costs for a low leverage firm ($c = 0.4$). As conjectured, costs of debt overhang are close to zero when the value of the growth option is either very low or high, but rise up for intermediate values. In that region, both delay in investment (relative to first best) and losses from underinvestment are significant. The agency costs are low without macroeconomic risk. For a firm with low leverage, the agency cost is very close to 0, and peaks at 0.8% of the first-best firm value. Consistent with the small agency cost, the delay in investment relative to the first best is also quite limited.

Once we take macroeconomic risk into account, the agency cost can become substantially higher. It peaks when PVGO is about 40%, where the agency cost rises to 2.6% in state G or 3.5% in state B. The investment boundaries with risky debt in states G and B become 42% and 48% higher than under the first best, respectively. These results show that macroeconomic risk indeed has

Table 2: COSTS OF DEBT OVERHANG: SYSTEMATIC RISK

The table reports the 5-year conditional investment probabilities in the two states $(p_G^5(x_0), p_B^5(x_0))$, the conditional costs of debt overhang in the two states $(ac_G(x_0), ac_B(x_0))$, and the average costs of debt overhang at $x_0 = 1$. In the benchmark case, $c = 0.4$, $f^a(G) = f^a(B) = 0$, $a(G) = 1.1$, $a(B) = 0.77$, $f^g(G) = f^g(B) = 0.14$, $g(G) = g(B) = 1$, ϕ and σ_f are calibrated to fix the PVGO at 40% and the average equity Sharpe ratio at 0.25. The rest of the parameters are in Table 1.

	invest prob (%)		agency costs (%)		average
	$p_G^5(x_0)$	$p_B^5(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$	agency costs
benchmark	73.2	68.0	2.6	3.5	2.9
$\kappa = \ln(3.0)$	61.7	57.9	3.7	4.3	3.9
$E[\eta(s_t)] = 0.35$	59.7	55.8	4.4	5.2	4.7
$\sigma(\eta(s_t)) = 0.16$	67.3	62.7	3.4	4.3	3.7
$\lambda(G) = 0.06, \lambda(B) = 0.14$	95.3	72.1	0.3	4.0	1.5

important effects on debt overhang. When the leverage of the firm gets higher, as we see in Panel B of Figure 3, the agency cost of debt peaks at 3.8% without macroeconomic risk, but rises to 7.2% and 8.6% respectively in states G and B with macroeconomic risk.

The results in Figure 3 also highlight the dynamic debt overhang effects, which are absent from the static model in Section 2. The conditional agency costs in the good and bad state, $ac_G(x_0)$ and $ac_B(x_0)$, are not that far apart, despite the fact that business cycle fluctuations in the level and conditional moments of cash flows imply that the benchmark firm is in a better than average condition in state G. When in state G, even though the cash flows are currently higher and are expected to growth faster, equity holders are still reluctant to invest because they are concerned that the state of the economy might change, which can make debt substantially more risky and raise the amount of wealth transfer from equity holders to debt holders through investment. Thus, debt overhang in this state comes mainly from concern of future wealth transfer in a state with worse conditions, which is different from the concern of immediate wealth transfer when debt is already under water. This dynamic overhang effect will become weaker when we make the two states more persistent.

We also examine the effects of systematic risk on the costs of debt overhang by raising the price of jump risks κ, the average price of Brownian risk $\eta(s_t)$ and its variation across the two states, and

the persistence of the two aggregate states. The results are reported in Table 2. In the benchmark case, the average agency cost across the two states is 2.9%. The probability of the firm making the investment in the next 5 years is 73.2% in state G, or 68.0% in state B. If we increase the price of jump risk κ from $\ln(2.5)$ to $\ln(3)$, the average agency cost rises to 3.9%, while the conditional investment probabilities in the two states fall to 51.7% and 47.6%, respectively. Similarly, when we increase either the mean or volatility of the risk price for Brownian shocks, $E[\eta(s_t)]$ and $\sigma(\eta(s_t))$, the agency costs in the two states will rise, while the investment probabilities will fall. Finally, if we increase the persistence of the two states by lowering $\lambda(G)$ to 0.06 and $\lambda(B)$ to 0.14 (making both states 5 times more persistent than before), the dynamic debt overhang effects start to diminish. The agency costs in the two states become further apart. The costs rise to 4% in state B, but fall to 0.3% in state G, making the average agency costs lower as well.

Having demonstrated the overall effect of business cycle risks on the costs of debt overhang, we next decompose the effects into two parts, one through assets in place, the other through growth option.

4.3 Assets in Place and Growth Option

As we discussed in the static model in Section 2, the cyclicality of assets in place and growth option have different effects on the agency costs of debt. To examine these effects in the dynamic model, we consider the following comparative statics in Table 3.

Throughout the table, we keep the fixed cost of investment $\phi = 12.4$ and idiosyncratic volatility $\sigma_f = 24.4\%$, which are the values that make the benchmark firm have 40% PVGO and equity Sharpe ratio of 0.25. As a reminder, if we turn off the business cycle fluctuations in cash flows and risk prices, the firm will set the investment threshold below the initial cash flow $x_0 = 1$, which means the firm will make the investment immediately, and the costs of debt overhang measured at x_0 will be 0.

