Quantifying Liquidity and Default Risks of Corporate Bonds over the Business Cycle*

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Abstract

We develop a structural credit risk model to examine how the interactions between default and liquidity affect corporate bond pricing. The model features debt rollover and bond-price dependent holding costs for illiquid corporate bonds. Both over the business cycle and in the cross section (across ratings), our model does a good job matching the average default rates and credit spreads in the data, and it captures important variations in bid-ask spreads and bond-CDS spreads. A structural decomposition reveals that the default-liquidity interactions can account for 10% to 24% of the level of credit spreads and 16% to 46% of the changes in spreads over the business cycle. We also apply our framework to evaluate the impact of liquidity frictions on the aggregate costs of corporate bond financing and the impact of liquidity-provision policies for the bond market.

Keywords: Liquidity-Default Feedback, Rollover Risk, Over-The-Counter Markets, Endogenous Default

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1. Introduction

This paper presents a tractable credit risk model that captures the interactions between default risks and liquidity frictions, and examines their effects on corporate bond pricing. We introduce secondary market search frictions together with business-cycle fluctuations in firm fundamentals and risk premia into a model of endogenous defaults. Besides providing a good fit of the default rates and credit spreads across different ratings, the model explains two general empirical patterns for the liquidity components of corporate bonds: (1) corporate bonds with higher credit ratings tend to be more liquid; (2) corporate bonds are less liquid during economic downturns, especially for riskier bonds.[1]

In the model, firms generate exogenous cash flows, and equity-holders optimally choose the timing of default. Investors face uninsurable idiosyncratic liquidity shocks, which impose holding costs on their corporate bond investments. These holding costs rise as bond prices fall (when firms get closer to default), which could reflect the shadow costs of bond-collateralized financing. Bid-ask spreads arise endogenously through the bargaining between investors and dealers in the OTC bond market. On the one hand, higher default risk raises the holding costs and thus the liquidity discount of corporate bonds. On the other hand, larger liquidity discounts make it more costly for firms to roll over their maturing debt, hence raising default risk. Thus, a *default-liquidity spiral* arises: when secondary market liquidity deteriorates, equity holders are more likely to default, which in turn worsens secondary bond market liquidity even further, and so on. This spiral is further amplified by the business-cycle fluctuations in fundamental cash-flow risks and liquidity frictions.

For calibration, we first pick the pricing kernel parameters to fit standard asset pricing moments. Firms have identical cash flow processes but differ in leverage, and the cash flow parameters are calibrated to the empirical moments of corporate profits, with the exception of the idiosyncratic volatility of cash flows, which is calibrated to match the average default rates. A part of the parameters governing secondary bond market liquidity are pre-fixed

[1] See e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhütter, and Lando (2012), and Friewald, Jankowitsch, and Subrahmanyam (2012).

based on the literature, anecdotal evidence, and moments of bond market turnover. The remaining parameters (3 parameters characterizing the holding costs) are calibrated to match the average bid-ask spreads across three rating classes and two aggregate states (6 moments in total). We then evaluate the model's performance by computing the model-implied average default probabilities, credit spreads, bid-ask spreads, and bond-CDS spreads across rating classes and the business cycle. Since these moments are nonlinear functions of firms' leverage, we integrate the firm-level moments over the empirical market leverage distribution within each rating class to capture the convexity effects.

The model provides a good fit for the average default rates and total credit spreads for bonds with 10-year maturity across four rating classes (Aaa/Aa, A, Baa, Ba). It also fits the bid-ask spreads and bond-CDS spreads reasonably well. Over business cycles, the model-implied variations in credit spreads and bid-ask spreads are also consistent with the data. The link between bond liquidity and firm's default risk, as generated by the price-dependent holding costs, is crucial for our model's ability to match the cross-sectional and business-cycle patterns for bond pricing. In contrast, the credit spreads, bid-ask spreads, and bond-CDS spreads (especially the latter two) show significantly less variation across firms and over time when we make the holding costs only depend on the aggregate state. Moreover, through comparative statics on the liquidity parameters, we show that bid-ask spreads and bond-CDS spreads capture very different aspects of bond illiquidity.

It is common practice in the empirical literature to decompose credit spreads into a liquidity and a default component, with the interpretation that these components are additively separable. In contrast, our model suggests that liquidity and default are inextricably linked. Such dynamic interactions are not easy to capture using reduced-form models (see, e.g., Duffie and Singleton (1999) and Liu, Longstaff, and Mandell (2006)) with exogenously imposed default and liquidity risk components. Our model enables us to perform a structural decomposition of credit spreads that quantifies these interactions.

First, we identify the default component in the credit spreads of a corporate bond by pricing the same bond in a hypothetical perfectly liquid market, while using the default thresholds that are optimal with liquidity frictions. The residual is then the liquidity component. Second, we decompose the default component into a pure default and liquidity-driven default component: The pure default component is the spread in a hypothetical setting with a perfectly liquid market and equity holders' re-optimized default decision (i.e., the default boundary implied by Leland (1994)), and the residual is the liquidity-driven default component. Third, we decompose the liquidity component into a pure liquidity and default-driven liquidity component: The pure liquidity component is the spread for default-free bonds when there are over-the-counter search frictions as in Duffie, Gârleanu, and Pedersen (2005), and the residual is the default-driven liquidity component. The two interaction terms, the liquidity-driven default and the default-driven liquidity component, capture the endogenous positive spiral between default and liquidity as discussed earlier. We also provide an analogous dollar-based decomposition.

Cross-sectionally, the two interaction terms account for 10% to 11% of the total credit spread of Aaa/Aa rated bonds and 17% to 24% of the total spread of Ba rated bonds across the two aggregate states. We also present a time-series default-liquidity decomposition using quarterly market leverage distributions and NBER-dated expansions and recessions from 1994 to 2012. These results demonstrate the relative importance of the four components for the time variation of credit spreads after taking into account the dynamics of macroeconomic conditions and leverage distributions. For example, the default-driven liquidity component is as large as the pure default component for Ba rated bonds.

To assess the impact of liquidity frictions on the aggregate costs of corporate bond financing, we perform a dollar-based decomposition similar to the spread decomposition above. Using the issuance data for the U.S. corporate bond market from SIFMA, we estimate that the cumulative dollar "losses" (the reduction in bond valuation due to both default and liquidity frictions) for new corporate bond issuances from 1996 to 2015 to be $2.9 trillion dollars (in 2015 dollars), about 14% of the total issuance amount. Together, the pure liquidity, liquidity-driven default, and default-driven liquidity components, which can be viewed as the added costs of capital due to liquidity frictions, account for 43% of these total losses.

By taking into account how individual firms' default decisions respond to changes in liquidity conditions, our model offers a way to evaluate the effects of government policies that aim at improving market liquidity. Consider a policy experiment in which the secondary market liquidity in a recession is improved to the level of normal times. In our model, such a policy would lower the average credit spreads of Ba rated bonds in recession by 102bps, or 28% of the original spread. The policy's direct impact on the pure liquidity component only accounts for 42% of the total reduction in credit spreads. In contrast, the liquidity-driven default component, which reflects the reduction in default risk when firms face smaller rollover losses, and the default-driven liquidity component, which captures the endogenous reduction in liquidity frictions as the bonds become safer, explain 9% and 49% of the reduction in spreads, respectively.

Furthermore, based on the notional amount of corporate bonds outstanding in 2008, we estimate that such a liquidity provision policy would raise the value of the aggregate U.S. corporate bond market by $256 billion. If one ignores the default-liquidity interactions and only considers the pure liquidity component, this estimate would be only $173 billion, which substantially understates the impact of such liquidity policies.

In summary, our paper makes the following three contributions to the literature. First, we introduce macroeconomic dynamics and bond-price dependent holding costs into He and Milbradt (2014), which significantly improve the model's ability to capture the cross-sectional and time-series patterns of both the default and non-default components of corporate bond pricing. Second, we provide a structural decomposition of the credit spreads that highlights the interactions between default risks and liquidity frictions. This decomposition helps us assess the full impact of liquidity frictions on the costs of capital for corporate bond financing. We find that these interaction effects are stronger for lower-rated firms and in recessions. Third, the model enables us to quantify the effects of a counter-cyclical liquidity provision policy on the corporate bond market.

Literature review. It is well known that a significant part of corporate bond pricing cannot be accounted for by default risk alone. For example, Longstaff, Mithal, and Neis

(2005) estimate that "non-default components" account for about 50% of the spread between the yields of Aaa/Aa-rated corporate bonds and Treasuries, and about 30% of the spread for Baa-rated bonds. Furthermore, Longstaff, Mithal, and Neis (2005) find that non-default components of credit spreads are strongly related to measures of bond liquidity, which is consistent with evidence of illiquidity in secondary corporate bond markets (e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011)).

Nonetheless, the literature on credit risk modeling has almost exclusively focused on the default component of credit spreads. A common way to take out the non-default component of the credit spreads is to focus on the differences between the spreads of bonds with different ratings, for example the Baa-Aaa spread. Such treatment relies on the assumption that the non-default components for bonds of different rating classes are the same, which is at odds with the empirical evidence.

The "credit spread puzzle," as defined by Huang and Huang (2012), refers to the finding that, after matching the observed default and recovery rates, traditional structural models produce credit spreads for investment grade bonds that are significantly lower than those in the data. By introducing macroeconomic risks into structural credit models, Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) are able to explain the default component of the spreads of investment-grade bonds. They are, however, silent on the non-default component of credit spreads, thus leaving a significant portion of credit spreads unexplained. In contrast, our model jointly studies the default and liquidity components of corporate bond pricing. By doing so, we are able to investigate a new set of liquidity-related moments such as bid-ask spreads and bond-CDS spreads.

Our model extends He and Milbradt (2014) in two key aspects. First, instead of a constant exogenous holding cost for investors experiencing liquidity shocks, we model holding costs that decrease with the endogenous bond price. We justify these holding costs through the friction of collateralized financing. In this mechanism, investors hit by liquidity shocks raise cash either via cheaper collateralized financing (using the bond as collateral, subject to haircuts) or more expensive uncollateralized financing. When the firm gets closer to default, a lower bond price together with a larger haircut pushes investors toward more expensive uncollateralized financing, which leads to higher effective holding costs.

Second, we introduce macroeconomic risks into the model through cyclical variations in firms' cash flows, aggregate risk prices, and liquidity frictions. This not only helps generate significant time variation in *default risk premium*, an important feature of the data, but also raises the *liquidity risk premium*, because market liquidity worsens in recessions (when investors' marginal utilities are high). Together, these two types of risk premia magnify the quantitative effect of the default-liquidity spiral on corporate bond pricing.

2. The Model

2.1 Aggregate States and the Firm

Aggregate states and stochastic discount factor. The aggregate state of the economy is described by a continuous time Markov chain, with the current Markov state denoted by s_t and the physical transition density between state i and state j denoted by $\zeta_{ij}^{\mathcal{P}}$. We assume an exogenous stochastic discount factor (SDF):

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)dt - \eta\left(s_t\right)dZ_t^m + \sum_{s_t \neq s_{t-}} \left(e^{\kappa\left(s_{t-},s_t\right)} - 1\right) dM_t^{(s_{t-},s_t)}, \tag{1}$$

where Z_t^m is a standard Brownian Motion under the physical probability measure $\mathcal{P}$, $r(\cdot)$ is the risk-free rate, $\eta(\cdot)$ is the state-dependent price of risk for aggregate Brownian shocks, $dM_t^{(i,j)}$ is a compensated Poison process capturing switches between states i and j, and $\kappa(i,j)$ determines the jump risk premia such that the jump intensity between states i and j under the risk neutral measure $\mathcal{Q}$ is $\zeta_{ij}^{\mathcal{Q}} = e^{\kappa(i,j)}\zeta_{ij}^{\mathcal{P}}$. We focus on the case of binary aggregate states to capture the notion of economic expansions and recessions, i.e., $s_t \in \{G, B\}$. In the Internet Appendix we provide the general setup for the case of $n > 2$ aggregate states.

Later on, we will introduce undiversifiable idiosyncratic liquidity shocks to investors. Upon receiving a liquidity shock, an investor who cannot sell the bond will incur some holding costs. In Appendix A we show that, in the presence of such undiversifiable liquidity shocks, bond investors can still price assets using the SDF in (1) provided that the bond holdings only make up an infinitesimal part of the representative investor's portfolio. Intuitively, if the representative agent's consumption pattern is not affected by the idiosyncratic shock (which is true if the bond holding is infinitesimal relative to the rest of the portfolio), then the representative agent's pricing kernel is independent of the idiosyncratic undiversified shocks. What is more, this is an empirically sound assumption: according to Flow of Funds, corporate bonds only accounts for 1.5% to 3.5% of households net worth.[2]

Firm cash flows and risk neutral measure. Consider a firm that generates cash flows at the rate of Y_t. Under the physical measure $\mathcal{P}$, the cash-flow rate Y_t dynamics, given the aggregate state s_t, follows

$$\frac{dY_t}{Y_t} = \mu_{\mathcal{P}}\left(s_t\right)dt + \sigma_m\left(s_t\right)dZ_t^m + \sigma_f dZ_t^f. \tag{2}$$

Here, dZ_t^m captures aggregate Brownian risk, while dZ_t^f captures idiosyncratic Brownian risk.

Given the stochastic discount factor Λ_t, the dynamics of the log cash-flows $y \equiv \log(Y)$ in aggregate state s_t under the risk-neutral measure $\mathcal{Q}$ are rewritten as

$$dy_t = \mu_{s_t}dt + \sigma_{s_t}dZ_t^{\mathcal{Q}}, \tag{3}$$

where $Z_t^{\mathcal{Q}}$ is a standard Brownian motion under $\mathcal{Q}$, and the drift and volatility are given by

$$\mu_{s_t} \equiv \mu_{\mathcal{P}}(s_t) - \sigma_m(s_t)\eta(s_t) - \frac{1}{2}\left[\sigma_m^2(s_t) + \sigma_f^2\right], \quad \sigma_{s_t} \equiv \sqrt{\sigma_m^2\left(s_t\right) + \sigma_f^2}.$$

We obtain valuations for any asset by discounting the expected cash flows under the risk neutral measure $\mathcal{Q}$ with the risk-free rate. The unlevered firm value, given aggregate state s

[2] At the end of 2015 the U.S. households' net worth sits around 87 trillion. The non-financial corporate bonds outstanding not held by non-US institutions is about 3 trillion. This implies that corporate bonds only account for about 3.4% of the U.S. households wealth. Furthermore, the majority of corporate bonds are held by insurance companies and pension funds who do not trade actively. If we exclude the holdings of these two types of institutions, then the fraction shrinks to only about 1.6%.

and cash-flow rate $e^y = Y$, is $v_U^s Y$, where the vector of price-dividend ratios $\mathbf{v}_U$ is

$$\mathbf{v}_U \equiv \begin{bmatrix} v_U^G \\ v_U^B \end{bmatrix} = \begin{bmatrix} r_G - \mu_G + \zeta_G & -\zeta_G \\ -\zeta_B & r_B - \mu_B + \zeta_B \end{bmatrix}^{-1} \mathbf{1}. \tag{4}$$

Firm's debt maturity structure and rollover frequency. The firm has a unit measure of bonds in place that are identical except for their time to maturity, with the aggregate and individual bond coupon and face value being c and p. As in Leland (1994) and Leland (1998), equity holders commit to keeping the aggregate coupon and outstanding face value constant before default, and thus issue new bonds of the same average maturity as the bonds maturing. The issuance of new bonds in the primary market incurs a proportional cost $\omega \in (0,1)$. Each bond matures with intensity m, and the maturity event is i.i.d. across individual bonds. Thus, by the law of large numbers over $[t, t+dt)$ the firm retires a fraction $m \cdot dt$ of its bonds. This implies an expected average debt maturity of $\frac{1}{m}$. The deeper implication of this assumption is that the firm adopts a "smooth" debt maturity structure with a constant refinancing/rollover frequency of m.[3]

2.2 Secondary Over-the-Counter Corporate Bond Market

Liquidity Shocks & Holding Cost. Bond investors can hold either zero or one unit of the bond and are in individual state $l \in \{H, L\}$. They start in the H state without any holding cost when holding a corporate bond. As time passes by, H-type bond holders are hit by idiosyncratic liquidity shocks with intensity ξ_s. These liquidity shocks lead them to become L-types who bear a positive holding cost hc_s per unit of time. We specify state-dependent holding costs that depend on the prevailing bond prices and aggregate state as follows:

$$hc_s(P^s(y)) = \chi_s \left[N - P^s(y) \right] \tag{5}$$

[3] Most of the literature follows the tradition of Leland (1998) by assuming that the firm can fully commit to the financing policy with a constant aggregate debt face value and a constant maturity structure. For recent papers that relax this stringent assumptions, see Dangl and Zechner (2006), DeMarzo and He (2014), He and Milbradt (2015).

where $N > 0, \chi_G$ and χ_B are positive constants and $P^s(y)$ is the endogenous market price of the bond (to be derived in the next section) as a function of the log cash-flow y.

