

**Internet Appendix for**  
*“Entrepreneurial Finance and Non-diversifiable Risk”*

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**Abstract**

This internet appendix contains supplemental materials for the published article. It is organized as follows. Section A considers the effect of borrowing constraints on the entrepreneur’s decisions. Section B models investment as a real option instead of a take-it-or-leave-it project. Section C examines the quantitative impact of under-diversified investors on debt pricing.

## A. Borrowing Constraints

In the paper, we do not model endogenous financial constraints. One way to capture the effects of financial constraints is to directly postulate some exogenously specified form of financing constraints. Consider the financing constraint that the total amount of corporate debt that the entrepreneur borrows cannot exceed an upper bound  $\bar{D}$ . Clearly, when the unconstrained solution is higher than  $\bar{D}$ , the constraint binds. In this case, the entrepreneur will choose different coupon and default/cash-out strategy than the unconstrained case. The lender prices debt in a competitive market. We summarize our results in Table IA-I.

The table shows that the borrowing constraint affects the entrepreneur's decisions in several ways. First, as the borrowing constraint tightens ( $\bar{D}$  drops), the optimal ownership becomes smaller, although the change is small. Both the value of private equity and the value of public equity become higher, but the total value of the private firm drops as the entrepreneur moves away from the optimal debt level. Second, the corresponding coupon and the private leverage both become smaller. Third, the default boundary drops, which is mainly driven by the borrowing constraint and lower coupon. However, the cash-out boundary (and cash-out probability) is not monotonic in the borrowing constraint. The 10-year cash-out probability is first decreasing and then increasing with the borrowing constraint. When the entrepreneur is limited in his ability to borrow, he seeks to diversify idiosyncratic risks through external equity, default, and cash-out. In our setting, external equity is initially the main channel of diversification. As the agency costs of external equity become higher, the entrepreneur becomes more willing to diversify via cash-out, which explains the rise in cash-out probability.

## B. Investment Option

In this section, we consider a setting where we replace the cash-out option at the upper boundary with a real option to increase the capital stock to a higher level. The specific setup we consider

Table IA-I: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: BORROWING CONSTRAINTS

This table reports the results for the setting where the entrepreneur is limited in its ability to borrow. We report the results for the unconstrained case, as well as the case when the total debt limit is 10, 5, and 2. The rest of the parameters are the same as in Table 5 of the paper.

public debt	ownership	coupon	public equity	private equity	private firm	private leverage (%)	default prob (%)	cash-out prob (%)
$F_0$	$\psi$	$b$	$(1 - \psi)E_0$	$\psi G_0$	$S_0$	$L_0$	$p_d(10)$	$p_u(10)$
unconstrained case								
11.49	0.67	0.43	10.22	11.52	33.22	34.6	0.6	8.9
constrained case								
10.00	0.67	0.36	10.76	12.45	33.21	30.1	0.3	8.7
5.00	0.66	0.17	12.36	15.64	33.02	15.2	0.0	8.9
2.00	0.66	0.07	13.17	17.59	32.84	6.3	0.0	9.4

is as follows. Starting with the setup that allows for both default and cash-out, we introduce an investment option that expands the size of total revenue flow of the firm from  $y_t$  to  $ay_t$  at a one-time cost  $I_p$ . At the time when the entrepreneur chooses to invest, we assume that she first recalls all the pre-existing debt at face value, and then optimally chooses the amount of new debt to issue. We focus on the cases where the investment boundary is below the cash-out boundary, which is achieved by setting the cash-out cost  $K$  sufficiently high.

We solve this problem in three steps. First, at the investment boundary  $y_i$ , the entrepreneur faces a problem essentially identical to the original problem without investment option, except that the initial revenue flow is  $ay_i$  instead of  $y_0$ , and that at the time of cash-out, the capital gain tax is based on total investment of  $I + I_p$ . We compute the total private value of the firm post investment  $S_p(y)$  for different values of  $y$ . Second, after initial investment but before the new round of investment is made, the entrepreneur faces a similar problem, and makes decisions on consumption, portfolio allocation, as well as the timing of default and new investment. Replacing the conditions (16c-d) in Theorem 1 of the paper are two new value-matching and smooth-pasting