For the benchmark firm, $a(B)/a(G) = 0.7$, $g(B)/g(G) = 1$, and the conditional mean and volatility of the growth rates of cash flow from assets in place are given in Table 1. The coupon rate $c = 0.4$ implies an initial leverage of 38% in state G, or 42% in state B. This firm delays making the investment significantly, as the probability of investment in the next 5 years is a mere 73.2% in the good state, or 68.0% in the bad state, as opposed to immediate investment under the first best. The average agency cost is 2.9% of the first best firm value, but the conditional agency

Table 3: CYCLICALITY OF ASSETS IN PLACE AND GROWTH OPTION

The table reports the 5-year conditional investment probabilities in the two states $(p_G^5(x_0), p_B^5(x_0))$, the conditional costs of debt overhang in the two states $(ac_G(x_0), ac_B(x_0))$, and the average costs of debt overhang at $x_0 = 1$. In the benchmark case, $a(G) = 1.1, a(B) = 0.77, c = 0.4$, $f^g(G) = f^g(B) = 0.14$, and $g(G) = g(B) = 1$. The remaining parameters are reported in Table 1. Throughout the table, we fix $\phi = 12.4$ and $\sigma_f = 24.4\%$.

	invest boundary		invest prob (%)		agency costs (%)		average
	$x_u(G)$	$x_u(B)$	$p_G^5(x_0)$	$p_B^5(x_0)$	$ac_G(x_0)$	$ac_B(x_0)$	agency costs
benchmark	1.23	1.30	73.2	68.0	2.6	3.5	2.9
$a(B)/a(G) = 1$	1.21	1.27	75.2	70.1	2.2	2.9	2.4
$a(B)/a(G) = 0.5$	1.25	1.32	71.5	66.5	3.1	4.0	3.4
$g(B)/g(G) = 0.7$	1.18	1.35	77.0	69.3	2.7	3.3	2.9
$g(B)/g(G) = 0.5$	1.14	1.42	80.0	70.2	2.6	3.0	2.7
$\sigma(\mu_t) = 3.5\%$	1.38	1.50	60.6	52.9	4.3	5.0	4.5
$\sigma(\sigma_{m,t}) = 7.0\%$	1.42	1.59	56.0	51.1	4.1	4.1	4.1
$f_t^g = 0.28, g_t = 0.68$	1.33	1.38	64.8	60.5	6.1	7.9	6.7

cost is considerably higher in state B (3.5%) than in state G (2.6%).

If we make the assets in place more cyclical (see the case $a(B)/a(G) = 0.5$), the investment boundaries rise and the 5-year probabilities of investment fall in both states. This result again highlights the dynamic debt overhang effect: the threat of a worse state B outweighs the better condition in the current state, making equity holders less willing to invest. Similarly, the conditional agency costs become higher in both states, rising from 2.6% to 3.1% in state G, and from 3.5% to 4.0% in state B. The opposite is true as we make the assets in place less cyclical than the benchmark (see the case $a(B)/a(G) = 1$).

The relation between the cyclicality of growth option and the costs of debt overhang is more complicated. On the one hand, stronger cyclicality raises the value of the growth option in the good state, but lowers it in the bad state, which has the effects of making default less likely in the good state but more likely in the bad state. This implies that *for a given value added by the investment* there will be more wealth transfer to the debt holders in the bad state, which tends

to exacerbate the debt overhang problem. On the other hand, a more cyclical growth option also lowers the potential loss of value in the bad state. This second effect tends to lower the costs of debt overhang.

As shown in Table 3, larger spread in g_t tends to lower the investment boundary in state G and raise the investment boundary in state B. However, the 5-year investment probabilities generally go up in both states. When the spread in g is not too big $(g(B)/g(G) = 0.75)$, the agency cost in state G becomes slightly higher than the benchmark case, reflecting the fact that the higher value of investment in state G is making delays more costly, even though the investment threshold has become lower. However, this effect is reversed when $g(B)/g(G) = 0.5$, suggesting that the effect of lower investment boundaries starts to dominate. On the other hand, the agency cost in state B falls as cash flow of growth option becomes more procyclical.

When we increase the variation of the conditional moments of cash flows, both the assets in place and growth option become more cyclical. Thus, while debt becomes more risky in state B, the conditional value of the growth option also becomes lower in state B. The conditional agency cost in the two states can be affected differently due to the competing effects. For example, when we increase the volatility of μ_t (the expected growth rate of cash flow) from 1.75% to 3.5% (without changing the average expected growth rate), the conditional agency costs in state B (5.0%) are still than that in state G (4.3%). When we raise the volatility of $\sigma_{m,t}$ from 3.5% to 7.0%, the conditional agency costs in the two states become about the same.

Finally, we examine what happens to debt overhang when the cash flow from growth option becomes less correlated with that of assets in place. To do so, we double the value of the riskfree component of the growth option to $f_t^g = 0.28$, while adjusting g_t downward to 0.68 to keep the average NPV of the growth option constant. Such a change substantially raises the investment thresholds in both states. The probabilities of investment in the next 5 years fall to 64.8% and 60.5%, respectively. The costs of debt overhang rise in both states, to 6.1% and 7.9%.

This result is quite intuitive. Part of the cash flow from the growth option loads on the same shock as assets in place (from x_t). Thus, when debt becomes risky (x_t is low), so will be the cash flow from the growth option, which reduces the wealth transfer to debt holders, hence limiting the costs of debt overhang. If the cash flow from growth option is uncorrelated with that from assets in place, in particular if the growth option is riskless, then the debt overhang problem will become more severe. Macroeconomic risk further strengthens this effect by (i) making debt more risky in

state B and (ii) the wealth transfer in state B more costly for equity holders ex ante.