In Appendix B, we show how relation (5), for simplicity without aggregate state switches, can be derived from costly collateralized financing. We interpret a liquidity shock as the urgent need for an investor to raise cash which exceeds the value of all the liquid assets that he holds, a common phenomenon for modern financial institutions. Bond investors first use their bond holdings as collateral to raise collateralized financing at the risk-free rate; and collateralized financing is subject to a haircut until they manage to sell the bonds. Any remaining gap must be financed through uncollateralized financing, which requires a higher interest rate. In this setting, the investor obtains less collateralized financing if (i) the current market price of the bond is lower, and/or (ii) the haircut for the bond is higher. In practice, (i) and (ii) often coincide, with the haircut increasing while the price goes down. The investor's effective holding cost is then given by the additional total uncollateralized financing cost, which increases when the bond price goes down. Under certain functional form assumptions on haircuts (see Appendix B), the holding cost takes the linear form in (5).

In Equation (5), if at issuance the bond is priced at par value p, a baseline holding cost of $\chi_s(N-p)$ applies (we will set $N > p$). With $\chi_s > 0$, the holding cost increases as the firm moves closer to default, and bond market value $P^s(y)$ declines further. This is the key channel through which our model captures the empirical pattern that lower rated bonds have significantly worse secondary market liquidity.

We further assume that the holding cost $hc_s(P^s(y))$ in (5) also depends on the aggregate state, through the following two channels. First, there is a direct effect, as we set $\chi_B > \chi_G$, which can be justified by the fact that the wedge between the collateralized and uncollateralized borrowing rates is higher in bad times. Second, there is an indirect effect, as the bond value $P^B(y) < P^B(y)$, giving rise to a higher holding cost for a given level of y.

Dealers and Equilibrium Prices. We assume a trading friction in moving bonds from L-type sellers to H-type potential buyers currently not holding the bond, in that trades have to be intermediated by dealers in an over-the-counter market. Sellers meet dealers with intensity λ_s, which we interpret as the intermediation intensity of the bond market. For simplicity, we assume that after L-type investors sell their holdings, they exit the market forever, and that there is a sufficient supply of H-type buyers on the sideline. [4] The buyers on the sideline currently not holding the bond also contact dealers with intensity λ_s. We follow Duffie, Gârleanu, and Pedersen (2007) to assume Nash-bargaining weights β for investors and $1-\beta$ for the dealer, constant across all dealer-investor pairs and aggregate states.

Dealers use the competitive (and frictionless) inter-dealer market to sell or buy bonds in order to keep a zero inventory position. When a contact between a L-type seller and a dealer occurs, the dealer can instantaneously sell the bond at the inter-dealer clearing price $M^s(y)$ to another dealer who is in contact with an H-type investor via the inter-dealer market. If a sale occurs, the bond travels from an L-type investor to an H-type investor with the help of the two dealers who are connected in the inter-dealer market.

Suppressing y, for any aggregate state s, denote by D_l^s the bond value for an investor of type $l \in \{H, L\}$. B^s is the bid price at which the L-type is selling his bond, A^s is the ask price at which the H-type is purchasing this bond, and M^s is the inter-dealer market price.

For simplicity, we assume that the flow of H-type buyers contacting dealers is greater than the flow of L-type sellers contacting dealers. Then, Bertrand competition, the holding restriction, and excess demand from buyer-dealer pairs in the inter-dealer market drive the surplus of buyer-dealer pairs to zero, resulting in a *seller's market*.

Proposition 1. *Fix valuations D_H^s and D_L^s. In equilibrium, the ask price A^s and inter-dealer market price M^s are equal to D_H^s, and the bid price is given by $B^s = \beta D_H^s + (1-\beta) D_L^s$. The dollar bid ask spread is given by $A^s - B^s = (1-\beta)(D_H^s - D_L^s)$.*

As there is no single "market price" in our over-the-counter market, we follow market-practice and define the "market price" in the endogenous holding cost in equation (5) as the

[4] This is an innocuous assumption made for exposition. Switching back from L to H is easily incorporated into the model. See the Appendix in He and Milbradt (2014) for details.

mid-price between the bid and ask prices, i.e.,

$$P^s(y) = \frac{A^s(y) + B^s(y)}{2} = \frac{(1+\beta)\, D_H^s(y) + (1-\beta)\, D_L^s(y)}{2}. \tag{6}$$

Finally, empirical studies often focus on the proportional bid-ask spread, defined as the dollar bid-ask spread divided by the mid price, which can be expressed as

$$ba^s\,(y) = \frac{2\,(1-\beta)\,[D_H^s(y) - D_L^s(y)]}{(1+\beta)\, D_H^s(y) + (1-\beta)\, D_L^s(y)}. \tag{7}$$

2.3 Bankruptcy and Effective Recovery Rates

When the firm's cash flows deteriorate, equity holders are willing to repay the maturing debt holders only when the equity value is still positive, i.e. the option value of keeping the firm alive justifies absorbing current rollover losses and coupon payments. Equity holders default in state s at the optimally chosen default threshold y_{def}^s, summarized by the vector $\boldsymbol{y}_{def} \equiv \left[y_{def}^G, y_{def}^B\right]^\top$. We assume that bankruptcy costs are a fraction $1-\alpha$ of the value of the unlevelered firm $v_U^s e^{y_\tau}$ at the time of default τ, where v_U^s is given in (4).

If bankruptcy leads investors to receive the bankruptcy proceeds immediately, then bankruptcy confers a "liquidity" benefit similar to a maturing bond. This "expedited payment" benefit runs counter to the fact that in practice bankruptcy leads to the freezing of assets within the company and a delay in the payout of any cash depending on court proceeding.[5] Moreover, investors of defaulted bonds may face a much more illiquid secondary market (e.g., Jankowitsch, Nagler, and Subrahmanyam (2013)), and potentially higher holding cost once liquidity shocks hit due to regulatory or charter restrictions which prohibit certain institutions from holding defaulted bonds. These practical features lead to a type- and

[5] For evidence on inefficient delay of bankruptcy resolution, see Gilson, John, and Lang (1990) and Ivashina, Smith, and Iverson (2013). The Lehman Brothers bankruptcy in September 2008 is a good case in point. After much legal uncertainty, payouts to the debt holders only started trickling out after over three years.

state-dependent bond recovery at the time of default:

$$\mathbf{D}^{def}(y) \equiv \begin{bmatrix} \alpha_H^G v_U^G \\ \alpha_L^G v_U^G \\ \alpha_H^B v_U^B \\ \alpha_L^G v_U^B \end{bmatrix} \times e^y. \tag{8}$$

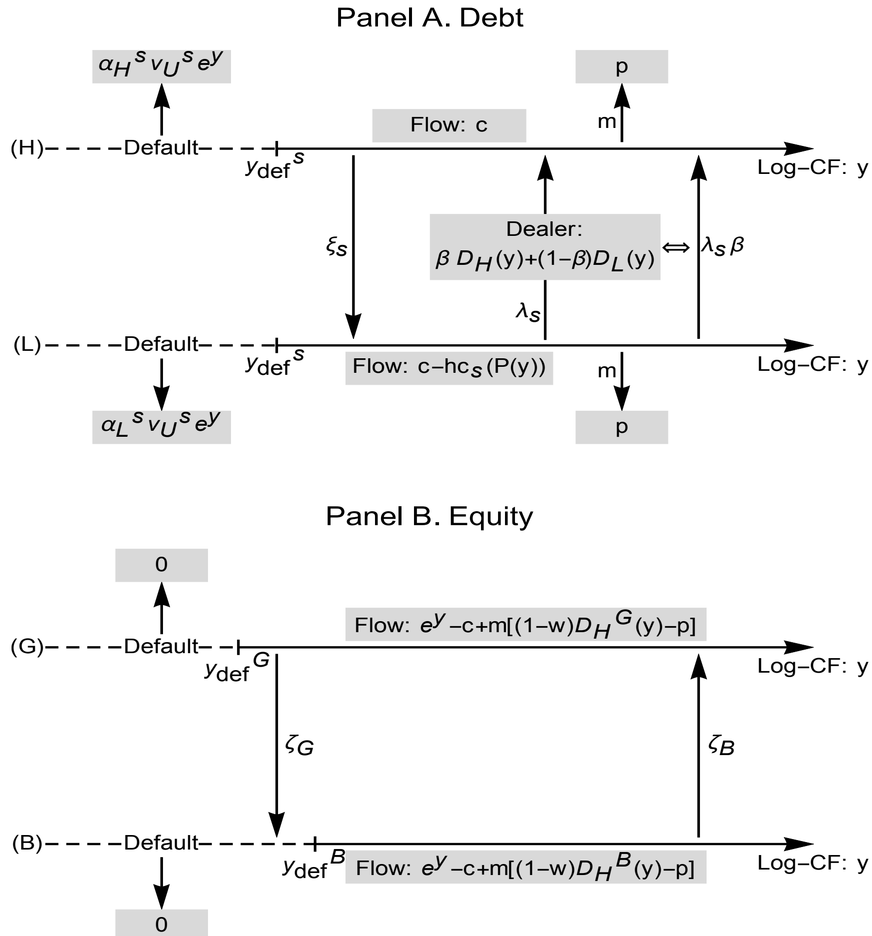
Here, $\boldsymbol{\alpha} \equiv \left[\alpha_H^G, \alpha_L^G, \alpha_H^B, \alpha_L^B\right]^\top$ are the *effective* bankruptcy recovery rates at default. As explained in Section 3.1, when calibrating $\boldsymbol{\alpha}$, we rely exclusively on the market price of defaulted bonds observed immediately after default, and the associated empirical bid-ask spreads, to pin down $\boldsymbol{\alpha}$.

2.4 Liquidity Premium of Treasury

It has been widely recognized (e.g., Duffie (1996), Krishnamurthy (2002), Longstaff (2004)) that Treasuries, due to their special role in financial markets, are earning returns that are significantly lower than the risk-free rate, which in our model is represented by r_s in equation (1). The risk-free rate is the discount rate for future deterministic cash flows, whereas Treasury yields also reflect the additional benefits of holding Treasuries relative to generic default-free and easy-to-transact bonds. The wedge between the two rates, which we term the "liquidity premium of Treasuries," represents the convenience yield that is specific to Treasury bonds. This is the ability to post Treasuries as collateral with a significantly lower haircut than other financial securities. Although this broad collateral-related effect is empirically relevant, our model is not designed to capture this economic force.

We accommodate this effect by simply assuming that there are (exogenous) state-dependent liquidity premia Δ_s for Treasuries. Specifically, given the risk-free rate r_s in state s, the yield of Treasury bonds is simply $r_s - \Delta_s$. When calculating credit spreads of corporate bonds, following the convention we use the Treasury yield as the benchmark.

Figure 1: **Schematic graphic of cash flows to debt and equity holders**



2.5 Summary of Setup

Figure 1 summarizes the cash flows to debt and equity holders. Panel A visualizes the cash flows to a debt holder in aggregate state s. The horizontal lines depict the current log cash flow y. The top half of the graph depicts an H-type debt holder who has not been hit by a liquidity shock yet. This bond holder receives a flow of coupon c each instant (all cash-flows in this figure are indicated by gray boxes). With intensity m, the bond matures and the investor receives the face value p. With intensity ξ_s the investor is hit by a liquidity shock and transitions to an L-type investor who receives cash flows net of holding costs of $[c - hc_s(P^s(y))]\, dt$ each instant, where $P^s(y) = [(1+\beta)\, D_H^s(y) + (1-\beta)\, D_L^s(y)]\,/2$ is the endogenous secondary market mid price. With intensity λ_s the L-type investor meets a dealer, sells the bond for $\beta D_H^s(y) + (1-\beta) D_L^s(y)$, and exits the market forever. To the debt holder, this is equivalent in value to losing the ability to trade but gaining an exogenous *recovery* intensity $\lambda_s\beta$ of transitioning back to being an H-type investor. Finally, when $y \leq y_{def}^s$, the firm defaults immediately and bond holders recover $\alpha_l^s v_U^s e^y$, which depends both on their individual type and on the aggregate state as well as the cash-flow state of the firm.

Panel B visualizes the cash flows to equity holders. The horizontal lines depict the current log cash flow y, where the top (bottom) line represents the aggregate G (B) state. Each instant, the equity holder receives a cash-flow $Y = e^y$ from the firm and pays the coupon c to debt holders. As debt is of finite average maturity, by the law of large numbers, a flow m of bonds comes due each instant and each bond requires a principal repayment of p. At the same time, the firm reissues these maturing bonds with their original specification and raises an amount (after issuance costs) of $(1-\omega)D_H^s(y)$ per bond depending on aggregate state $s \in \{G, B\}$. With intensity ζ_G the state switches from G to B and the primary bond market price decreases from $D_H^G(y)$ to $D_H^B(y)$, reflecting a higher default probability as well as a worsened liquidity in the market. In cases where $y \in \left(y_{def}^G, y_{def}^B\right)$ (as shown), the cash flows to equity holders are so low that they declare default immediately following a jump, receiving a payoff of 0. Finally, with intensity ζ_B, the state jumps from B to G. Implicit in the model is that equity holders are raising new equity frictionlessly to cover negative cash flows before default.

Panel A and Panel B are connected via the primary market prices of newly issued bonds, i.e. $D_H^s(y)$. Although the firm is able to locate and place newly issued bonds to H-type investors in the primary market, the issuance prices reflect the secondary market illiquidity in Panel A, simply because forward-looking H-type investors take into account that they will face the illiquid secondary market in the future if hit by liquidity shocks. Through this channel, the secondary market illiquidity enters the firm's rollover cash flows in Panel B and affects the firm's default decision.

2.6 Model Solutions

For the individual state $l \in \{H, L\}$ and the aggregate state $s \in \{G, B\}$, denote by D_l^s the l-type bond value in aggregate state s, E^s the equity value in aggregate state s. We derive the closed-form solution for debt and equity valuations as a function of the log cash flow y for given default boundaries $\boldsymbol{y}_{def}$, along with the characterization of the optimally chosen $\boldsymbol{y}_{def}$.

Because equity holders default earlier in state B, i.e., $y_{def}^G < y_{def}^B$, the domains on which bonds and equity are "alive" change when the aggregate state switches. We deal with this issue by the method described below; see the Internet Appendix for the technical proof, and Appendix C for a more detailed discussion including the HJBs.

Define two intervals $I_1 = \left[y_{def}^G, y_{def}^B\right]$ and $I_2 = \left[y_{def}^B, \infty\right)$, and denote by $D_l^{s,i}$ the restriction of D_l^s to the interval I_i, i.e., $D_l^{s,i}(y) = D_l^s(y)$ for $y \in I_i$, and analgously for equity. The bond value on interval I_1 when the aggregate state is B is given by $D_l^{B,1}(y) = \alpha_l^B v_U^B e^y$ – the bond is "dead"in that state, as the firm immediately defaults on interval I_1 when switching into state B. Similarly, equity value is given by $E^{B,1}(y) = 0$. In contrast, on interval $I_2 = \left[y_{def}^B, \infty\right)$, all bond and equity valuations are alive.