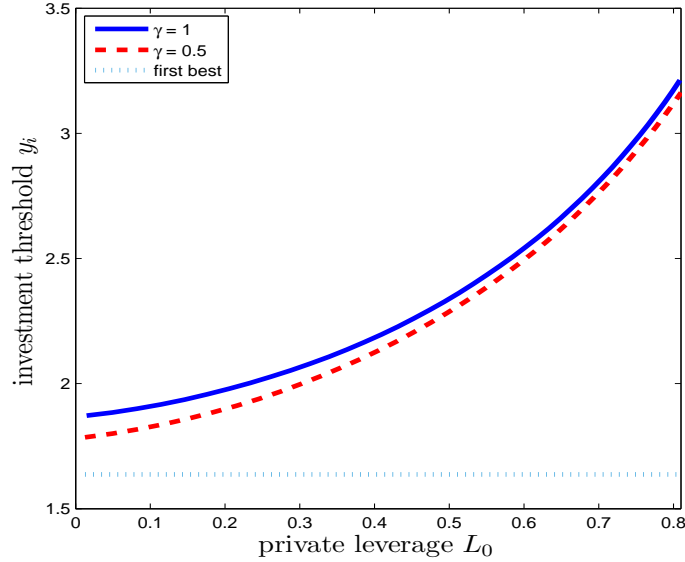


Figure IA-1: **Entrepreneurial risk aversion, private leverage, and investment threshold.**

This figure plots the optimal investment threshold as function of the private leverage  $L_0$  for entrepreneurs with different risk aversion. The dotted line gives the first best investment threshold for an all-equity public firm.

conditions at the investment boundary  $y_i$ , which depend on the value and first derivative of the post-investment firm value  $S_p(y)$  computed in the previous step. Finally, we can search for the optimal coupon  $b$  at  $t = 0$  to maximize the initial value of the firm.

The results are plotted in Figure IA-1. We assume that  $a = 2$ ,  $I_p = 20$ , and  $K = 100$ . Rather than focusing on the case with optimal coupon, we plot the optimal investment boundary as a function of the initial private leverage  $L_0$  (corresponding to different initial coupon  $b$ ). The solid line is for the case where the entrepreneur has risk aversion  $\gamma = 1$ , while the dash line is for  $\gamma = 0.5$ . As a benchmark for comparison, we also plot the first best investment boundary (for an all-equity public firm), specified by the dotted line.

Figure IA-1 shows that lack of diversification leads to underinvestment. Even when there is no debt, the investment boundary can be significantly higher for the entrepreneurial firm than for

the public firm, and the boundary increases with the entrepreneurial risk aversion. This result is consistent with the finding that the breakeven investment cost decreases with the entrepreneurial risk aversion, both suggesting that the more risk-averse the entrepreneur gets, the more reluctant she is to make investments that increase her idiosyncratic risk exposure.

The investment boundary is also rising with private leverage, which is mainly due to the standard debt overhang problem (Myers (1977)). Interestingly, the difference between the investment boundaries for the two levels of risk aversion shrinks as leverage becomes higher. This is because more risky debt helps the entrepreneur reduce her idiosyncratic risk exposure, thus making her more willing to invest. Thus, on the one hand, higher leverage makes the standard debt overhang problem more severe. On the other hand, it helps alleviate the underinvestment problem due to lack of diversification.

### C. Private Value of Debt

In our model, external investors (e.g. the lender for the risky debt) are assumed to be fully diversified and hence only demand a risk premium for the systematic component of the risk. We now consider the case where debt is held by under-diversified investors. The qualitative implication of under-diversified lenders for the optimal leverage is clear. Since debt would be priced at a lower value due to idiosyncratic risk, the diversification benefit of risky debt drops. As a result, the entrepreneur will issue less debt. Thus, we focus on the quantitative impact of under-diversified investors on debt pricing. Specifically, we consider two scenarios.