5   Optimal Leverage

In this section, we investigate the endogenous choice of capital structure based on the trade-off between tax benefits and agency costs. The capital structure that maximizes the market value received by the initial owners for sale of equity and debt in state s can be determined from the coupon rate c^* solving

$$\sup_c \{e_s(x, c) + d_s(x, c)\} \tag{51}$$

We set the tax rate $\tau = 0.2$, a value commonly used in dynamic tradeoff models, which reflects the fact that the tax benefits of debt at the corporate level are partially offset by individual tax disadvantages of interest income. Unlike standard tradeoff models, we assume full recovery at default ($\alpha = 1$).[10] Notice that "full recovery" here refers to the fact that there is no dead-weight loss for the firm due to bankruptcy. It does not mean that debt holders will recover in full in bankruptcy.

Table 4 illustrates the impact of macroeconomic risk on optimal leverage. Panel A1 shows that, under our benchmark parameters (with PVGO calibrated to 40% of total firm value), when macroeconomic risk is taken into account, the optimal coupon decreases from 0.92 to 0.45 or 0.33 depending on the current macroeconomic state being good or bad respectively. This translates into a rise in the interest coverage from 1.1 to 2.4 in state G or 2.3 in state B, and a decrease in the optimal market leverage from 60% to 45% in state G or 40% in state B. It might be surprising to see that the costs of debt overhang at optimal leverage are higher in the case without macroeconomic risk. However, that is precisely because when there is no macroeconomic risk, the low costs of debt overhang for a given leverage induces the firm to issue more debt, which in turn leads to higher costs of debt overhang. In fact, the magnitude of the agency costs with macroeconomic risk is comparable to that in Panel A (low leverage case) of Figure 3, while the magnitude of the agency costs without macroeconomic risk is comparable to that in Panel B (high leverage case). Finally, it is also interesting to see that the firm optimally chooses lower leverage in state B (compared to

---

[10]By assuming full recovery rate, we exclude the effect of countercyclical bankruptcy costs on optimal leverage as analyzed in Chen (2009), and instead entirely focus on the costs of debt overhang. Since we assume debt holders do not relever the firm after bankruptcy, there will still be losses in tax benefits when $\tau > 0$. However, such losses will be small.

Table 4: OPTIMAL LEVERAGE AND DEBT OVERHANG

The table reports the optimal coupon, initial interest coverage and market leverage, agency cost, and the 5-year conditional investment and default probabilities in the two states $(p_G^5(x_0), p_B^5(x_0))$. In the case "no macro", there are no business cycle variations in the cash flows, and the market Sharpe ratio is 0.35. In the benchmark case, $a(G) = 1.10$, $a(B) = 0.77$, $f^a(G) = f^a(B) = 0$, $g(G) = g(B) = 1$, and $f^g(G) = f^g(B) = 0.14$.

	coupon	coverage ratio	mkt leverage	agency cost	inv prob	def prob
		A1. Effects of macroeconomic risk: High PVGO				
no macro	0.92	1.09	0.60	2.19	53.37	6.51
G	0.45	2.43	0.45	2.07	85.10	0.64
B	0.33	2.31	0.40	1.82	86.96	0.40
		A2. Effects of macroeconomic risk: Low PVGO				
no macro	1.36	0.74	0.82	0.39	7.79	27.37
G	1.00	1.10	0.75	1.48	13.73	17.23
B	0.93	0.83	0.77	1.32	13.89	19.79
		B1. Cyclicality of assets in place: $a(G) = a(B) = 1$				
G	0.58	1.72	0.50	2.71	80.25	1.28
B	0.49	2.04	0.48	3.05	79.16	1.27
		B2. Cyclicality of growth option: $g(G) = a(G), g(B) = a(B)$				
G	0.52	2.12	0.51	4.05	70.55	1.72
B	0.57	1.36	0.57	5.15	60.98	4.01

state G) in order to avoid high costs of debt overhang, so much so that at optimal leverage the conditional probability of making the investment in the next 5 years is slightly higher than in state G.

Panel A2 of Table 4 shows the impact of macroeconomic risk on optimal leverage when the value of the growth option is only around 20% of the total market value on average rather than around 40% as in the benchmark scenario.[11] Agency costs of debt are smaller than in the benchmark model, and consequently the optimal coupon and market leverage are higher than in the benchmark model. Macroeconomic risk still has a meaningful impact on the optimal choice of coupon and leverage, albeit the difference is smaller than in Panel A1. When macroeconomic risk is taken into account, the optimal coupon decreases from 1.36 to 1.0 or 0.93 depending on the current macroeconomic

---

[11]The value of the growth option is reduced by doubling the size of the fixed cost of investment (from $\phi = 11$ to $\phi = 22$).

state being good or bad respectively.

Panel B1 of Table 4 illustrates how the impact of macroeconomic risk on optimal leverage depends on the cyclicality of assets in place. Compared to our benchmark scenario, the case $a(G) = a(B) = 1$ leads to less cyclical assets in place. Consistent with our earlier findings, less cyclical assets in place imply lower costs of debt overhang, allowing the firm to take on more leverage. When compared to our benchmark scenario, the optimal coupon indeed increases from 0.45 and 0.33 in the good and bad states to 0.58 and 0.49 when we have $a(G) = a(B) = 1$.