Proposition 2. *Given default boundaries $\mathbf{y}_{def}$, the bond values on interval i are given by*

$$\underbrace{\mathbf{D}^{(i)}(y)}_{2i\times 1} = \mathbf{G}^{(i)} \cdot \exp\left(\boldsymbol{\Gamma}^{(i)} y\right) \cdot \mathbf{b}^{(i)} + \mathbf{k}_0^{(i)} + \exp(y)\mathbf{k}_1^{(i)}, \tag{9}$$

and the equity values are given by

$$\underbrace{\mathbf{E}^{(i)}(y)}_{i\times 1} = \mathbf{GG}^{(i)} \cdot \exp\left(\boldsymbol{\Gamma\Gamma}^{(i)} y\right) \cdot \mathbf{bb}^{(i)} + \mathbf{KK}^{(i)} \exp\left(\boldsymbol{\Gamma}^{(i)} y\right) \mathbf{b}^{(i)} + \mathbf{kk}_0^{(i)} + \exp(y)\,\mathbf{kk}_1^{(i)} \text{ for } y \in I_i \tag{10}$$

The constant matrices $\mathbf{G}^{(i)}$, $\boldsymbol{\Gamma}^{(i)}$, $\mathbf{GG}^{(i)}$, $\boldsymbol{\Gamma\Gamma}^{(i)}$, $\mathbf{KK}^{(i)}$, and the vectors $\mathbf{k}_0^{(i)}$, $\mathbf{k}_1^{(i)}$, $\mathbf{b}^{(i)}$, $\mathbf{kk}_0^{(i)}$, $\mathbf{kk}_1^{(i)}$ and $\mathbf{bb}^{(i)}$ are given in the Internet Appendix.

For the bond values, the second term given by the vector $\mathbf{k}_0^{(i)}$ summarizes the expected value of each bond absent default-risk. The third term summarizes the expected value stemming from bankruptcy after a jump to default induced by an aggregate state jump, i.e., a cash flow independent intensity-based default.[6] The first term consequently summarizes the impact that distance to default, i.e., $y - y^s_{def}$, has on the valuation of the bond.

For the equity values, the fourth term is the sum of the expected (unlevered) value of the direct cash flows from assets, and the indirect valuation impact of the recovery of bonds from jumps to default. The first term summarizes the direct valuation impact of distance to default on equity holders. In contrast, the second and third term summarize the indirect impact of default via the cash-flows arising from the firm's bond issuance and rollover activity.

Finally, equity holders choose the bankruptcy boundaries $\boldsymbol{y}_{def} = \left[y^G_{def}, y^B_{def}\right]^\top$ optimally, which is characterized by a smooth-pasting condition:

$$\left(\mathbf{E}^{(1)}\right)' \left(y^G_{def}\right)_{[1]} \;=\; 0, \text{ and } \left(\mathbf{E}^{(2)}\right)' \left(y^B_{def}\right)_{[2]} = 0. \tag{11}$$

3. Calibration

3.1 Benchmark Parameters

We calibrate the model parameters to a set of empirical moments of on firm cash flows, asset prices, historical default rates, bond turnover rates, and bond bid-ask spreads. The benchmark parameter values are reported in Table 1. Below we explain the details of the calibration procedure.

[TABLE 1 ABOUT HERE]

SDF and cash flow parameters. Start with the pricing kernel. To abstract away from any term structure effects, we set the risk free rate $r_G = r_B = 5\%$ in both aggregate states. Transition intensities for the aggregate state give the average durations of expansions and recessions over the business cycle (10 years for expansions and 2 years for recessions). The

[6] Note that $\mathbf{k}^{(2)}_1 = \mathbf{0}$ as both bonds are alive on I_2.

price of risk η for Brownian shocks and the jump risk premium $\exp(\kappa)$ are calibrated to match key asset pricing moments including the equity premium and price-dividend ratio.

Next, on the firm side, the cash-flow growth is matched to the average (nominal) growth rate of aggregate corporate profits. State-dependent systematic volatilities σ_m^s are calibrated to match the model-implied equity return volatilities with the data. We set the debt issuance cost ω in the primary corporate bond market to be 1%. Based on the empirical median debt maturity (including bank loans and public bonds), we set $m = 0.2$ implying an average debt maturity of 5 years. The idiosyncratic volatility σ_f is chosen to match the average default probability across firms. There is no state-dependence of σ_f as we do not have data counterparts for state-dependent default probabilities. As explained later, the firm's current cash-flow level is chosen to match the empirical leverage in Compustat at the firm-quarter frequency. Finally, our calibration implies an equity Sharpe ratio of 0.11 in state G and 0.20 in state B, which are close to the mean firm-level Sharpe ratio for the universe of CRSP firms (0.17) reported in Chen, Collin-Dufresne, and Goldstein (2009).

Secondary bond market liquidity. Recall that in Section 2.4 we allow Treasuries to enjoy extra state-dependent liquidity premium Δ_s. We set them based on the average observed repo-Treasuries spread, as measured by the difference between the 3-month general collateral repo rate and the 3-month Treasury rate. During the period from October 2005 to September 2013 (excluding the crisis period of October 2008 to March 2009), the daily average of the repo-Treasury spread is 15bps during the non-recession periods and 40bps during recessions, leading us to set $\Delta_G = 15$bps and $\Delta_B = 40$bps.[7] These estimates are roughly consistent with the average liquidity premium reported in Longstaff (2004) based on Refcorp bond rates.

The liquidity parameters describing the secondary corporate bond market are less standard in the literature. We first fix the state-dependent intermediary meeting intensity based on anecdotal evidence, so that it takes a bond holder on average a week ($\lambda_G = 50$) in the good state and 2.6 weeks ($\lambda_B = 20$) in the bad state to find an intermediary to divest of all bond

[7] Over a given horizon, the state-dependent instantaneous liquidity premium suggests that the average liquidity premium is horizon-dependent, but we ignore this effect for simplicity.

holdings. We interpret the lower λ in state B as a weakening of the financial system and its ability to intermediate trades. We then set bond holders bargaining power $\beta = 0.05$ independent of the aggregate state, based on empirical work that estimates search frictions in secondary corporate bond markets (Feldhütter (2012)).

We choose the intensity of liquidity shocks, ξ_s, to match the average bond turnover in the secondary market. In the TRACE sample from 2005 to 2012, the value-weighted turnover of corporate bonds during NBER expansion periods is about 70% per year, which leads us to set $\xi_G = 0.7$. This is because given the relative high meeting intensities ($\lambda_G = 50$ and $\lambda_B = 20$), the turnover rate is almost entirely determined by the liquidity shock intensity ξ_s.[8] Although in the data there is no significant difference in bond turnover over the business cycle, in the baseline calibration we set $\xi_B = 1$ to capture the idea that during economic downturns institutional holders of corporate bonds are more likely to be hit by liquidity shocks.

By calibrating ξ_s to the bond turnover rate, we are assuming that the majority of the corporate bond transactions are driven by liquidity shocks. Trading driven by "liquidity shocks" in our model admits a broad interpretation. In essence, an idiosyncratic liquidity event in the model refers to any event that reduces the private valuation of an investor for the bond, thus generating the need for trade. It not only captures the selling needs of institutions after funding shocks, but also represents portfolio rebalancing needs (e.g., due to some exogenous shifts of asset allocations, like in Duffie, Gârleanu, and Pedersen (2007)), or even changes in beliefs. Anecdotally, these considerations seem to be the predominant trading motives for relatively sophisticated investors in secondary corporate bond market.

The parameters χ_G, χ_B and N in equation (5) are central to determining the bond-price dependent holding costs and thus the illiquidity of corporate bonds in the secondary market. We calibrate them to target the bid-ask spreads for superior grade, investment grade, and junk bonds in both aggregate states (3 free parameters and 6 moments).[9]

[8]The model implied expected turnover is $\frac{\xi_s \lambda_s}{\xi_s + \lambda_s} \simeq \xi_s$ when $\lambda_s \gg \xi_s$. Of course, we implicitly assume that all turnover in the secondary corporate bond market is driven by liquidity trades in our setting, while in practice investors trade corporate bonds for reasons other than liquidity shocks.

[9]While we do check the model's performance in explaining bond-CDS spreads, we do not target them in the calibration. Alternatively, we could use χ_G, χ_B and N to simultaneously target the moments for bid-ask spreads and bond-CDS spreads.

Recovery rates. Our model features type- and state-dependent recovery rates α_l^s for $l \in \{L, H\}$ and $s \in \{G, B\}$. We first borrow from the existing structural credit risk literature, specifically Chen (2010), who treats the traded prices right after default as bond recovery rates, and estimates firm-level recovery rates of $57.55\% \cdot v_U^G$ in normal times and $30.60\% \cdot v_U^B$ in recessions (recall v_U^s is the unlevered firm value at state s). Assuming that post-default prices are bid prices at which investors are selling, then Proposition 1 implies:

$$0.5755 \quad = \quad \alpha_L^G + \beta(\alpha_H^G - \alpha_L^G), \text{ and } 0.3060 = \alpha_L^B + \beta(\alpha_H^B - \alpha_L^B). \tag{12}$$

We need two more pieces of information on bid-ask spreads of defaulted bonds to pin down the α_l^s's. Edwards, Harris, and Piwowar (2007) report that in normal times (2003-2005), the transaction cost for defaulted bonds for median-sized trades is about 200bps. To gauge the bid-ask spread for defaulted bonds during recessions, we take the following approach. Using TRACE, we first follow Bao, Pan, and Wang (2011) to calculate the implied bid-ask spreads for low rated bonds (C and below) for both non-recession and recession periods. We find that relative to the non-recession period, during recessions the implied bid-ask spread is higher by a factor of 3.1. Given a bid-ask spread of 200bps for defaulted bonds, this multiplier implies that the bid-ask spread for defaulted bonds during recessions is thus about $3.1 \times 200bps = 620bps$. Hence we have

$$2\% \quad = \quad \frac{2\,(1-\beta)\left(\alpha_H^G - \alpha_L^G\right)}{\alpha_L^G + \beta(\alpha_H^G - \alpha_L^G) + \alpha_H^G}, \text{ and } 6.2\% = \frac{2\,(1-\beta)\left(\alpha_H^B - \alpha_L^B\right)}{\alpha_L^B + \beta(\alpha_H^B - \alpha_L^B) + \alpha_H^B}. \tag{13}$$

Solving (12) and (13) gives us the estimates of:[10]

$$\boldsymbol{\alpha} = \left[\alpha_H^G = 0.5871, \alpha_L^G = 0.5749, \alpha_H^B = 0.3256, \alpha_L^B = 0.3050\right]^\top. \tag{14}$$

These default recovery rates determine the bond recovery rate, a widely-used measure defined as the defaulted bond price divided by its promised face value. In our calibration,

[10]This calculation assumes that bond transactions at default occur at the bid price. If we assume that transactions occur at the mid price, these estimates are $\alpha_H^G = 0.5813, \alpha_L^G = 0.5691, \alpha_H^B = 0.3140, \alpha_L^B = 0.2972$.

the implied bond recovery rate is 49.7% in state G and 24.5% in state B. The unconditional average recovery rate is 44.6%. These values are consistent with the average issuer-weighted bond recovery rate of 42% in Moody's recovery data over 1982-2012 (Emery (2007)), and they capture the cyclical variations in recovery rates.

Degrees of freedom in calibration. Although there are a total of 28 parameters in our model, most of them are "pre-fixed parameters" in that they are not chosen to improve our model's fit for the set of moments used to evaluate the model's performance (default rates, credit spreads, bid-ask spreads, and bond-CDS spreads). Instead, they are picked based on the literature or to target other moments closely related to the parameter. We report these parameters in Panel A of Table 1. After these parameter values are set, we are left with 4 parameters, the idiosyncratic volatility σ_f, and the holding cost parameters, N, χ_G, χ_B, shown in Panel B of Table 1. As explained above, they are picked to target the average 10-year default rates across firms, and the bid-ask spreads across ratings and across states. Thus, the degrees of freedom (4) are far below the number of empirical moments that we aim to explain (4 moments for 10-year default rates, 8 for 10-year credit spreads, 6 for bid-ask spreads, and 8 for bond-CDS spreads).

3.2 Target Moments

We consider four rating classes: Aaa/Aa , A, Baa, and Ba; the first three rating classes are investment grade, while Ba is speculative grade. We combine Aaa and Aa together because there are few observations for Aaa firms. Furthermore, we report the model performance conditional on macroeconomic states. We classify each quarter as either in "state G" or "state B" based on NBER recessions. As the "B" state in our model only aims to capture normal recessions in business cycles, we exclude two quarters during the 2008 financial crisis, which are 2008Q4 and 2009Q1, to mitigate the effect caused by the unprecedented disruption in financial markets during crisis.[11]

[11] For recent empirical research that study the corporate bond market during the 2007/08 crisis, see Dick-Nielsen, Feldhütter, and Lando (2012) and Friewald, Jankowitsch, and Subrahmanyam (2012).

We primarily focus on the model's performance in explaining the default rates, credit spreads, and liquidity measures for bonds with 10-year maturity rather than the entire term structure. This is partly because the average maturity of newly issued corporate bonds is 11 years (according to SIFMA), and partly due to the difficulty in explaining the term structure of default risks and credit spreads, as discussed by Duffie and Lando (2001), Bhamra, Kuehn, and Strebulaev (2010), Feldhütter and Schaefer (2014), and others.

[TABLE 2 ABOUT HERE]

Default rates. The default rates for 5-year and 10-year bonds in Panel A in Table 2 are taken from Moody's (2012), which provides cumulative default probabilities over the period of 1920-2011. Unfortunately, state-dependent measures of default probabilities over the business cycle are unavailable.

Credit spreads. Our data of bond spreads are from the Mergent Fixed Income Securities Database (FISD) from January 1994 to December 2004, and TRACE data from January 2005 to June 2012. We exclude utility and financial firms.[12] For each transaction, we calculate the bond credit spread by taking the difference between the bond yield and the treasury yield with corresponding maturity. Within each rating class, we average these observations in each month to form a monthly time series of credit spreads for that rating. We then calculate the time-series average for each rating conditional on the macroeconomic state (whether the month is classified as a NBER recession) and the standard deviation for the conditional mean estimates. These moments are reported in Panel B of Table 2.

Bid-ask spreads. One of our measures related to the non-default components of credit spreads is bid-ask spreads in the secondary market, whose model counterpart is given in (7). We use the rating classes and average bid-ask spread estimates in Edwards, Harris, and Piwowar (2007): superior grade (Aaa/Aa) with a bid-ask spread of 40bps, investment grade (A/Baa) with a bid-ask spread of 50bps, and junk grade (Ba and below) with a bid-ask spread

[12] We follow Collin-Dufresne, Goldstein, and Martin (2001) and Dick-Nielsen (2009) to clean the Mergent FISD and TRACE data.