First, we consider the case where the lender is also risk averse and has the same exponential utility function but with a potentially different risk aversion from the entrepreneur's. The under-diversified lender solves a similar optimization and certainty equivalent valuation problem as the entrepreneur does. The lender's certainty equivalent value of the risky debt  $\widehat{D}(y)$  satisfies the

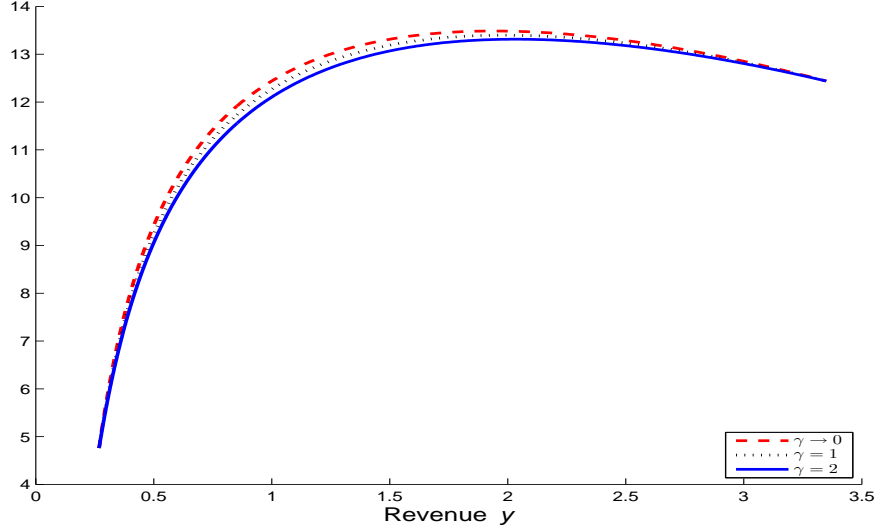


Figure IA-2: **Debt valuation for under-diversified lenders.**

following differential equation:

$$r\widehat{D}(y) = b + (\mu - \omega\eta)y\widehat{D}'(y) + \frac{\sigma^2 y^2}{2}\widehat{D}''(y) - \frac{\gamma r \epsilon^2 y^2}{2}\widehat{D}'(y)^2, \quad (1)$$

subject to the same boundary conditions at default and cash-out as the public debt:  $\widehat{D}(y_d) = \alpha A(y_d)$  and  $\widehat{D}(y_u) = F_0$  (see Appendix C of the paper). We consider three levels of risk aversion for the lender,  $\gamma = 0, 1, 2$ . In each case, we hold the coupon and the entrepreneur's default/cash-out strategy  $(b, y_d, y_u)$  as given, which are solutions from the version of the model in Section 5, with entrepreneur's risk aversion  $\gamma = 1$  (see Table 3, Panel B in the paper).

The more risk averse the lender is, the lower his subjective debt value  $\widehat{D}(y)$  is. However, as shown in Figure IA-2, the quantitative effects of the lender's risk aversion  $\gamma$  on  $\widehat{D}(y)$  are small. The reason for the small effect is that the idiosyncratic risk exposure to the lender is limited as he does not receive the business income from the firm, and that debt income is less risky than equity. This suggests that our baseline calculation where the lender is diversified is a reasonable approximation for under-diversified lenders as well.

Second, we compute the private value the entrepreneur would assign to the debt if he holds both the inside equity and the debt component. Similarly to the preceding analysis, we hold the coupon and the entrepreneur's default strategy to be the same. The entrepreneur's subjective valuation of debt is given by

$$\tilde{D}(y) = H(y) - G(y), \quad (2)$$

where the private value of equity  $G(y)$  is given by (15) in the paper, and the private value of firm  $H(y)$  satisfies

$$rH(y) = (1 - \tau_e)(y - b) + b + (\mu - \omega\eta)yH'(y) + \frac{\sigma^2 y^2}{2}H''(y) - \frac{\gamma r \epsilon^2 y^2}{2}H'(y)^2. \quad (3)$$

The note on the next page provides other details of the solution (e.g. boundary conditions) and sketches out the derivation for equation (3).

Figure IA-3 shows that the subjective debt value for the entrepreneur  $\tilde{D}(y)$  is lower than  $\hat{D}(y)$ , the subjective debt value held by outside under-diversified investors, which in turn is lower than the public value of debt  $D(y)$ . Note that quantitatively, the gap between the entrepreneur's subjective debt valuation and the lender's subjective debt valuation is significant, while the difference between the subjective debt value for the lender and the public value of debt is much smaller. This again suggests that even when the lenders are under-diversified, issuing risky debt still provides significant diversification benefits for the entrepreneur. Intuitively, the entrepreneur is already under-diversified with his inside equity, which makes him demand a significantly higher idiosyncratic risk premium for holding the debt. The entrepreneur's subjective valuation of debt captures this effect.