Panel B2 illustrates how the impact of macroeconomic risk on optimal leverage depends on the cyclicality of growth option. Compared to our benchmark scenario, the case $g(G) = a(G)$ and $g(B) = a(B)$ leads to a more cyclical growth option. As argued in the two-period model, the effect of more cyclical growth options on agency costs of debt is ambiguous. When compared to our benchmark scenario, the optimal coupon increases from 0.45 and 0.33 in the good and bad states respectively to 0.52 and 0.57 when we have $g(G) = a(G)$ and $g(B) = a(B)$. The rise in coupon is more dramatic in state B, which is because a more cyclical growth option lowers the transfer to debt holders in state B for a given leverage. At the same time, the agency costs of debt increase from 2.07 and 1.86 in the good and bad states respectively to 4.05 and 5.15. The agency cost in state B now exceeds that in state G due to the higher leverage.

6   Concluding Remarks

Using a dynamic model of capital structure with investment decisions and macroeconomic risk, we show that the agency cost of debt due to debt overhang increases substantially when macroeconomic risk is taken into account. For example, in our benchmark case, the debt overhang costs for a low leverage firm peak at 0.7% when macroeconomic risk is not taken into account, while these costs peak at 2.7% or 3.5% in booms and recessions respectively when macroeconomic risk is taken into account.

We also show that investment and capital structure decisions as well as debt overhang costs depend on the cyclicality of cash flows from assets in place and growth opportunities. More cyclical cash flows from assets in place make underinvestment more likely in bad times, exacerbating the costs of debt overhang when macroeconomic risk is taken into account. More cyclical cash flows from growth opportunities also make underinvestment more likely in bad times, but the overall

effect on the costs of debt overhang when macroeconomic risk is taken into account is ambiguous. Moreover, among the growth options that are not too profitable (so that debt is still risky), equity holders would prefer to invest in ones that have the same cyclicality as their assets in place. Finally, we also show that macroeconomic risk significantly impacts the optimal capital structure of the firm.

Several questions remain unanswered. For example, what is the effect of macroeconomic risk on different agency conflicts, such as asset substitution (Jensen and Meckling (1976)) or free cash-flow (Jensen (1986))? Because in bad times firms are usually closer to default, the asset substitution problem may be more prevalent in bad times. If this is indeed the case, asset substitution costs will be amplified by macroeconomic risk as well. On the other hand, the free cash flow problem may be more prevalent in good times, when there is more cash available to be diverted. If this is the case, free cash flow costs are reduced if macroeconomic risk is taken into account. We leave these questions to future research.

Appendix

A Value of Equity

After investment, for $x \in J_1$, the solution to the homogeneous equation in the ODE (20) is

$$E_G(x) = w_{1,1}^E x^{\alpha_1} + w_{1,2}^E x^{\alpha_2},$$

where

$$\alpha_1, \alpha_2 = -\sigma^{-2}(G) \left[\left(\widetilde{\mu}(G) - \frac{\sigma^2(G)}{2} \right) \pm \sqrt{\left(\widetilde{\mu}(G) - \frac{\sigma^2(G)}{2} \right)^2 + 2r(G)\sigma^2(G)} \right], \tag{52}$$

and it is easy to verify that the particular solution is $h_1^E(G)x + k_1^E(G)$, where

$$h_1^E(G) = \frac{(1-\tau)(a(G)+g(G))}{r(G)+\widetilde{\lambda}(G)-\widetilde{\mu}(G)}, \tag{53}$$

$$k_1^E(G) = \frac{(1-\tau)(f^a(G)+f^g(G)-c)}{r(G)+\widetilde{\lambda}(G)}. \tag{54}$$

For $x \in J_2$, the homogeneous equations from the ODE system (22a-22b) can be formulated as a quadratic eigenvalue problem (see Chen 2009 for details), and the solution is given by

$$E_s(x) = \sum_{j=1}^4 w_{2,j}^E \theta_j(s) x^{\beta_j},$$

where β_j and θ_j are the j-th eigenvalue and (part of the) eigenvector for the following standard eigenvalue problem:

$$\begin{bmatrix} 0 & \mathbf{I} \\ -2\boldsymbol{\Sigma}^{-1}\left(\widetilde{\boldsymbol{\Lambda}} - \mathbf{r}\right) & -\left(2\boldsymbol{\Sigma}^{-1}\widetilde{\mu} - \mathbf{I}\right) \end{bmatrix} \begin{bmatrix} \theta_j \\ \varphi_j \end{bmatrix} = \beta_j \begin{bmatrix} \theta_j \\ \varphi_j \end{bmatrix}, \tag{55}$$

where $\mathbf{I}$ is a 2×2 identity matrix, $\mathbf{r} = \mathbf{diag}\left([r(G), r(B)]'\right)$, $\widetilde{\mu} = \mathbf{diag}\left([\widetilde{\mu}(G), \widetilde{\mu}(B)]'\right)$, and $\boldsymbol{\Sigma} = \mathbf{diag}\left([\sigma^2(G), \sigma^2(B)]'\right)$. From Barlow, Rogers, and Williams (1980), we know that there are exactly 2 eigenvalues with negative real parts, and 2 with positive real parts.