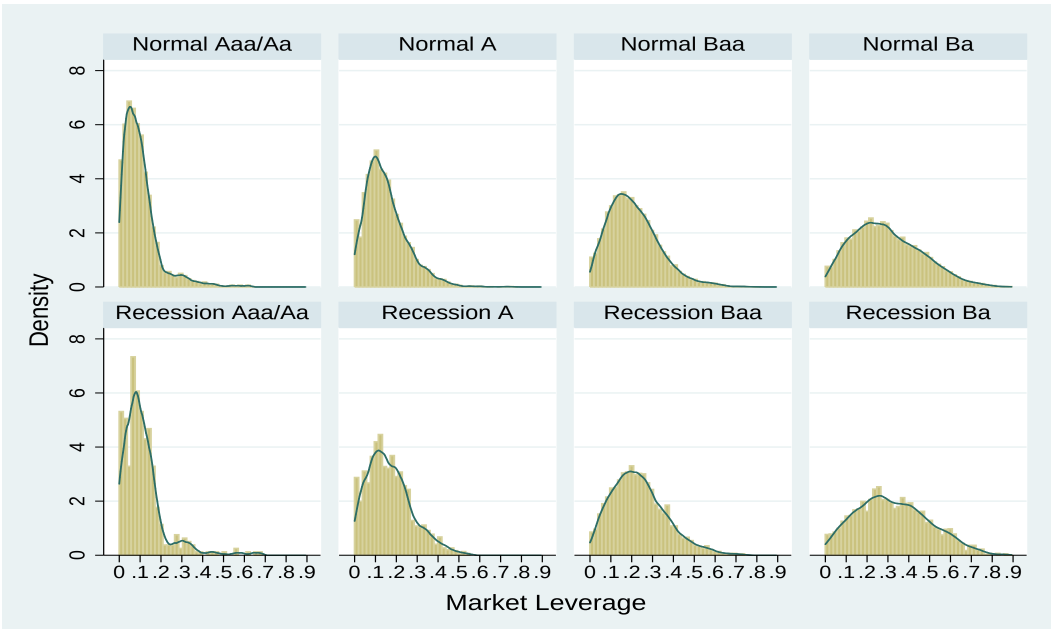
of 70bps. As these bid-ask spreads estimates only for non-recession times (2003-2005), we construct our recession counterparts as follows: For each grade, we compute the measure of liquidity in Roll (1984) as in Bao, Pan, and Wang (2011), which we use to back out the bid-ask spread ratio between B-state and G-state. We then multiply this ratio by the G state bid-ask spread estimated by Edwards, Harris, and Piwowar (2007) to arrive at a bid-ask spread measure for the B state. These estimates are reported in Table 3 Panel A.

Bond-CDS spreads. Longstaff, Mithal, and Neis (2005) argue that because the market for CDS contracts is much more liquid than the secondary market for corporate bonds, the CDS spread should mainly reflect the default risk of a bond, while the credit spread also includes a liquidity premium to compensate for the illiquidity in the corporate bond market. Following Longstaff, Mithal, and Neis (2005), we take the difference between the bond credit spread and the corresponding CDS spread to get the Bond-CDS spread. The CDS spreads are from Markit, and the data sample period starts from 2005 when CDS data become available. These estimates are reported in Table 3 Panel B.

3.3 Calibration Results

To map the model's predictions on various moments at firm level to their counterparts in the data, which are aggregated by rating classes, it is important to take into account firm heterogeneity in market leverage. For example, David (2008) argues that model-implied default probabilities and credit spreads based on the average market leverage within a rating category will be lower than the average model-implied default probabilities and credit spreads across firms with the same rating, due to the fact that credit spreads are convex function of leverage. As Figure 2 shows, the empirical distributions of market leverage within each rating category (after excluding financials, utilities, and firms with zero leverage) are indeed wide spread. To account for such heterogeneity, we use the model to translate firms' observed market leverages at a given point in time one-to-one into log cash-flow y. Then, for firms with various leverage ratios, we compute the default probabilities, credit spreads, bid-ask spreads, and bond-CDS spreads for bonds with fixed maturity using Monte-Carlo method.

Figure 2: **Empirical Distribution of Market Leverage for Compustat Firms by Aggregate State and Rating classes.** We compute market leverage for each firm-quarter observation in Compustat from 1994-2012, excluding financials, utilities, and firms with zero leverage. State B is classified as quarters for which at least two months are in NBER recession; the remaining quarters are G state. We exclude the financial crisis quarters 2008Q4 and 2009Q1.



Finally, in each quarter we average these moments over the empirical leverage distribution for each rating class and each aggregate state. A more detailed description of this procedure is available in the Internet Appendix.

3.3.1 Default probabilities and credit spreads

Table 2 presents our calibration results on default probabilities (Panel A) and credit spreads across four rating classes (Panel B), for both 5-year and 10-year bonds, with 10-years being the targeted horizon of our calibration.

10-year default probabilities and credit spreads. For 10-year bond maturities, our quantitative model is able to deliver decent matching of both cross-sectional and state-dependent patterns in default probabilities and credit spreads. Overall, relative to the data the model implied credit spread tends to overshoot in state G and undershoot in state B, but the match remains reasonable.

Our model delivers a satisfactory fit for 10-year Baa credit spreads: in state G, the model predicts 182bps while the data counterpart is 150bps; in state B, we have 261bps in the model versus 262bps in the data. The fit of default rates for Baa-rated bonds is also good: the 10-year cumulative default probability is 7.9% in the model, compared to 7% in the data.

Our model also produces reasonable default rates for Aaa/Aa bonds (1.6%, slightly below the data counterpart of 2.1%), but the model-implied credit spreads are somewhat high compared to the data. This result indicates that the model-implied liquidity frictions are likely too strong for Aaa/Aa bonds. Nonetheless, we see the potential of the model to properly account for the pricing of superior grade bonds, which have been a bigger challenge for the existing credit risk models than the other rating classes (see e.g., Chen (2010)).

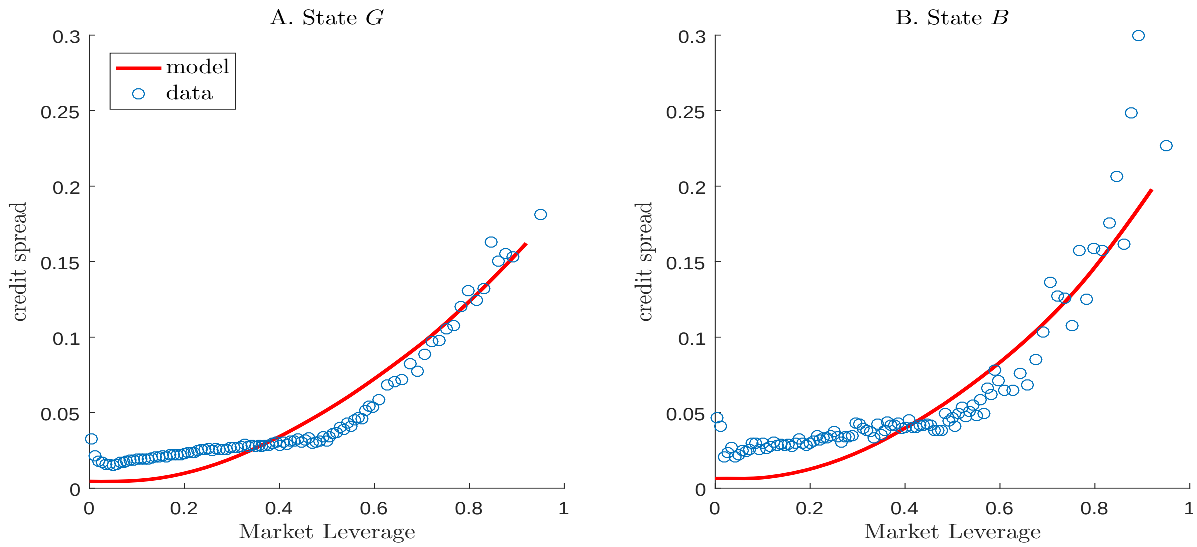
5-year default probabilities and credit spreads. Previous studies (e.g., Huang and Huang (2012)) reveal that the class of structural models typically imply a much steeper term structure of credit spreads than reflected in the data, i.e., for relatively safe corporate bonds (above Ba rated, say), the model-implied difference between 5-year and 10-year credit spreads is greater than its data counterpart. Our model suffers from the same issue; for instance, our model undershoots the 5-year Baa rated credit spreads (114bps in the model versus 149bps in the data in state G, and 191bps in the model versus 275bps in the data in state B). Certain interesting extensions of our model (e.g., introducing jumps in cash flows) could help in this dimension, and we leave it to future research to address this issue.[13]

[TABLE 3 ABOUT HERE]

Credit spreads vs. leverage. Since the only source of heterogeneity across firms in our model is leverage, it is informative to check our model's cross-sectional performance regarding the joint distribution of leverage and credit spreads. We compare the model-implied joint

[13]In unreported results, we find that the method of David (2008) which addresses the nonlinearity in the data (caused by the diverse distribution in leverage) helps our model to deliver a flatter term structure. This finding is consistent with Bhamra, Kuehn, and Strebulaev (2010) and Feldhütter and Schaefer (2014). Nevertheless, this treatment is not strong enough to get the term structure to match the data.

Figure 3: **Credit spreads vs. market leverage.** This figure compares the relation between market leverage and credit spread in the model (solid line) with the data (circles).



distribution with the data in Figure 3. In the data, we first compute the firm-level spread as the value-weighted average spread of all bonds outstanding each month, and then compute the average spreads for all firms in different leverage bins. As the figure shows, the model fits the data quite well overall in both aggregate states. One limitation of the model is that it under-predicts the spreads for low-leverage firms. This is a well-known problem for structural credit risk models driven (primarily) by diffusion shocks (see e.g., Duffie and Lando (2001) – jumps in the cash-flow process can bring about default even for firms with low leverage ratios) and models that do not allow for sufficiently flexible leverage adjustments (see e.g., DeMarzo and He (2014) – credit spreads can be high today even with low leverage due to expectations of future debt issuance).

3.3.2 Bond market liquidity

Bid-ask spreads. Table 3 reports the empirical bid-ask spreads for bonds with different ratings across aggregate states. To calculate our model implied bid-ask spreads, again we correct for the convexity bias by relying on the empirical leverage distribution in Compustat of firms across ratings and aggregate states. Since the average maturity in TRACE data is around 8 years, the model implied bid-ask spread is calculated as the weighted average between the bid-ask spread of a 5-year bond and a 10-year bond.[14]

Our model is able to generate both cross-sectional and state-dependent patterns that quantitatively match what we observe in the data, especially in normal times. We calibrate three state-dependent holding cost parameters (χ_G, χ_B, and N) to the bid-ask spreads of three rating classes (superior, investment, junk) over two macroeconomic states. Overall, we observe a satisfactory fit for the cross-sectional pattern of bond market illiquidity, especially during normal times. One weakness is that the model does not generate as much cross-sectional variation in bid-ask spreads during recessions.

Bond-CDS spreads. Another reasonable bond market liquidity measure is the Bond-CDS spread, i.e., the credit spread minus the CDS spread. We assume a perfectly liquid CDS market, and Appendix D explains how we calculate the model-implied CDS spread. Since we do not specifically target the bond-CDS spreads in our calibration (unlike the bid-ask spreads), these moments provide a tougher test for the model.

Panel B in Table 3 presents the model implied Bond-CDS spread together with its data counterpart, for both normal and distress states, for 10-year bonds. In the data Bond-CDS spreads are higher for lower rated bonds and in bad times, a qualitative pattern that our model can capture. Quantitatively, our model undershoots the Bond-CDS spread in bad time while overshoots in good time. The fact that our model ignores the secondary market liquidity of CDS contracts is likely to cause the poor performance on the 10-year Bond-CDS spreads, and we wait for future research to address this issue.[15]

[14] Although not reported here, our model-implied bid-ask spread of longer-maturity bonds is higher than that of shorter-maturity bonds, which is consistent with previous empirical studies (eg. Edwards, Harris, and Piwowar (2007); Bao, Pan, and Wang (2011)).

[15] In practice, 5-year CDS contracts are traded with the most secondary market liquidity, rather than 10-year contracts. However, our calibration has focused on 10-year bonds to be more consistent with the existing literature on credit risk. Finally, since the CDS market is a zero-net-supply derivative market, how the secondary market liquidity of CDS market affects the pricing of CDS depends on market details; Bongaerts, De Jong, and Driessen (2011) show that the sellers of CDS contract earn a liquidity premium.

3.4 Comparative Statics

In this section, we perform several comparative static exercises to assess the importance of secondary market (il)liquidity for the model implied credit spreads of corporate bonds. Again, we focus on the results at the 10-year maturity.

[TABLE 4 ABOUT HERE]

What if there are state-dependent constant holding costs? The calibration takes the same baseline parameters, but chooses hc_s so that the implied bid-ask spreads across the ratings and macro states are consistent with the benchmark case. We then report the model-implied credit spreads, bid-ask spreads, and bond-CDS spreads in the rows labeled as "hc_s" in Table 4.

The results show that, relative to our model that features distance-to-default dependent holding costs, the "hc_s" model without this distance to default component fails to deliver a sizable cross-sectional differences in bond illiquidity across different ratings. *Qualitatively*, the endogenous default-illiquidity relation does not rely on the assumption of holding costs being decreasing in the firm's distance-to-default. An endogenous default-illiquidity loop arises as long as bond investors face a worse liquidity in the post-default secondary bond market. However, our results indicate the importance of matching default probabilities and leverage distributions in *quantitative* exercises. The relatively rating-insensitive bond illiquidity of the "hc_s" model translates to too flat credit spreads across ratings compared to the our baseline model, as shown in Table 4. Together, these results highlight the importance of our assumption of (default) risk-sensitive holding costs in explaining the cross section of credit spreads and bond liquidity.

What aspects of illiquidity do bid-ask spreads and bond-CDS spreads capture? Bid-ask spreads and bond-CDS spreads are commonly used proxies for corporate bond illiquidity. Our model helps demonstrate a key distinction between the two. The bid-ask spreads in our model depend crucially on the gap between the valuations of H- and L-type investors (see (7)); the bond-CDS spreads depend on the severity of the liquidity frictions in the form of (expected) holding costs, which directly affects the valuations of L-type investors and indirectly the H-type. This distinction is important for understanding the different aspects of bond illiquidity that bid-ask spreads and bond-CDS spreads are meant to capture, which can help us better use these different measures to monitor market liquidity in practice. It can also be used for empirical identification of different bond liquidity parameters.

We use a pair of comparative statics in Table 4 to illustrate the above effects. In the first case, the liquidity shock intensities in both stats are twice as high as the benchmark case, $\xi_G = 1.4, \xi_B = 2$. In the second case, the meeting intensities in both states are 50% higher than in the benchmark case, i.e., $\lambda_G = 75, \lambda_B = 30$.

It is intuitive that higher liquidity shock intensities increase the liquidity frictions. More subtly, while raising the average expected holding costs ex ante and thus driving the bond-CDS spreads higher, they would also tend to make the valuations of H-type investors closer to those of L-type investors, which would tend to reduce bid-ask spreads. That's indeed what we see in Table 4. Compared to the benchmark case, in the case "$\xi = 1.4, 2$", total credit spreads and bond-CDS spreads become higher across ratings, while the bid-ask spreads become lower. Moreover, the (unreported) implied default probabilities for all ratings also go up, as worse secondary market liquidity leads firms to default earlier (and thus more frequently) due to the rollover risk channel.

Next, higher meeting intensities in the case "$\lambda = 75, 30$" imply that L-type investors are expected to be able to find a dealer faster and thus incur smaller holding costs. This reduces the liquidity frictions, which significantly lowers the total credit spreads and bond-CDS spreads compared to the benchmark. At the same time, they also reduce the bid-ask spreads because the valuation of H-type and L-type investors again become more similar due to faster reversion from L to H-state.

Recall that we calibrate ξ_s to match the average bond turnover rates. In light of the results of the comparative statics for ξ_s, we also examine the model's performance under an alternative calibration with lower liquidity shock intensities. The results are reported in the Internet Appendix. Under this calibration, the model matches the moments of bid-ask spreads well, but significantly undershoots total credit spreads and bond-CDS spreads. The reason is that lowering liquidity shock intensities, all else equal, raises bid-ask spreads. To still match the bid-ask spreads in the data, the calibration then reduces the holding costs by lowering N, χ_G, χ_B, which, together with lower liquidity shock intensities, reduce the bond-CDS spreads and total credit spreads.