### C.1 Derivation for the Entrepreneur's Subjective Valuation of Debt

The entrepreneur's private value of an asset depends on his wealth and other sources of income. To determine his private valuation of the debt issued by the entrepreneurial firm, we assume that

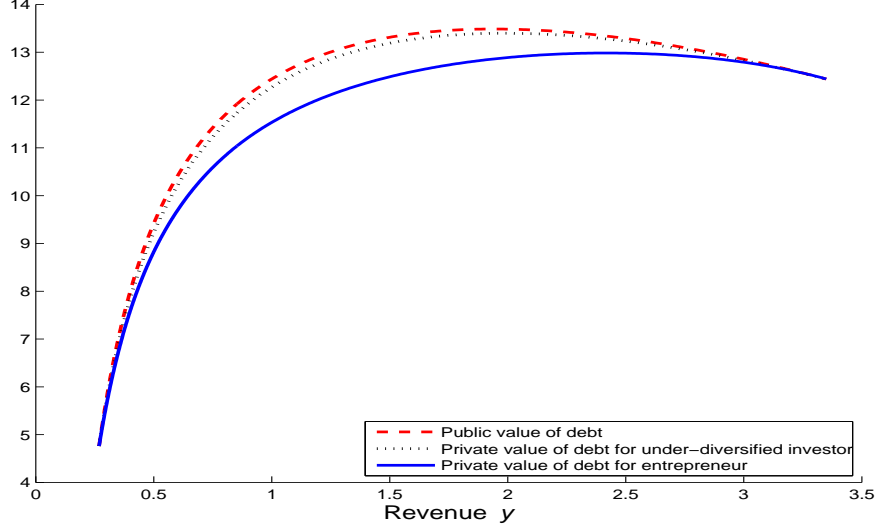


Figure IA-3: Debt valuation for the entrepreneur.

the entrepreneur takes the default and cash-out decisions as given, and that his income from the firm remains the same. His wealth process is given by:

$$dx_t = (r(x_t - \phi_t) + (1 - \tau_e)(y - b) + b - c_t) dt + \phi_t (\mu_p dt + \sigma_p dB_t), \quad 0 < t < \min(T_d, T_u). \quad (4)$$

The entrepreneur's value function  $J^b(x, y)$  satisfies the following HJB equation:

$$\begin{aligned} \delta J^b(x, y) = \max_{c, \phi} & u(c) + (rx + \phi(\mu_p - r) + (1 - \tau_e)(y - b) + b - c) J_x^b(x, y) \\ & + \mu y J_y^b(x, y) + \frac{(\sigma_p \phi)^2}{2} J_{xx}^b(x, y) + \frac{\sigma^2 y^2}{2} J_{yy}^b(x, y) + \phi \sigma_p \omega y J_{xy}^b(x, y). \end{aligned} \quad (5)$$

The first-order conditions (FOCs) for consumption  $c$  and portfolio allocation  $\phi$  are as follows:

$$u'(c) = J_x^b(x, y), \quad (6)$$

$$\phi = \frac{-J_x^b(x, y)}{J_{xx}^b(x, y)} \left( \frac{\mu_p - r}{\sigma_p^2} \right) + \frac{-J_{xy}^b(x, y)}{J_{xx}^b(x, y)} \frac{\omega y}{\sigma_p}. \quad (7)$$



We conjecture that the value function takes the following exponential form:

$$J^b(x, y) = -\frac{1}{\gamma r} \exp \left[ -\gamma r \left( x + H(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right) \right],$$

where  $H(y)$  satisfies the differential equation (3). The boundary conditions are:

$$H(y_d) = \alpha A(y_d), \tag{8}$$

$$H(y_u) = V^*(y_u) - K - \tau_g (V^*(y_u) - K - I). \tag{9}$$

Finally, the entrepreneur's private value of debt  $\tilde{D}(y)$  is given by (2), which is obtained by taking the difference between  $H(y)$  and the private value of equity  $G(y)$  given in the text.