Next, one can verify that the particular solutions will be in the form $\mathbf{h}_2^E x + \mathbf{k}_2^E$, where

$$\mathbf{h}_2^E = (1-\tau)\left(\mathbf{r} - \widetilde{\mu} - \widetilde{\boldsymbol{\Lambda}}\right)^{-1}(\mathbf{a}+\mathbf{g}), \tag{56}$$

$$\mathbf{k}_2^E = (1-\tau)\left(\mathbf{r} - \widetilde{\boldsymbol{\Lambda}}\right)^{-1}(\mathbf{f}^a + \mathbf{f}^g - c\mathbf{1}). \tag{57}$$

The coefficients $\left\{\mathbf{w}_1^E, \mathbf{w}_2^E\right\}$ are determined by the boundary conditions (24-29) for given default bound-

aries $x_D(G), x_D(B)$.

Before investment, for $x \in I_1$, we can solve for $e_G(x)$ the same way as for $E_G(x)$. The particular solution is $h_1^e(G)x + k_1^e(G)$, where

$$h_1^e(G) = \frac{(1-\tau)\, a(G)}{r(G) + \widetilde{\lambda}(G) - \widetilde{\mu}(G)}, \tag{58}$$

$$k_1^e(G) = \frac{(1-\tau)\,(f^a(G) - c)}{r(G) + \widetilde{\lambda}(G)}. \tag{59}$$

Similarly, for $x \in I_2$, the particular solution will be in the form $\mathbf{h}_2^e x + \mathbf{k}_2^e$, where

$$\mathbf{h}_2^e = (1-\tau)\left(\mathbf{r} - \widetilde{\mu} - \widetilde{\mathbf{\Lambda}}\right)^{-1} \mathbf{a}, \tag{60}$$

$$\mathbf{k}_2^e = (1-\tau)\left(\mathbf{r} - \widetilde{\mathbf{\Lambda}}\right)^{-1} (\mathbf{f}^a - c\mathbf{1}). \tag{61}$$

Finally, for $x \in I_3$, the solution to the homogeneous equation in ODE (32) is

$$e_B(x) = w_{3,1}^e x^{\gamma_1} + w_{3,2}^e x^{\gamma_2},$$

where

$$\gamma_1, \gamma_2 = -\sigma^{-2}(L)\left[\left(\widetilde{\mu}(B) - \frac{\sigma^2(B)}{2}\right) \pm \sqrt{\left(\widetilde{\mu}(B) - \frac{\sigma^2(B)}{2}\right)^2 + 2r(B)\sigma^2(B)}\right], \tag{62}$$

and we can verify that the particular solution is $h_3^e(B)x + k_3^e(B) + \sum_{j=1}^{4} \omega_{3,j}^e x^{\beta_j}$, where

$$h_3^e(B) = \frac{(1-\tau)\, a(B) + \widetilde{\lambda}(B)\, h_2^E(G)}{r(B) + \widetilde{\lambda}(B) - \widetilde{\mu}_B}, \tag{63}$$

$$k_3^e(B) = \frac{(1-\tau)\,(f_B^a - c) + \widetilde{\lambda}(B)\left(k_2^E(G) - \phi\right)}{r(B) + \widetilde{\lambda}(B)}, \tag{64}$$

$$\omega_j = \frac{\widetilde{\lambda}(B) w_{2,j}^E \theta_j(G)}{r(B) + \widetilde{\lambda}(B) - \widetilde{\mu}(B)\beta_k - \frac{1}{2}\sigma_B^2 \beta_k(\beta_k - 1)}. \tag{65}$$

The coefficients $\{\mathbf{w}_1^e, \mathbf{w}_2^e\}$ are determined by the boundary conditions (34-41) for given default and investment boundaries $\{x_d(G), x_d(B), x_u(G), x_u(B)\}$, which leads to a system of linear equations that can be solved in closed form.

B Value of Debt and the Firm under First Best

B.1 First Best Firm Value

Since we assume that the recovery value of debt is a fraction of the first best firm value, let us summarize the solution of the first-best value of the firm, which can be obtained the same way as equity while forcing $c = 0$.

After investment, the value of the firm under the first best is $V_s^{AE}(x)$,

$$V_s^{AE}(x) = h^V(s) x + k^V(s),$$ (66)

where

$$\mathbf{h}^V = (1-\tau)\left(\mathbf{r} - \widetilde{\mu} - \widetilde{\mathbf{\Lambda}}\right)^{-1}(\mathbf{a} + \mathbf{g})$$ (67a)

$$\mathbf{k}^V = (1-\tau)\left(\mathbf{r} - \widetilde{\mathbf{\Lambda}}\right)^{-1}(\mathbf{f}^a + \mathbf{f}^g)$$ (67b)

Before investment, in Region $I_1 = [0, x_u^*(G))$,

$$v_s^{AE}(x) = \sum_{j=1}^{4} w_{1,j}^v \theta_j(s) x^{\beta_j} + h_1^v(s) x + k_1^v(s),$$ (68)

where θ and β are given in Proposition 1,

$$\mathbf{h}_1^v = (1-\tau)(\mathbf{r} - \mu - \mathbf{\Lambda})^{-1}\mathbf{a}$$ (69a)

$$\mathbf{k}_1^v = (1-\tau)(\mathbf{r} - \mathbf{\Lambda})^{-1}\mathbf{f}^a$$ (69b)

and the boundary condition at $x = 0$ implies that $w_{1,1}^v = w_{1,2}^v = 0$.