Other comparative statics. In the case "$m = 1/3$" in Table 4, we increase the average debt rollover frequency from 0.2 (an average debt maturity of 5 years) to 1/3 (an average debt maturity of 3 years). We are still studying a bond with a 10-year maturity; what we are changing is the firm's rollover risk: the faster the firms refinance themselves via the bond market, the greater the firms are exposed to the bond market liquidity risk, and hence the greater the default risk. Quantitatively, we observe that the default probability (not reported in the table) for the same Ba-rated 10-year bond increase from 15.9% to 19.6%. Credit spreads and bid-ask spreads both become higher than in the benchmark case, while bond-CDS spreads remain largely the same.

In another case (unreported), we remove all liquidity frictions by setting $\chi_s = 0$ (no holding costs). Obviously, the model implied bid-ask spreads are now identically zero. Both default probabilities and credit spreads become lower, but especially for credit spreads, and more so for higher rated bonds. For highly rated Aaa/Aa firms, in state G the spread falls from 86bps to 33bps, while in state B it falls from 136bps to 60bps.

4. Structural Default-Liquidity Decomposition

4.1 A Spread-based Decomposition Scheme

We propose a structural decomposition that nests the additive default-liquidity decomposition common in the literature. To focus on studying the credit spread and its default and liquidity parts relative to the risk-free rate, we first take out the exogenous liquidity premium of

Treasuries. Denote the credit spread relative to the risk-free rate by $\hat{cs}_{rf}$. Then, we use the following decomposition scheme

$$\hat{cs}_{rf} = \overbrace{\hat{cs}_{pureDEF} + \hat{cs}_{LIQ\to DEF}}^{\text{Default Component } \hat{cs}_{DEF}} + \overbrace{\hat{cs}_{pureLIQ} + \hat{cs}_{DEF\to LIQ}}^{\text{Liquidity Component } \hat{cs}_{LIQ}}. \tag{15}$$

We start by considering the decomposition of the spread into a "Default" component and a "Liquidity" component. Imagine a hypothetical small investor who is not subject to liquidity frictions and consider the spread that this investor demands for the bond over the risk-free rate. The resulting spread, denoted by $\hat{cs}_{DEF}$, only prices the default event given the unchanged default boundaries y^s_{def} from our model with liquidity frictions in equation (11). Then, the "Liquidity" component is defined as the residual between the total spread $\widehat{cs}$ and the default component $\hat{cs}_{DEF}$, $\hat{cs}_{LIQ} \equiv \hat{cs} - \hat{cs}_{DEF}$.[16].

Next, we define the "Pure-Default" component $\hat{cs}_{pureDEF}$ as the spread implied by the benchmark Leland model without secondary market liquidity frictions (e.g., setting $\xi_s = 0$ or $\chi_s = 0$) with the re-optimized default boundary $y^{Leland,s}_{def} < y^s_{def}$ as a perfectly liquid bond market leads to less rollover losses and thus less frequent default. This implies a smaller pure-default component $\hat{cs}_{pureDEF}$ relative to the default component $\hat{cs}_{DEF}$. The difference $\hat{cs}_{DEF} - \hat{cs}_{pureDEF}$ gives the "Liquidity-driven Default" component, which quantifies the increase in default risk due to the illiquidity of the secondary bond market.

Similarly, we decompose the liquidity component $\hat{cs}_{LIQ}$ into a "Pure-Liquidity" component and a "Default-driven Liquidity" component. Let $\hat{cs}_{pureLIQ}$ be the spread of a default-free bond that is only subject to liquidity frictions as in Duffie, Gârleanu, and Pedersen (2005).[17] The resulting spread now captures the pure-liquidity component. The residual $\hat{cs}_{LIQ} - \hat{cs}_{pureLIQ}$ is what we term the "Default-driven Liquidity" part of our credit spread: when distance-to-default falls, lower bond prices give rise to higher holding costs, which contribute to the

[16]This two-way decomposition is roughly in line with the methodology of Longstaff, Mithal, and Neis (2005) who use the spreads of the relatively liquid CDS contract on the same firm to proxy for the default component in corporate bond spreads and attribute the residual to the liquidity component

[17]Let p_{df} be this default-free price. Then the holding costs become a constant as we plug in $P^s = p_{df}$ in equation (5).

default-driven liquidity part.

4.2 Default-Liquidity Decomposition

The four-way decomposition scheme helps us separate *causes* from *consequences*, and emphasizes that lower liquidity (higher default risk) can lead to a rise in the credit spread via the default (liquidity) channel. Recognizing and further quantifying this endogenous interaction between liquidity and default is important in evaluating the economic consequence of policies that are either improving market liquidity (e.g., Term Auction Facilities or discount window loans) or alleviating default issues (e.g, direct bailouts).

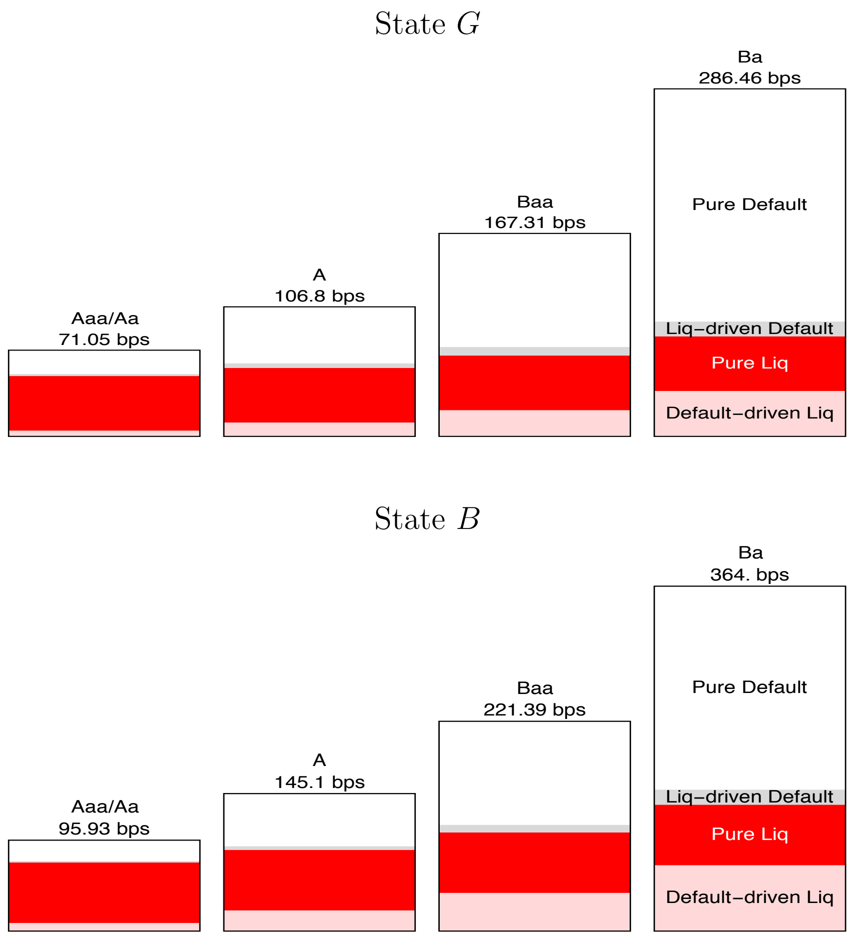
Cross-sectional spread decomposition. We perform the above default-liquidity decomposition for 10-year bonds at firm level, and then aggregate the results over firms and quarters based on the empirical leverage distribution for each rating category, as we did for default rates and credit spreads above. The results are presented in Table 5 and Figure 4. As discussed above, the credit spreads reported here are relative to the risk-free rate instead of Treasury yields. For each component, we report its absolute level in basis points, as well as the percentage contribution to the total credit spread (in parenthesis).

[TABLE 5 ABOUT HERE]

As expected, the "pure default" component rises for lower rated bonds. For example, in state G, the fraction of credit spreads explained by the " pure default" component starts from only 27% for Aaa/Aa rated bonds and monotonically increases to about 67% for Ba rated bonds. The "pure default" component for each rating category also increases during recessions, but it becomes a smaller fraction of total spreads because the other components of the spreads increase even more. The "pure liquidity" component, which is identical across ratings (by our definition it is based on a hypothetical default-free bond), is higher in state B (63bps) than state G (45bps).

The remainder of the credit spreads, which is around 10%$\sim$17% in state G and 11%$\sim$24% in state B, can be attributed to the two interaction terms, "liquidity-driven default" and

Figure 4: **Graphical illustrations of Structural Liquidity-Default Decomposition for 10-Year Bonds Across Ratings.** For numbers and explanations, see Table 5.



"default-driven liquidity." The "liquidity-driven default" part captures how endogenous default decisions are affected by secondary market liquidity frictions via the rollover channel, which is quantitatively small for the highest rating firms (3% or less) for Aaa/Aa rated bonds. Its quantitative importance rises for lower rating bonds. For example, for Ba rated bonds, the liquidity-driven default component on average accounts for 13bps (16bps) of the spreads in state G (B), which is 4% (5%) of the total spreads.

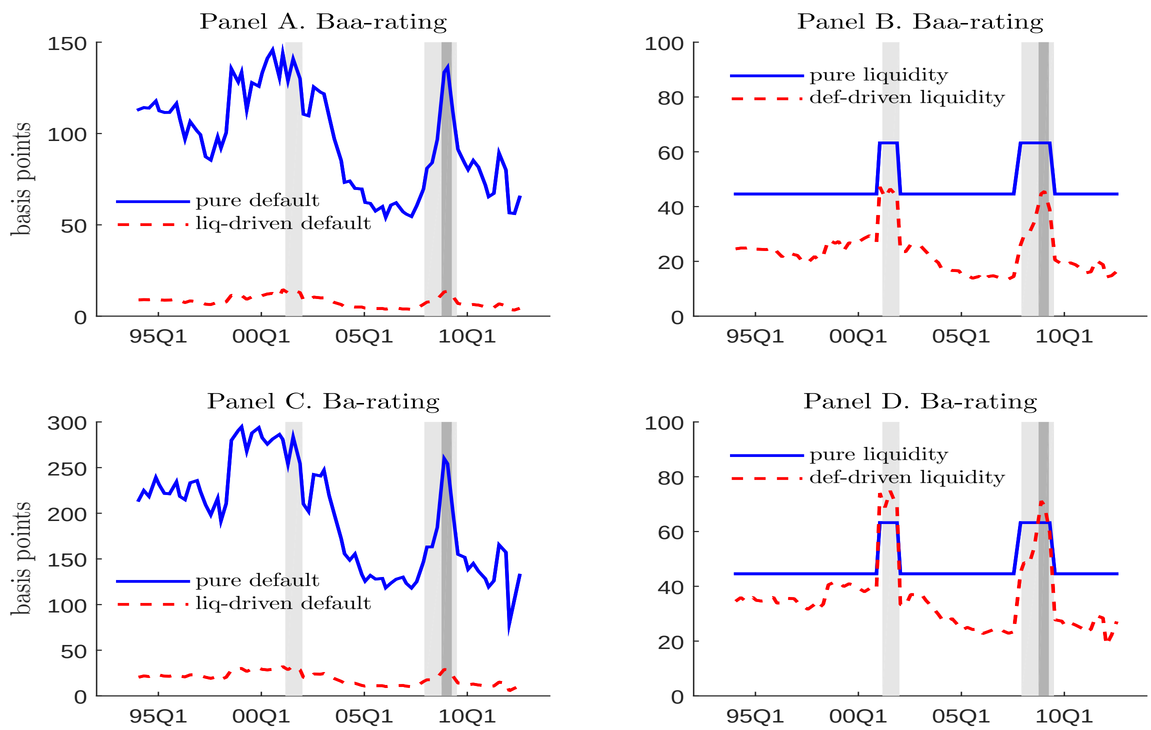
The "default-driven liquidity" component captures how secondary market liquidity endogenously worsens when a bond is closer to default. Given a more illiquid secondary market for defaulted bonds, a lower distance-to-default leads to a worse secondary market liquidity because of the increased holding cost in (5). The "default-driven liquidity" component is significant across all ratings: it accounts for about 7% to 13% (9% to 19%) of the credit spread in state G (B) from Aaa/Aa to Ba ratings.

Next, we apply the default-liquidity decomposition to the time series of credit spreads. For a given credit rating, each quarter we use the observed leverage distribution of firms within that rating class to compute the average credit spread and its four components in equation (15). We treat the NBER expansions and recessions as states G and B in our model, respectively. One caveat of this assumption is that the model treats the severity of the 2001 recession and the 2008-09 recession as the same (we have excluded 2008Q4 and 2009Q1 in this study so far), even though the latter is arguably more severe in reality.

Time-series spread decomposition. Figure 5 plots the time-series decomposition of credit spreads for Baa and Ba-rated bonds. To highlight the relative importance of the two interaction terms, in the left panels we plot the pure default spreads together with the liquidity-driven default spreads, while in the right panels we plot the pure liquidity spreads and the default-driven liquidity spreads. The four components of the credit spreads are driven by both the time series variation in the leverage distribution and the aggregate state, with recessions identified by grey bars. Thus, relative to Table 5, Figure 5 demonstrates the additional impact from the time series variation in the cross-sectional leverage distribution.

Consider the default components in Panel A first. For both the Baa-rated and Ba-rated firms, the liquidity-driven default spreads have meaningful magnitudes, but they are significantly smaller than the pure default spreads. Not surprisingly, both default components rise in the two recessions in the sample. The model predicts that the pure default part for Baa spreads is lower in 2008-09 than in 2001. In reality, the credit spreads in 2008-09 recession were much higher than in the 2001 recession (especially in the financial crisis period from late 2008 to early 2009, which is marked in dark grey in the plots), potentially due to capital-deprived financial intermediaries around that time (He and Krishnamurthy (2013) and Chen, Joslin, and Ni (2016)). A more fine-tuned model with a "deep recession"—in

Figure 5: **Time-series Structural Decomposition of Credit Spreads for Baa and Ba-rated Firms.** For each firm-quarter observation, we locate the corresponding cashflow level y that delivers the observed market leverage in Compustat (excluding financial and utility firms) and perform the structural liquidity-default decomposition for a 10-year bond following the procedure discussed in Section 4.1. For a given credit rating (Baa or Ba), we average across firms to obtain each component for each quarter from 1994 to 2012. Recessions are highlighted in grey. For completeness, we also calculate the model implied decomposition results for the crisis period from 2008Q4 to 2009Q1 in dark grey (which is excluded from the rest of this paper).



addition to "normal recession" modeled here—would help on this front.[18]

[18] Our model, given its current calibration, misses by a wide margin when confronted with the crisis quarters. For instance, for Aaa/Aa ratings, the model-implied credit spread is 120bps while it is 173bps in the data. For A-rated bonds, the numbers are 170bps (model) and 296bps (data); for Baa-rated bonds, the numbers are 257bps (model) and 544bps (data); and for Ba-rated bonds they are 419bps (model) and 932bps (data). Recall that we have targeted normal business expansion/recession moments to calibrate our key asset pricing parameters that governs the associated liquidity premium and risk premium. However, these premia are probably one order of magnitude smaller than those in the 2008-09 financial crisis. For instance, during 2008Q4 and 2009Q1, the financial intermediation sector got severely disrupted, and VIX even quadrupled from Aug 2008 (VIX around 20) to Jan 2009 (VIX around 80). Therefore, not surprisingly, our model–which is calibrated based on relatively normal periods–misses a wide margin when confronted by the crisis quarters. We are awaiting future projects to tie more closely the financial intermediary sector and the credit spread movement over these two crisis quarters.

Moving on to the liquidity components in Panel B, we observe that by definition the pure liquidity parts only depend on the aggregate state and are identical across ratings. In contrast, the default-driven liquidity spreads show significant variation over time and across ratings. For Baa-rated bonds, the default-driven liquidity spreads have a slightly lower magnitude than the pure liquidity spreads and similar time-series properties. For Ba-rated bonds, the default-driven liquidity spreads account for roughly half of the total liquidity spread on average, and for noticeably more in recessions.