In Region $I_2 = [x_u^*(G), x_u^*(B)]$, the firm would have already made the investment in state G, so that

$$v_G^{AE}(x) = V_G^{AE}(x) - \phi.$$ (70)

In state B,

$$v_B^{AE}(x) = w_{2,1}^v x^{\gamma_1} + w_{2,2}^v x^{\gamma_2} + h_2^v(B) x + k_2^v(B),$$ (71)

where

$$h_2^v(B) = \frac{(1-\tau)\,a\,(B) + \widetilde{\lambda}\,(B)\,H^V\,(G)}{r\,(B) + \widetilde{\lambda}\,(B) - \widetilde{\mu}\,(B)}, \tag{72a}$$

$$k_2^v(B) = \frac{(1-\tau)\,f^a\,(B) + \widetilde{\lambda}\,(B)\,\left(K^V\,(G) - \phi\right)}{r\,(B) + \widetilde{\lambda}\,(B)}. \tag{72b}$$

Finally, the optimal investment boundaries $x_u^*(G)$ and $x_u^*(B)$ are solutions to a pair of smooth-pasting conditions similar to equations (46-47).

B.2   Debt

B.2.1   Debt Value After Investment

For $x \in J_1 = [x_D\,(G), x_D\,(B))$, the firm has not defaulted yet in state G, but has already defaulted in state B. Thus,

$$D_B(x) = \alpha\,(B)\,V_B^{AE}\,(x), \tag{73}$$

and $D_G\,(x)$ satisfies

$$r\,(G)\,D_G = c + \widetilde{\mu}\,(G)\,xD_G' + \frac{1}{2}\sigma^2\,(G)\,x^2 D_G'' + \widetilde{\lambda}\,(G)\,(D_B - D_G). \tag{74}$$

The solution is

$$D_G\,(x) = w_{1,1}^D x^{\alpha_1} + w_{1,2}^D x^{\alpha_2} + h_1^D\,(G)\,x + k_1^D\,(G), \tag{75}$$

with

$$h_1^D\,(G) = \frac{\widetilde{\lambda}\,(G)\,\alpha\,(B)\,H_1^V\,(B)}{r\,(G) + \widetilde{\lambda}\,(G) - \widetilde{\mu}\,(G)} \tag{76a}$$

$$k_1^D\,(G) = \frac{c + \widetilde{\lambda}\,(G)\,\alpha\,(B)\,K_1^V\,(B)}{r\,(G) + \widetilde{\lambda}\,(G)} \tag{76b}$$

Next, for $x \in J_2 = [x_D\,(B), +\infty)$, the firm is not in default yet in either state, hence

$$r\,(G)\,D_G = c + \widetilde{\mu}\,(G)\,xD_G' + \frac{1}{2}\sigma^2\,(G)\,x^2 D_G'' + \widetilde{\lambda}\,(G)\,(D_B - D_G), \tag{77}$$

$$r\,(B)\,D_B = c + \widetilde{\mu}\,(B)\,xD_B' + \frac{1}{2}\sigma^2\,(B)\,x^2 D_B'' + \widetilde{\lambda}\,(B)\,(D_G - D_B). \tag{78}$$

The solutions are:

$$D_s\,(x) = \sum_{j=1}^{4} w_{2,j}^D \theta_j\,(s)\,x^{\beta_j} + h_2^D\,(s)\,x + k_2^D\,(s), \tag{79}$$

where $\mathbf{h}_2^D$ and $\mathbf{k}_2^D$ are given by

$$\mathbf{h}_2^D = 0 \tag{80a}$$

$$\mathbf{k}_2^D = \left(\mathbf{r} - \widetilde{\boldsymbol{\Lambda}}\right)^{-1} c\mathbf{1} \tag{80b}$$

The values of the coefficients $\mathbf{w}^D$ are determined by the following boundary conditions. First, there are the value-matching at default:

$$\lim_{x \downarrow x_D(G)} D_G(x) = \alpha(G) V_G^{AE}(x_D(G)), \tag{81a}$$

$$\lim_{x \downarrow x_D(B)} D_B(x) = \alpha(B) V_B^{AE}(x_D(B)). \tag{81b}$$

Next, $D_G(x)$ needs to be piecewise $\mathcal{C}^2$, which implies

$$\lim_{x \uparrow x_D(B)} D_G(x) = \lim_{x \downarrow x_D(B)} D_G(x) \tag{82}$$

$$\lim_{x \uparrow x_D(B)} D_G'(x) = \lim_{x \downarrow x_D(B)} D_G'(x) \tag{83}$$

Finally, to rule out bubbles, we have

$$\lim_{x \uparrow +\infty} \frac{D_G(x)}{x} < \infty,$$

$$\lim_{x \uparrow +\infty} \frac{D_B(x)}{x} < \infty,$$

which imply:

$$w_{2,3}^D = w_{2,4}^D = 0. \tag{84}$$

The remaining unknowns are $\left\{w_{1,1}^D, w_{1,2}^D, w_{2,1}^D, w_{2,2}^D\right\}$, which can solved via a system of linear equations implied by the boundary conditions above.

B.2.2 Debt Value Before Investment

We will focus our analysis on the case where $x_d(G) < x_d(B) < x_u^*(G) < x_u^*(B) < x_u(G) < x_u(B)$. Cases with different orderings of the default and investment thresholds can be solved similarly.