In our baseline calibration, we set $\xi_G = 0.7$ and $\xi_B = 1$ to match the average secondary corporate bond market turnover rate in the entire TRACE sample. We could also choose ξ to match the bond market turnover rate for firms with both bonds and CDS contracts (a sample that Longstaff, Mithal, and Neis (2005) focus on). The higher turnover rates of these bonds would roughly double the liquidity shock intensities in our model to $\xi_G = 1.4$ and $\xi_B = 2$. Under this alternative calibration (the results are presented in Figure A1 in the Appendix), the two interaction terms become significantly larger, especially the default-driven liquidity component.

Cross-sectional price decomposition. So far we have been focusing our analysis on corporate bond spreads. An analogous decomposition applies to corporate bond prices, which in turn allows us to do a back-of-the-envelope calculation to determine the different sources of the costs of capital in the aggregate U.S. corporate bond market.[19]

Absent liquidity frictions and given $r_G = r_B = r$, a default-free bond sells at its face value if its coupon rate c is equal to the riskfree rate r. Thus, the gap between the face value and the price of the defaultable bond captures the value lost due to default and liquidity frictions. We decompose the total loss in value into four components in a similar way as we do for credit spreads. The details of the decomposition of bond prices are given in the Internet Appendix. Among the different components, the pure default component would be present even in the absence of liquidity frictions, and strictly speaking does not represent a true cost of capital.[20] In contrast, the remaining three can be viewed as the added costs of capital due

[19] We thank an anonymous referee for this suggestion.

[20] The pure default component can be strictly positive even in a frictionless Modgliani-Miller world, as

to liquidity frictions.

The results of this dollar-based decomposition are reported in Table A1 in the Appendix. Percentage wise, the contribution of each component to the price gap of 10-year bonds is quite close to the percentage wise contribution of each component to the credit spreads of 10-year bonds. For superior grade bonds (Aaa/Aa), the majority of value lost is due to the pure liquidity component, while for junk bonds (Ba), the majority of the value lost is due to the pure default component.

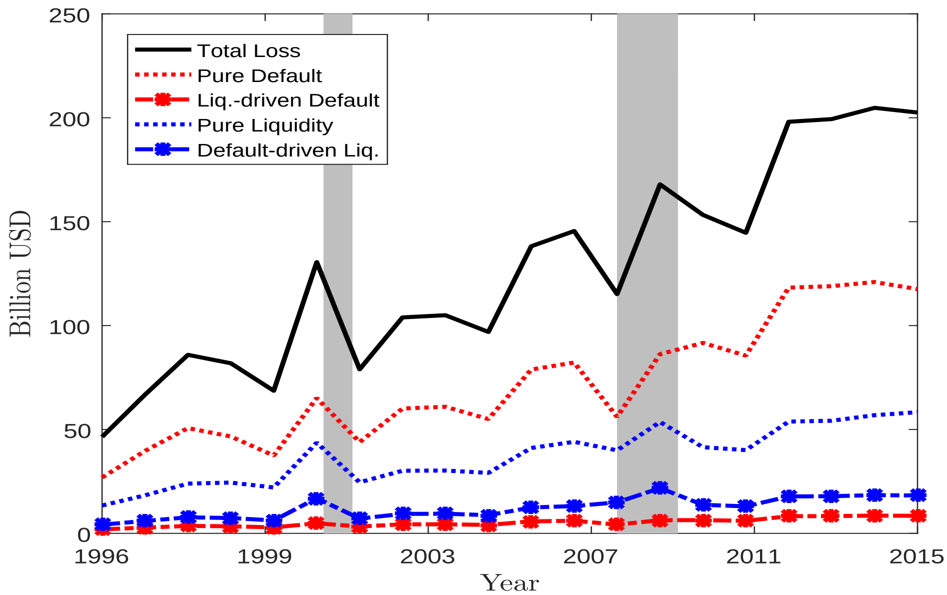
Aggregate bond-market value decomposition. Next, we assess the aggregate value of the four components for the entire U.S. corporate bond market. Based on the annual issuance data of the U.S. bond markets for the period of 1996 to 2015 by SIFMA, we produce an estimate of the total value lost for new issuances each year, and plot the time series of the four components in Figure 6.[21] The annual losses from corporate bond issuance (due to default and liquidity frictions) for the 20-year period sum up to \$2.9 trillion dollars (in 2015 dollars), which is about 14% of the total amount issued (\$20.6 trillion). Among the total losses, the pure default component accounts for \$1.65 trillion (56.8%) of the total losses. The liquidity-driven default, pure liquidity, and default-driven liquidity components account for \$0.12 trillion (4.1%), \$0.86 trillion (29.4%), and \$0.28 trillion (9.6%) of the total loss, respectively.

At this point, it might be tempting to add up the three liquidity-related components to estimate the total savings in the costs of capital if one were to remove liquidity frictions in the corporate bond market. An important caveat of such an estimation is that it ignores the potential endogenous responses of firms, consumers, and investors to such a change in the market environment, as well as the potential general equilibrium effects. Assessing the resulting impact of these effects is beyond the scope of our paper.

what the debt holders lose is just what equity gains.

[21] The issuance volume from SIFMA is separated into investment grade category and high yield category, but not by ratings. We use the average rating distribution for all bonds in our sample to proxy for the rating distribution within the investment grade category and treat all the bonds in the high yield category as Ba bonds (which will understate the total losses).

Figure 6: **Annual total loss in value for new issuances in the corporate bond market and its decomposition.** This figure plots our estimates of the total loss in value for new issuances in the corporate bond market due to default risk and liquidity frictions. At bond level, the loss in value is measured as the gap between the price of a defaultable bond and the price of the same bond if it is default-free and not subject to liquidity frictions.



4.3 Evaluating a liquidity provision policy

Our model-based decomposition of credit spreads is informative for evaluating policies that target lowering the borrowing cost of corporations in recession by injecting liquidity into the secondary market. For evaluating the effectiveness of such a policy, it is important to realize that firms' default policies respond to liquidity conditions and liquidity conditions respond to default risks. These endogenous forces are what our structural model is aiming to capture.

We consider the class of policies that improve the secondary market liquidity of corporate bonds. In practice, facing the deteriorating funding liquidity in the securities lending market during financial crisis, the Federal Reserve in US and European Central Bank created a series of liquidity provision polices, e.g., Term Asset-Backed Securities Loan Facility (TALF by U.S. Federal Reserve) or Securities Markets Program (SMP by Europen Central Bank, ECB later on). In essence, these facilities allow(ed) private financial institutions (including banks or dealers) to obtain funding from the central bank using a wide range of securities as collateral, with certain haircuts. For instance, in May 2010, ECB introduced the Phase I of SMP which started to accept sovereign bonds issued by Greece as eligible collateral. More recently, on March 10th 2016, the QE program by ECB announced that "investment grade euro-denominated bonds issued by non-bank corporations established in the euro area will be included in the list of assets that are eligible for regular purchases."

These facilities were created to improve the depth and liquidity of the secondary market liquidity of the targeted securities, and various policy reports and academic research suggest that these facilities indeed achieved their intended goals (e.g., Sack (2010) and Aggarwal, Bai, and Laeven (2015)). Although it is beyond the scope of this paper to model the details of how these lending facilities improve liquidity in our OTC search framework, a plausible mechanism is by making dealers more willing to intermediate trades in the secondary corporate bond market. For instance, in light of the micro-foundation of bond-price dependent holding cost in Appendix B, that these corporate bonds can be used as collateral to obtain financing should directly reduce the holding costs χ. Knowing that, dealers with backstop liquidity provision should be more willing to buy bonds from the low-type investors who demand liquidity, which increases the meeting intensity λ.[22]

Suppose that the government is committed to launching certain liquidity enhancing programs whenever the economy falls into a recession, and suppose that the policy is effective in making the secondary market in state B as liquid as that in state G. That is to say, the policy helps increase the meeting intensity between L-type investors and dealers in state B, so that λ_B rises from 20 to 50 (equal to λ_G), and reduce the state B holding cost parameter χ_B from 0.11 to 0.06 (equal to χ_G). Such a "liquidity provision policy" is admittedly simplistic and incomplete compared to the real world policy interventions. Our partial remedy here is to benchmark the hypothetical policy intervention in state B to the liquidity condition in state G and then judge the magnitude of policy intervention through certain observable market outcomes (e.g., credit spreads and bid-ask spreads).

[22] This effect can be micro-founded in a directed search framework (e.g., Guerrieri, Shimer, and Wright (2010)) which is more appealing than exogenously treating λ as a parameter in our random search framework.

Estimating the impact of the policy on spreads. Following the same procedure as in Table 5, we compare the credit spreads with and without the state-B liquidity provision policy. The results are shown in Table 6. The state-B liquidity provision policy lowers state-B credit spreads by about 52bps for Aaa/Aa rated bonds and up to 102bps for Ba rated bonds, which are about 54% and 28% of the credit spreads without the policy. Moreover, the state-B-only liquidity provision affects credit spreads in state G as well: the state-G credit spreads for Aaa/Aa (Ba) rated bonds go down by 29bps (52bps), or about 41% (18%).

[TABLE 6 ABOUT HERE]

We further investigate the underlying driving forces for the effectiveness of this liquidity provision policy. By definition, the "pure default" component remains unchanged (the default policy in that case is given any policy that only affects the secondary market liquidity. In Table 6, we observe that the pure-liquidity component accounts for about 83% (83%) of the drop in spread for Aaa/Aa rated bonds in state G (B). However, the quantitative importance of the pure-liquidity component diminishes significantly as we walk down the rating spectrum: for Ba rated bonds, it only accounts for about 46% (42%) in state G (B) of the decrease in the credit spread.

The market-wide liquidity provision not only reduces the investors' required compensation for bearing liquidity risk, but also alleviates some default risk. A better functioning financial market helps mitigate a firm's rollover risk and thus lowers its default risk — this force is captured by the "liquidity-driven default" part. Table 6 shows that it accounts for around 5% (3%) of credit spread change in Aaa/Aa rated bonds, and goes up to 12% (9%) for lower Ba rated bonds in state G (B).

Given that the hypothetical policy was limited to only improving secondary market liquidity, the channel of "default-driven liquidity" is more intriguing. Such an interaction term only exists in our model with endogenous liquidity featuring a positive feedback loop between corporate default and secondary market liquidity. Interestingly, this interaction is more important quantitatively: it accounts for around 12% (14%) of credit spread change in Aaa/Aa rated bonds, and goes up to 42% (49%) for Ba rated bonds in state G (B).

Estimating the dollar impact of the policy. Following the procedure in Section 4.2, Table A2 in the Appendix presents the dollar decomposition of the liquidity provision policy. The average prices of 10-year bonds rise by 2.3% $\sim$ 4% in state G and 4.1% $\sim$ 8% in state B. The breakdowns of the sources of the value increase (by the pure liquidity component, the liquidity-driven default component, and the default-driven liquidity component) are similar to those for credit spread changes.

In addition, we consider the following "back-of-the-envelop" estimation of the total impact that a liquidity provision policy we have considered in the paper would have for the aggregate corporate bond market. Take, for example, the year 2008, when the economy is in a recession.[23] According to SIFMA, the total book value of corporate debt outstanding in 2008 is $5.42 trillion. Since SIFMA does not provide additional distributional information of the bonds outstanding by rating or maturity, we simply assume that half of the bonds are of 10-year maturity and the other half are of 5-year maturity. In addition, we assume the rating distribution in our sample for 2008 also applies to the whole U.S. corporate debt market that year, and to be conservative, we treat all the bonds rated B or below and all the non-rated as Ba bonds. Based on these assumptions, a liquidity provision policy as we considered in the paper would raise the value of the aggregate U.S. corporate bond market by $256 billion. Had we only considered the direct impact of such a policy on bond prices (as captured by the pure liquidity component), the estimate would drop to $173 billion.

We again note that one of the important caveats of the calculation above is that it ignores any of the general equilibrium effects that such a liquidity provision policy would have (on banking lending, investment, consumption, stochastic discount factor, etc).

4.4 Implications on accounting recognitions of credit-related losses

The interaction between liquidity and default as documented above has important implications for the ongoing debate regarding how accounting standards should recognize credit losses on

[23]The Fed announced the Term Asset-Backed Securities Loan Facility (TALF) in November 2008. We are not assuming that TALF had the same impact as the thought experiment we are conducting.

financial assets. The interesting interplay between liquidity and default and their respective accounting recognitions have been illustrated in the collapse of Asset-Backed-Securities market during the second half of 2007. As Acharya, Schnabl, and Suarez (2013) document, because market participants are forward-looking, the liquidity problems (i.e., these conduits cannot roll over their short-term financing) occur before the actual credit-related losses (assets in the conduit start experiencing default). In a news release by Financial Accounting Standards Board (FASB) on 12/20/2013 (see FASB (2012), the FASB Chairman Leslie F. Seidman noted that "[t]he global financial crisis highlighted the need for improvements in the accounting for credit losses for loans and other debt instruments held as investment ... the FASB's proposed model would require more timely recognition of expected credit losses." However, there is no mentioning of the "liquidity" of these debt instrument at all. Our model not only suggests that (il)liquidity can affect the credit losses for these debt instruments, but more importantly offers a framework on how to evaluate the expected credit losses while taking into account the liquidity information.

5. Concluding Remarks

We build an over-the-counter search friction into a structural model of corporate bonds. In the model, default risk interacts with time varying macroeconomic and secondary market liquidity conditions. We calibrate the model to historical moments of default probability, bond yields, and empirical measures of bond liquidity. The model is able to match the conditional observed credit spreads across different rating classes and aggregate states. We propose a structural decomposition that captures the interaction of liquidity and default risks of corporate bonds over the business cycle and use this framework to evaluate the effects of liquidity provision policies during recessions. Our results identify quantitatively important economic forces that were previously overlooked in empirical researches on corporate bonds.

To focus on the interaction of liquidity and default, our model is cast in a partial equilibrium. Nevertheless, we believe these interactions have profound macroeconomic real impact, and the recent progress of general equilibrium models with credit risk is the path for future research.

References

Acharya, V. V., P. Schnabl, and G. Suarez, 2013, "Securitization without risk transfer," Journal of Financial Economics, 107, 515–536.

Aggarwal, R., J. Bai, and L. Laeven, 2015, "Sovereign Debt, Securities Lending, and Financing During Crisis," Unpublished working paper, Georgetown University and European Central Bank.

Bao, J., J. Pan, and J. Wang, 2011, "The illiquidity of corporate bonds," The Journal of Finance, 66, 911–946.

Berk, J., and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets," Journal of Political Economy, 112, 1269–1295.

Bhamra, H., L. Kuehn, and I. Strebulaev, 2010, "The levered equity risk premium and credit spreads: A unified framework," Review of Financial Studies, 23, 645–703.

Bongaerts, D., F. De Jong, and J. Driessen, 2011, "Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market," Journal of Finance, 66, 203–240.

Chen, H., 2010, "Macroeconomic conditions and the puzzles of credit spreads and capital structure," The Journal of Finance, 65, 2171–2212.

Chen, H., S. Joslin, and S. X. Ni, 2016, "Demand for Crash Insurance, Intermediary Constraints, and Risk Premia in Financial Markets," Working Paper.

Chen, H., Y. Xu, and J. Yang, 2015, "Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads," Discussion paper, National Bureau of Economic Research.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein, 2009, "On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle," Review of Financial Studies, 22, 3367–3409.

Cheng, I., and K. Milbradt, 2012, "The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts," Review of Financial Studies, 25, 1070–1110.

Collin-Dufresne, P., R. S. Goldstein, and S. J. Martin, 2001, "The Determinants of Credit Spread Changes," Journal of Finance, 56, 2177–2207.