In region $I_1 = [x_d(G), x_d(B))$, the firm has already defaulted in state B. Thus,

$$d_B(x) = \alpha(B) v_B^{AE}(x). \tag{85}$$

In state G, $d_G(x)$ satisfies

$$r(G) d_G = c + \widetilde{\mu}(G) x d_G' + \frac{1}{2} \sigma^2(G) x^2 d_G'' + \widetilde{\lambda}(G) \left(\alpha(B) v_B^{AE} - d_G \right). \tag{86}$$

Due to the nonlinear term introduced by $v_B^{AE}(x)$, the solution will also have a slightly different form:

$$d_G(x) = w_{1,1}^d x^{\alpha_1} + w_{1,2}^d x^{\alpha_2} + h_1^d(G) x + k_1^d(G) + \sum_{j=1}^4 \omega_{1,j}^d x^{\beta_j} \tag{87}$$

where

$$h_1^d(G) = \frac{\widetilde{\lambda}_G \alpha(B) h_1^v(B)}{r_G + \widetilde{\lambda}_G - \widetilde{\mu}_G}, \tag{88a}$$

$$k_1^d(G) = \frac{c + \widetilde{\lambda}_G \alpha(B) k_1^v(B)}{r_G + \widetilde{\lambda}_G}, \tag{88b}$$

$$\omega_{1,j}^d = \frac{\widetilde{\lambda}_G \alpha(B) w_{1,j}^v \theta_j(B)}{r_G + \widetilde{\lambda}_G - \widetilde{\mu}_G \beta_j - \frac{1}{2} \sigma_G^2 \beta_j (\beta_j - 1)}. \tag{88c}$$

The last equality follows from:

$$r(G) \omega_{1,j}^d = \widetilde{\mu}(G) \omega_{1,j}^d \beta_j + \frac{1}{2} \sigma^2(G) \omega_{1,j}^d \beta_j (\beta_j - 1) + \widetilde{\lambda}(G) \left(\alpha(B) w_{1,j}^v \theta_j(B) - \omega_{1,j}^d \right).$$

In region $I_2 = [x_d(B), x_u(G))$, the solutions are similar to the case of affine contingent claims:

$$d_s(x) = \sum_{j=1}^4 w_{2,j}^d \theta_j(s) x^{\beta_j} + h_2^d(s) x + k_2^d(s), \tag{89}$$

where

$$\mathbf{h}_2^d = \mathbf{0}, \tag{90a}$$

$$\mathbf{k}_2^d = \left(\mathbf{r} - \widetilde{\mathbf{\Lambda}} \right)^{-1} c \mathbf{1}. \tag{90b}$$

In region $I_3 = [x_u(G), x_u(B)]$, the firm will have already made the investment in state G. Thus,

$$d_G(x) = D_G(x). \tag{91}$$

In state B, $d_B(x)$ satisfies:

$$r(B) d_B = c + \widetilde{\mu}(B) x d_B' + \frac{1}{2} \sigma^2(B) x^2 d_B'' + \widetilde{\lambda}(B) (D_G - d_B). \tag{92}$$

The solution is

$$d_B(x) = w_{3,1}^d x^{\gamma_1} + w_{3,2}^d x^{\gamma_2} + h_3^d(B) x + k_3^d(B) + \sum_{j=1}^{4} \omega_{3,j}^d x^{\beta_j}, \tag{93}$$

where

$$h_3^d(B) = \frac{\widetilde{\lambda}_B h_2^D(G)}{r_B + \widetilde{\lambda}_B - \widetilde{\mu}_B}, \tag{94a}$$

$$k_3^d(B) = \frac{c + \widetilde{\lambda}_B k_2^D(G)}{r_B + \widetilde{\lambda}_B}, \tag{94b}$$

$$\omega_{3,j}^d = \frac{\widetilde{\lambda}_B w_{2,j}^D \theta_j(G)}{r_B + \widetilde{\lambda}_B - \widetilde{\mu}_B \beta_j - \frac{1}{2}\sigma_B^2 \beta_j(\beta_j - 1)}. \tag{94c}$$

The last equality follows from:

$$r(B)\omega_{3,j}^d = \widetilde{\mu}(B)\omega_{3,j}^d \beta_j + \frac{1}{2}\sigma^2(B)\omega_{3,j}^d \beta_j(\beta_j - 1) + \widetilde{\lambda}(B)\left(w_{2,j}^D \theta_j(G) - \omega_{3,j}^d\right).$$

Again, the values of the coefficients $\mathbf{w}^d$ are determined by a set of boundary conditions.

- Value-matching conditions at default threshold:

$$\lim_{x \downarrow x_d(G)} d_G(x) = \alpha(G) v_G^{AE}(x_d(G)), \tag{95a}$$

$$\lim_{x \downarrow x_d(B)} d_B(x) = \alpha(B) v_B^{AE}(x_d(B)). \tag{95b}$$

- Smoothness of $d_G(x)$:

$$\lim_{x \uparrow x_d(B)} d_G(x) = \lim_{x \downarrow x_d(B)} d_G(x) \tag{96}$$

$$\lim_{x \uparrow x_d(B)} d_G'(x) = \lim_{x \downarrow x_d(B)} d_G'(x) \tag{97}$$

- Value-matching conditions at investment threshold:

$$\lim_{x \uparrow x_u(G)} d_G(x) = \lim_{x \downarrow x_u(G)} D_G(x) \tag{98}$$

$$\lim_{x \uparrow x_u(B)} d_B(x) = \lim_{x \downarrow x_u(B)} D_B(x) \tag{99}$$

40

- Smoothness of $d_B(x)$:

$$\lim_{x \uparrow x_u(G)} d_B(x) = \lim_{x \downarrow x_u(G)} d_B(x) \tag{100}$$

$$\lim_{x \uparrow x_u(G)} d'_B(x) = \lim_{x \downarrow x_u(G)} d'_B(x) \tag{101}$$

These conditions translate into a system of linear equations for $\mathbf{w}^d$ which is solved in closed form.