Dangl, T., and J. Zechner, 2006, "Debt maturity and the dynamics of leverage," Working Paper.

David, A., 2008, "Inflation uncertainty, asset valuations, and the credit spreads puzzle," Review of Financial Studies, 21, 2487–2534.

DeMarzo, P., and Z. He, 2014, "Leverage dynamics without commitment," Working Paper.

Dick-Nielsen, J., 2009, "Liquidity Biases in TRACE," Journal of Fixed Income, 19, 43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando, 2012, "Corporate bond liquidity before and after the onset of the subprime crisis," Journal of Financial Economics, 103, 471–492.

Duffie, D., 1996, "Special Repo Rates," Journal of Finance, 51, 493–526.

Duffie, D., N. Gârleanu, and L. Pedersen, 2005, "Over-the-Counter Markets," Econometrica, 73, 1815–1847.

Duffie, D., N. Gârleanu, and L. H. Pedersen, 2007, "Valuation in Over-the-Counter Markets," Review of Financial Studies, 20, 1865–1900.

Duffie, D., and D. Lando, 2001, "Term Structures of Credit Spreads with Incomplete Accounting Information," Econometrica, 69, 633–664.

Duffie, D., and K. J. Singleton, 1999, "Modeling Term Structures of Defaultable Bonds," Review of Financial Studies, 12, 687–720.

Edwards, A., L. Harris, and M. Piwowar, 2007, "Corporate bond market transaction costs and transparency," The Journal of Finance, 62, 1421–1451.

Emery, K., 2007, "Special Comment: Moody's Ultimate Recovery Database," Discussion paper, Moody's Investors Services.

FASB, 2012, "News Release 12/20/12," http://www.fasb.org/jsp/FASB/FASBContent_C/NewsPage%26cid%3D1176160587833.

Feldhütter, P., 2012, "The same bond at different prices: identifying search frictions and selling pressures," Review of Financial Studies, 25, 1155–1206.

Feldhütter, P., and S. M. Schaefer, 2014, "The Credit Spread Puzzle - Myth or Reality?," Working Paper.

Friewald, N., R. Jankowitsch, and M. G. Subrahmanyam, 2012, "Illiquidity or credit deterioration: A study of liquidity in the US corporate bond market during financial crises," Journal of Financial Economics, 105, 18–36.

Gilson, S. C., K. John, and L. H. P. Lang, 1990, "Troubled debt restructurings*1: An empirical study of private reorganization of firms in default," Journal of Financial Economics, 27, 315–353.

Gomes, J. F., and L. Schmid, 2010, "Equilibrium Credit Spreads and the Macroeconomy," working paper.

Guerrieri, V., R. Shimer, and R. Wright, 2010, "Adverse Selection in Competitive Search Equilibrium," Econometrica, 78, 1823–1862.

He, Z., and A. Krishnamurthy, 2013, "Intermediary asset pricing," American Economic Review, 103, 732–770.

He, Z., and K. Milbradt, 2014, "Endogenous Liquidity and Defaultable Bonds," Forthcoming, Econometrica.

He, Z., and K. Milbradt, 2015, "Dynamic Debt Maturity," Working Paper.

He, Z., and W. Xiong, 2012, "Dynamic debt runs," Review of Financial Studies, 25, 1799–1843.

Huang, J.-Z., and M. Huang, 2012, "How much of the Corporate-Treasury Yield Spread is Due to Credit Risk," Review of Asset Pricing Studies, 2, 153–202.

Ivashina, V., D. C. Smith, and B. Iverson, 2013, "The Ownership and Trading of Debt Claims in Chapter 11 Restructurings," Working Paper.

Jankowitsch, R., F. Nagler, and M. G. Subrahmanyam, 2013, "The Determinants of Recovery Rates in the US Corporate Bond Market," Working Paper.

Jobert, A., and L. C. G. Rogers, 2006, "Option Pricing With Markov-Modulated Dynamics," SIAM Journal on Control and Optimization, 44, 2063–2078.

Krishnamurthy, A., 2002, "The bond/old-bond spread," Journal of Financial Economics, 66, 463–506.

Leland, H., 1994, “Bond prices, yield spreads, and optimal capital structure with default risk,” Working Paper.

Leland, H., 1998, “Agency costs, risk management, and capital structure,” Journal of Finance, 53, 1213–1243.

Liu, J., F. A. Longstaff, and R. E. Mandell, 2006, “The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks,” The Journal of Business, 79, 2337–2360.

Longstaff, F., S. Mithal, and E. Neis, 2005, “Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market,” The Journal of Finance, 60, 2213–2253.

Longstaff, F. A., 2004, “The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices,” The Journal of Business, 77, 511–526.

Moody’s, 2012, “Annual Default Study: Corporate Default and Recovery Rates, 1920-2011,” Discussion paper, Moody’s Investors Services.

Oehmke, M., and A. Zawadowski, 2013, “The anatomy of the CDS market,” Working Paper.

Roll, R., 1984, “A simple implicit measure of the effective bid-ask spread in an efficient market,” Journal of Finance, 39, 1127–39.

Sack, B. P., 2010, “Reflections on the TALF and the Federal Reserve’s role as liquidity provider,” Speech, Federal Reserve Bank of New York.

Suarez, G., E. Schroth, and L. Taylor, 2014, “Dynamic Debt Runs and Financial Fragility: Evidence from the 2007 ABCP Crisis,” Journal of Financial Economics, 112, 164–189.

Appendix

A SDF Approach with Undiversifiable Idiosyncratic Liquidity Shocks

As emphasized in the main text, this paper essentially treats the investor-level liquidity shocks as the asset-level payoff shocks—that is, modeling the low-type bond valuation as a state-dependent holding costs. We then price the asset based on the standard discounted cash-flow framework in which the pricing kernel is exogenously given by the representative agent's consumption process, i.e., we adopt a partial equilibrium approach. For general equilibrium credit risk models with an endogenous pricing kernel, see, e.g., Gomes and Schmid (2010). Obviously, there is a theoretical issue whether the undiversified idiosyncratic liquidity shocks will affect the pricing kernel itself, simply because in theory, idiosyncratic shocks are supposed to hit both the asset payoffs and the agent's consumption in the same time.

Our simplified treatment can be considered as a first-order approximation when the holdings of corporate bonds that are subject to liquidity shocks constitute only a small part of the representative agent's aggregate wealth. This is an empirically sound assumption; in fact, Flow of Funds data suggest that corporate bonds only accounts for 1.5% to 3.5% of households net worth (for detailed calculation, see footnote 2).

To see this point rigorously, let us consider a simple two-period framework. Consider the standard endowment economy with a representative agent whose preference is

$$U\left(C_0\right)+\mathbb{E}\left[\frac{1}{1+\beta}U\left(\widetilde{C}_1\right)\right],$$

in which C_0 and $\widetilde{C}_1$ denote his consumption at date 0 and 1, respectively.

In this economy there is an asset with exogenous supply $\widehat{x}>0$, which is initially equally owned by all agents with measure one. Besides this asset, the agent i is also endowed with c_0 and $\widetilde{c}_1$ of consumption goods, which are homogeneous across all agent i's.

The asset has a date 1 payoff of

$$1+\widetilde{\epsilon}^i.$$

Here, for simplicity we normalize the systematic component of the asset payoffs to be 1, and $\widetilde{\epsilon}^i$ captures the idiosyncratic shocks that are specific to agent i. In our context, $\widetilde{\epsilon}_i$ can be interpreted as the payoff shocks driven by idiosyncratic liquidity shocks (and hence with a subscript of i).

At date 0 all agents are identical. Denote p the endogenous date-0 price, and Δx^i the units of asset sold by the agent i. The agent i is hence solving

$$\max_{\Delta x^i} U\left(c_0 + p \underbrace{\Delta x^i}_{\text{amount sold at } t=0}\right) + \mathbb{E}\left[\frac{1}{1+\beta} U\left(\widetilde{c}_1 + \underbrace{\left(\widehat{x} - \Delta x^i\right)}_{\text{asset holdings at } t=1}\left(1+\widetilde{\epsilon}^i\right)\right)\right].$$

Since agents are identical ex ante, the equilibrium condition is $\Delta x^i = 0$ for all i. The first-order condition, evaluated at the equilibrium condition $\Delta x^i = 0$, is

$$pU'(c_0) = \frac{1}{1+\beta}\mathbb{E}\left[U'\left(\widetilde{c}_1 + \widehat{x} + \widehat{x}\widetilde{\epsilon}^i\right)\left(1+\widetilde{\epsilon}^i\right)\right]. \tag{16}$$

To a first order approximation where $\widetilde{\epsilon}^i$ is small, for any $\widetilde{c}_1$ we can expand the term inside the bracket of (16) as

$$U'\left(\widetilde{c}_1 + \widehat{x} + \widehat{x}\widetilde{\epsilon}^i\right) = U'\left(\widetilde{c}_1 + \widehat{x}\right) + U''\left(\widetilde{c}_1 + \widehat{x}\right)\widehat{x}\widetilde{\epsilon}^i. \tag{17}$$

Using the law of large numbers on individual idiosyncratic shocks $\int \widetilde{\epsilon}^i di = 0$, the equilibrium aggregate consumption is

$$c_0 = C_0 \text{ and } \widetilde{c}_1 + \widehat{x} = \widetilde{C}_1.$$

This implies that (17) is

$$U'\left(\widetilde{c}_1 + \widehat{x} + \widehat{x}\widetilde{\epsilon}^i\right) = U'\left(\widetilde{C}_1\right) + U''\left(\widetilde{C}_1\right)\widehat{x}\widetilde{\epsilon}^i. \tag{18}$$

Plug (18) back to (16), and devide both sides by $U'(C_0)$, we obtain (once certain integrability condition imposed to ensure the expecation is well-defined):

$$\begin{aligned} p &= \frac{1}{1+\beta}\mathbb{E}\left[\frac{U'\left(\widetilde{C}_1\right)}{U'(C_0)}\left(1+\widetilde{\epsilon}^i\right) + \frac{U''\left(\widetilde{C}_1\right)}{U'(C_0)}\widehat{x}\widetilde{\epsilon}^i\left(1+\widetilde{\epsilon}^i\right)\right] \\ &= \underbrace{\frac{1}{1+\beta}\mathbb{E}\left[\frac{U'\left(\widetilde{C}_1\right)}{U'(C_0)}\left(1+\widetilde{\epsilon}^i\right)\right]}_{\text{standard asset-pricing equation, } \widetilde{\epsilon}^i \text{ as dividend shocks}} + \underbrace{\frac{\widehat{x}}{1+\beta}\mathbb{E}\left[\frac{U''\left(\widetilde{C}_1\right)}{U'(C_0)}\widetilde{\epsilon}^i\left(1+\widetilde{\epsilon}^i\right)\right]}_{\text{idiosyncratic shocks affect pricing kernel}}. \end{aligned} \tag{19}$$

As indicated, the first part in (19) is exactly our treatment: the price of the asset today equals the discounted future asset dividends, with standard pricing kernel (independent of idiosyncratic shock $\widetilde{\epsilon}^i$) but treating liquidity shocks $\widetilde{\epsilon}^i$ as dividend shocks. The second term captures the fact that idiosyncratic shocks will affect the pricing kernel of any agent who cannot diversify his/her idiosyncratic shocks.

Nevertheless, notice that the second term in (19) is proportional to the equilibrium holding $\widehat{x}$.

As a result, the second term vanishes if $\widehat{x} \to 0$ so that the asset (in our context, the corporate bonds that are subject to liquidity shocks) is infinitesimal relative to the representative agent's aggregate consumption. This justifies our simplified treatment of taking an exogenous and homogeneous pricing kernel to price corporate bonds with idiosyncratic liquidity shocks.

B Holding Costs Microfoundation

This section gives the details of the derivation of bond-price dependent holding.[24] For simplicity, we ignore the time-varying aggregate state. Suppose that investors can only borrow at the riskfree rate r if the loan is collateralized; otherwise the borrowing rate is $r + \chi$ for all uncollateralized amounts. Suppose further that, when an investor is hit by a liquidity shock, he needs to raise an amount of cash that is large relative to his financial asset holdings. This implies that the investor will borrow at the uncollateralized rate $r + \chi$ in addition to selling all of his liquid assets.

The investor can reduce the financing cost of uncollateralized borrowing by using the bond as collateral to raise an amount $(1 - h(y)) P(y)$, where $h(y)$ is the haircut on the collateral and $P(y) = \frac{A(y)+B(y)}{2}$ is the midpoint bond price. Then, the ownership of the bond conveys a marginal value of $\chi (1 - h(y)) P(y)$ per unit of time (equaling to the net savings on financing cost) until the time of sale. At the time of sale, which occurs with intensity λ, on top of the sale proceeds equal to the bid price $B(y)$, the bond conveys a marginal value of $\chi B(y)$ per unit of time perpetually, or $\frac{\chi B(y)}{r}$ in present value. Notice that there is no haircut on the cash proceeds. Intuitively, a more risky collateral asset, due to a greater haircut, lowers its marginal value for an investor hit by liquidity shocks. This is the channel that generates bond-price dependent holding costs in our model.

We now characterize the value of the bond for a financially constrained investor, which can be different from the market price of the bond when the investor's marginal value of cash is above 1.

$$
\begin{aligned}
rV_H(y) &= c + \mathcal{L}V_H(y) + \xi_{HL}\left[V_L(y) - V_H(y)\right], &(20)\\
rV_L(y) &= c + \chi(1 - h(y)) P(y) + \mathcal{L}V_L(y) + \lambda\left[B(y) + \frac{\chi B(y)}{r} - V_L(y)\right], &(21)
\end{aligned}
$$

where $\mathcal{L}$ stands for the standard differential operator for the geometric Brownian motion of cashflows. Suppose that with probability β, the investor can make a take-it-or-leave-it offer to the dealer, and

[24] While we provide one micro-foundation for $hc_s(P^s)$ based on collateralized financing, there are other mechanisms via which institutional investors hit by liquidity shock incur extra losses if the market value of their bond holdings has dropped. For instance, suppose that corporate bond fund managers face some unexpected withdrawals when hit by a liquidity shock. As models with either learning managerial skills or coordination-driven runs would suggest, the deteriorating bond portfolios can trigger even greater fund outflows and extra liquidation costs. Models that analyze these issues include, for example, Berk and Green (2004), He and Xiong (2012), Cheng and Milbradt (2012), and Suarez, Schroth, and Taylor (2014).

with probability $(1-\beta)$ the dealer can make the offer to the investor. If the dealer gets to make the offer, his offering price, denoted by $B_d(y)$, should satisfy $B_d(y) + \frac{\chi B_d(y)}{r} - V_L(y) = 0$, which implies that

$$B_d(y) = \frac{r}{r+\chi} V_L(y). \tag{22}$$

The dealer's outside option is 0, and his valuation of the bond is simply $V_H(y)$, the price at which he can sell the bond on the secondary market to H-type investors. If the investor gets to make the offer, his offering price, denoted by $B_i(y)$, will be

$$B_i(y) = V_H(y). \tag{23}$$

Thus, with probability β, a surplus of $\left[(1+\frac{\chi}{r})V_H(y) - V_L(y)\right]$ accrues to the investor, and with probability $(1-\beta)$, zero surplus accrues to the investor. We can also see that cash has a Lagrange multiplier $1+\frac{\chi}{r} > 1$ in the liquidity state L.