References

Almeida, H., and T. Philippon, 2007, The Risk-Adjusted Cost of Financial Distress, *Journal of Finance* 62, 2557–2586.

Arnold, M., A. Wagner, and R. Westermann, 2010, Macroeconomic Conditions, Growth Opportunities and the Cross-Section of Credit Risk, Working Paper, Swiss Finance Institute.

Bansal, R., and A. Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, *Journal of Finance* 59, 1481 – 1509.

Barlow, M., L. Rogers, and D. Williams, 1980, *Wiener-Hopf Factorization for Matrices, in Séminaire de Probabilités XIV, Lecture Notes in Math. 784* Springer-Verlag, Berlin pp. 324–331.

Bhamra, H., L. Kuehn, and I. Strebulaev, 2009, The Levered Equity Risk Premium and Credit Spreads: A Unified Framework, *Review of Financial Studies,* forthcoming.

Chen, H., 2009, Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure, *Journal of Finance,* forthcoming.

Chen, L., P. Collin-Dufresne, and R. Goldstein, 2009, On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle, *Review of Financial Studies* 22, 3367–3409.

Childs, P., D. Mauer, and S. Ott, 2005, Interactions of Corporate Financing and Investment Decisions: The Effects of Agency Conflicts, *Journal of Financial Economics* 76, 667–690.

Décamps, J., and A. Faure-Grimaud, 2002, Excessive Continuation and Dynamic Agency Cost of Debt, *European Economic Review* 46, 1623–1644.

Diamond, D., and R. Rajan, 2010, Fear of Fire Sales and the Credit Freeze, Working paper, University of Chicago.

Dixit, A., 1989, Entry and Exit Decisions Under Uncertainty, *Journal of Political Economy* 97, 620–638.

Dixit, A., and R. Pindyck, 1994, *Investment Under Uncertainty.* (Princeton University Press Princeton).

Guo, X., J. Miao, and E. Morellec, 2005, Irreversible Investment with Regime Shifts, *Journal of Economic Theory* 122, 37–59.

Hackbarth, D., and D. Mauer, 2009, Optimal Priority Structure, Capital Structure, and Investment, Working paper, Univesity of Illinois at Urbana-Champaign.

Hackbarth, D., J. Miao, and E. Morellec, 2006, Capital Structure, Credit Risk, and Macroeconomic Conditions, *Journal of Financial Economics* 82, 519–550.

Hennessy, C., 2004, Tobin's Q, Debt Overhang, and Investment, *Journal of Finance* 59, 1717–1742.

Jensen, M., 1986, Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers, *American Economic Review* 76, 323–329.

Jensen, M., and W. Meckling, 1976, Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure, *Journal of Financial Economics* 3, 305–360.

Ju, N., and H. Ou-Yang, 2006, Asset Substitution and Underinvestment: A Dynamic View, Working paper, Duke University.

Lamont, O., 1995, Corporate-Debt Overhang and Macroeconomic Expectations, *American Economic Review* 85, 1106–1117.

Leland, H., 1998, Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* 53, 1213–1243.

Lobanov, S., and I. Strebulaev, 2007, A Theory of Dynamic Financing and Investment, Working paper, Stanford GSB.

Manso, G., 2008, Investment Reversibility and Agency Cost of Debt, *Econometrica* 76, 437–442.

Mauer, D., and S. Ott, 2000, Agency Costs, Underinvestment, and Optimal Capital Structure: The Effect of Growth Options to Expand, in M. Brennan, and L. Trigeorgis, eds.: *Project Flexibility, Agency, and Competition* (Oxford University Press, Oxford).

Mauer, D., and A. Triantis, 1994, Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework, *Journal of Finance* 49, 1253–1277.

McDonald, R., and D. Siegel, 1986, The value of waiting to invest, *Quarterly Journal of Economics* 101, 707–728.

Mello, A., and J. Parsons, 1992, Measuring the Agency Cost of Debt, *Journal of Finance* 47, 1887–1904.

Morellec, E., and N. Schuerhoff, 2010, Dynamic Investment and Financing Under Personal Taxation, *Review of Financial Studies* 23, 101–146.

Moyen, N., 2007, How Big is the Debt Overhang Problem?, *Journal of Economic Dynamics and Control* 31, 433–472.

Myers, S., 1977, Determinants of Corporate Borrowings, *Journal of Financial Economics* 5, 147–175.

Sundaresan, S., and N. Wang, 2007, Dynamic Investment, Capital Structure, and Debt Overhang, Working paper, Columbia University.

Titman, S., S. Tompaidis, and S. Tsyplakov, 2004, Market Imperfections, Investment Flexibility, and Default Spreads, *Journal of Finance* 59, 165–205.