Further, the mid-point bond price is

$$P(y) = \frac{A(y)+B(y)}{2} = \frac{V_H(y) + (1-\beta)V_H(y) + \beta\frac{r}{r+\chi}V_L(y)}{2} = \left(1-\frac{\beta}{2}\right)V_H(y) + \frac{\beta}{2}B_d(y). \tag{24}$$

Multiplying the V_L equation (21) by $\frac{r}{r+\chi}$, we rewrite to get

$$rB_d(y) = \frac{r}{r+\chi}\left[c + \chi(1-h(y))P(y)\right] + \mathcal{L}B_d(y) + \lambda\beta\left[V_H(y) - B_d(y)\right] \tag{25}$$

From (21) to (25), we have simply re-expressed the bond valuation in state L from being in utility terms into dollar terms through the Lagrange multiplier, which allows us to express the effective holding cost in dollars. Specifically, we can rewrite the flow term in (25) as

$$\frac{r}{r+\chi}\left[c + \chi(1-h(y))P(y)\right] = c - \underbrace{\frac{\chi}{r+\chi}\left[c - r(1-h(y))P(y)\right]}_{\text{holding cost}},$$

where the second term can be interpreted as the holding cost. Under appropriate parameterization, this holding cost is increasing in the spread for uncollateralized financing χ and the haircut $h(y)$.

While we have left the haircut function $h(y)$ as exogenous, it is intuitive that it should become larger when the bond becomes more risky, which is when the bond price is lower. Consider the following functional form,

$$h(y) = \frac{a_0}{P(y)} - a_1.$$

By choosing $a_0 = (N(r+\chi)-c)/r$ and $a_1 = \chi/r$, we obtain the holding cost $hc(y) = \chi(N - P(y))$ as in equation (5).

C Details of Model Solutions

We will see that the HJB's for the value functions are 2nd-order linear Matrix ODEs, which can be solved in closed form using the techniques of Jobert and Rogers (2006) (the technical proof of the value functions is relegated to the Internet Appendix). We apply the pricing kernel (1) without risk adjustments for the liquidity shocks to derive the HJBs describing the value functions.

Debt value function. Bond prices are given by $\mathbf{D}^{(2)} = \left[D_H^{G,2}, D_L^{G,2}, D_H^{B,2}, D_L^{B,2}\right]^\top$ on interval I_2 and by $\mathbf{D}^{(1)} = \left[D_H^{G,1}, D_L^{G,1}\right]^\top$ on interval I_1. Holding costs given liquidity shocks can be interpreted as negative dividends, which effectively lower the coupon flows that bond investors are receiving. Take the bond prices $\mathbf{D}^{(2)}$ on interval I_2 for example. The bond valuation equation can then be written in matrix form

$$\begin{aligned}\overbrace{\hat{\mathbf{R}} \cdot \mathbf{D}^{(2)}(y)}^{Discounting,4\times1} \quad &= \overbrace{\boldsymbol{\mu}\left(\mathbf{D}^{(2)}\right)'(y) + \frac{1}{2}\boldsymbol{\Sigma}\left(\mathbf{D}^{(2)}\right)''(y)}^{y\,Dynamics,4\times1} + \overbrace{\hat{\mathbf{Q}} \cdot \mathbf{D}^{(2)}(y)}^{Transition,4\times1} \\ &+ \underbrace{c\mathbf{1}_4}_{Coupon,4\times1} + \underbrace{m\left[p\mathbf{1}_4 - \mathbf{D}^{(2)}(y)\right]}_{Maturity,4\times1} - \underbrace{\mathbf{hc}(y)}_{Holding\,Cost,4\times1}. \end{aligned} \tag{26}$$

with boundary conditions $\mathbf{D}^{(2)}\left(y_{def}^B\right)_{[1,2]} = \mathbf{D}^{(1)}\left(y_{def}^B\right)$ and $\left(\mathbf{D}^{(2)}\left(y_{def}^B\right)\right)'_{[1,2]} = \left(\mathbf{D}^{(1)}\left(y_{def}^B\right)\right)'$ (value-matching and smooth-pasting across y_{def}^B in state G), $\mathbf{D}^{(2)}\left(y_{def}^B\right)_{[3,4]} = \mathbf{D}^{def}\left(y_{def}^B\right)_{[3,4]}$ and $\mathbf{D}^{(1)}\left(y_{def}^G\right) = \mathbf{D}^{def}\left(y_{def}^G\right)_{[1,2]}$ (value-matching at default boundary for defaulting bonds).

Here, $\hat{\mathbf{R}} \equiv \text{diag}\left([r_G, r_G, r_B, r_B]\right)$ is the diagonal matrix summarizing the state-dependent discount rate used by the bond holders: there is a possibly different discount rate for each aggregate state, but not for the individual state. Thus, the left-hand side of the equation gives the required rate-of-return for holding the bond.

The right-hand side gives the expected return of the bond. Here $\boldsymbol{\mu} \equiv \text{diag}\left([\mu_G, \mu_G, \mu_B, \mu_B]\right)$ and $\boldsymbol{\Sigma} \equiv \text{diag}\left(\left[\sigma_G^2, \sigma_G^2, \sigma_B^2, \sigma_B^2\right]\right)$, so the first (second) term on the right-hand side summarizes the impact of the different drifts (volatilities) of the process y on the bond price for the different aggregate states. These first two terms together thus summarize the movement in prices caused by movements in y. The third-term on the right-hand side summarizes the stochastic price jumps caused by state transitions facing each agent. A state transition is either reflecting an aggregate shock or an individual liquidity shock (including trading-induced "recovery" from L to H). The transition matrix $\hat{\mathbf{Q}}$ summarizes these transition intensities:

$$\hat{\mathbf{Q}} = \hat{\mathbf{Q}}^{(2)} = \begin{bmatrix} -\xi_G - \zeta_G & \xi_G & \zeta_G & 0 \\ \beta\lambda_G & -\beta\lambda_G - \zeta_G & 0 & \zeta_G \\ \zeta_B & 0 & -\xi_B - \zeta_B & -\xi_B \\ 0 & \zeta_B & \beta\lambda_B & -\beta\lambda_B - \zeta_B \end{bmatrix},$$

The fourth term on the right-hand side reflects the coupon payment and the fifth term captures the effect of debt maturing at an intensity m. Finally, the last term reflects the holding costs facing the agent, which is identically zero for H-type agents.

Equity value function. Equity prices are given by $\mathbf{E}^{(2)} = \left[E^{G,2}, E^{B,2}\right]^{\top}$ on interval I_2 and by $\mathbf{E}^{(1)} = \left[E^{G,1}\right]$ on interval I_1. Recall that when the firm refinances its maturing bonds, it can place newly issued bonds with H investors in a competitive primary market subject to proportional issuance costs ω, summarized by the matrix $\mathbf{S}^{(i)}$.[25] This implies that there are rollover gains/losses of $m\left[\mathbf{S}^{(i)} \cdot \mathbf{D}^{(i)}(y) - p\mathbf{1}_i\right] dt$ as a mass $m \cdot dt$ of debt holders matures at each instant. Here, bonds are reissued at $\mathbf{S}^{(i)} \cdot \mathbf{D}^{(i)}(y)$, while principal p is paid out on the maturing bonds. Denote by double letters (e.g. $\mathbf{xx}$) a constant for equity that takes an analogous place to the single letter (i.e. $\mathbf{x}$) constant for debt. Then, we can write down the equity valuation equation on interval I_i. For instance, on interval I_2 we have

$$\begin{aligned} \overbrace{\mathbf{RR} \cdot \mathbf{E}^{(2)}(y)}^{Discounting,2\times 1} \quad &= \quad \overbrace{\boldsymbol{\mu\mu}\left(\mathbf{E}^{(2)}\right)'(y) + \frac{1}{2}\boldsymbol{\Sigma\Sigma}\left(\mathbf{E}^{(2)}\right)''(y)}^{y\ Dynamics,2\times 1} + \overbrace{\hat{\mathbf{Q}}\mathbf{Q} \cdot \mathbf{E}^{(2)}(y)}^{Transition,2\times 1} \\ &\quad + \underbrace{\exp(y)\,\mathbf{1}_2}_{Cashflow,2\times 1} - \underbrace{(1-\pi)\,c\mathbf{1}_2}_{Coupon,2\times 1} + \underbrace{m\left[\mathbf{S}^{(2)} \cdot \mathbf{D}^{(2)}(y) - p\mathbf{1}_2\right]}_{Rollover,2\times 1} \end{aligned} \tag{27}$$

where π is the marginal tax rate. The boundary conditions, in addition to the optimality conditions for $\boldsymbol{y}_{def}$ given in the main text, are $\mathbf{E}^{(2)}\left(y^B_{def}\right)_{[1]} = \mathbf{E}^{(1)}\left(y^B_{def}\right)$, $\left(\mathbf{E}^{(2)}\left(y^B_{def}\right)\right)'_{[1]} = \left(\mathbf{E}^{(1)}\left(y^B_{def}\right)\right)'$ (value-matching and smooth-pasting across y^B_{def} in state G), and $\mathbf{E}^{(2)}\left(y^B_{def}\right)_{[2]} = \mathbf{E}^{(1)}\left(y^G_{def}\right) = 0$ (value-matching at default boundary for defaulting equity).

Again, the left-hand side gives the required rate of return of the equity holders, summarized by the discount rate matrix $\mathbf{RR} = \text{diag}\left([r_G, r_B]\right)$. The right-hand side summarizes the different terms that make up the expected return on equity: The first two terms are the price changes caused by the dynamics of y that are summarized by the drift matrix $\boldsymbol{\mu\mu} = \text{diag}\left([\mu_G, \mu_B]\right)$ and volatility matrix $\boldsymbol{\Sigma\Sigma} = \text{diag}\left(\left[\sigma^2_G, \sigma^2_B\right]\right)$. The matrices enter in the same fashion as their debt holders counterparts. As

[25] For instance, for $y \in I_2$ and state-independent issuance costs ω, we have $\mathbf{S}^{(2)} = \begin{bmatrix} (1-\omega) & 0 & 0 & 0 \\ 0 & 0 & (1-\omega) & 0 \end{bmatrix}$.

equity holders do not face individual liquidity risk, and are only exposed to aggregate shocks, the third term's state transition matrix only reflects aggregate jumps: $\hat{\mathbf{QQ}} = \hat{\mathbf{QQ}}^{(2)} = [-\zeta_G, \zeta_G; \zeta_B, -\zeta_B]$. The fourth term on the right-hand side (the first-term of the second row) reflects the cash flow from the assets in place that accrue to equity holders directly every instant (remember that $Y = e^y$). The fifth term reflects the (before-tax) coupon payout from servicing the interest on the debt. The final term reflects the rollover payoff to the equity holders: a mass of $m \cdot dt$ bonds mature between t and $t + dt$, and each require a payment of p, while new bonds with face-value p are issued for proceeds of $\mathbf{S}^{(2)} \cdot \mathbf{D}^{(2)}(y)$ by the equity holders.

D Model Implied Credit Default Swap

Since the CDS market is much more liquid than that of corporate bonds, following Longstaff, Mithal, and Neis (2005) we compute the model implied CDS spread under the assumption that the CDS market is perfectly liquid.[26] Let τ (in years from today) be the time of default. Formally, if today is time u, then $\tau \equiv \inf\{t : y_{u+t} \leq y_{def}^{s_{u+t}}\}$ can be either the first time at which the log cash-flow rate y reaches the default boundary y_{def}^s in state s, or when $y_{def}^G < y_t < y_{def}^B$ so that a change of state from G to B triggers default. Thus, for a T-year CDS contract, the required flow payment f is the solution to the following equation:

$$\mathbb{E}^{\mathcal{Q}}\left[\int_0^{\min[\tau,T]} \exp(-rt) f dt\right] = \mathbb{E}^{\mathcal{Q}}\left[\exp\left(-r\tau 1_{\{\tau \leq T\}}\right) LGD_\tau\right], \tag{28}$$

where LGD_τ is the loss-given-default, which is the bond face value p minus its recovery value, where the recovery value is defined as the mid transaction price at default. If there is no default, no loss-given-default is paid out by the CDS seller. We calculate the flow payment f that solves (28) using a simulation method. The CDS spread, f/p, is defined as the ratio between the flow payment f and the bond's face value p.

[26] Arguably, the presence of the CDS market will in general affect the liquidity of the corporate bond market; but we do not consider this effect. A recent theoretical investigation by Oehmke and Zawadowski (2013) shows ambiguous results in this regard. Further, there is some ambiguity in the data about which way the illiquidity in the CDS market affects the CDS spread. Bongaerts, De Jong, and Driessen (2011) show that the sellers of CDS contracts earn a liquidity premium.

Figure A1: **Time-Series Structural Decomposition of Credit Spreads for Baa and Ba-rated Firms with More Frequent Liquidity Shocks.** Liquidity shock intensities are $\xi_G = 1.4$ and $\xi_B = 2$ which double the benchmark liquidity shock intensities in Table 1. We also adjust the holding cost intercept down from $N = 115$ to $N = 110$ to deliver similar total credit spreads for Baa ratings.

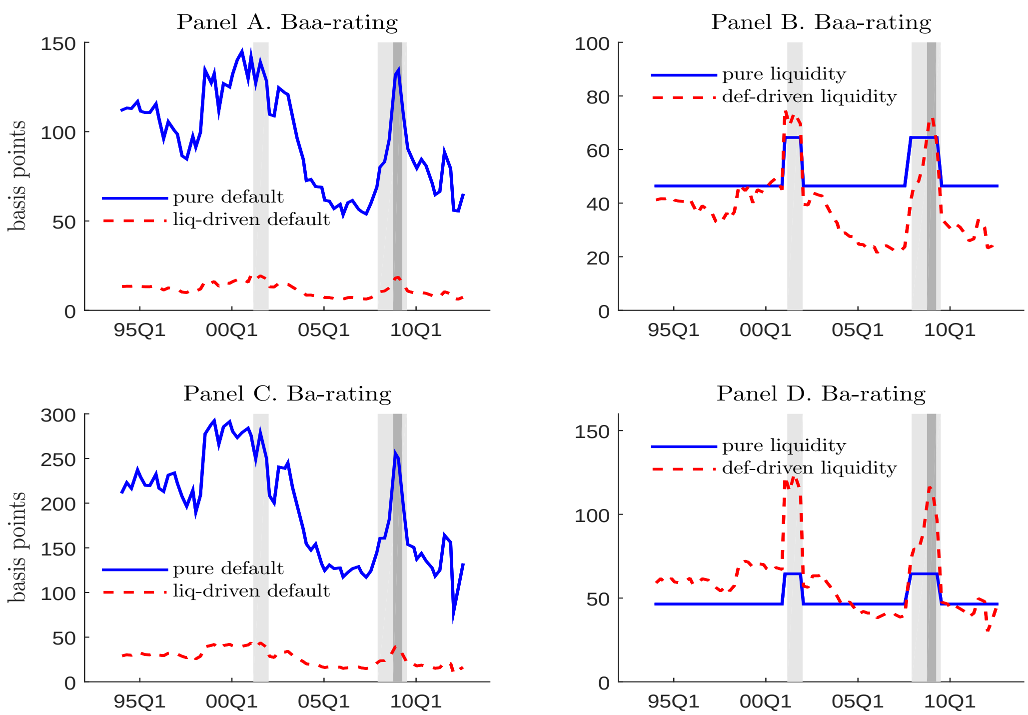


Table 1: **Benchmark Parameters.** Panel A reports pre-fixed parameters. We explain how we pick these parameter values in Section 3.1. Panel B reports four calibrated parameters. The idiosyncratic volatility σ_f, the holding cost intercept N, and holding cost slopes χ_s are set to target Baa default probability, investment grade bid-ask spreads in both states, and superior grade bid-ask spread in state G. Unreported parameters are the tax rate $\pi = 0.35$ and bond face value $p = 100$.

Symbol	Description	State G	State B	Justification / Target
		A. Pre-fixed parameters		
$\zeta^{\mathbb{P}}$	Transition density	0.1	0.5	Chen (2010)
$\exp(\kappa)$	Jump risk premium	2.0	0.5	Chen (2010)
η	Risk price	0.17	0.22	Chen (2010)
r	Risk free rate	0.05		average nominal riskfree rate
$\mu_{\mathbb{P}}$	Cash flow growth	0.045	0.015	corporate profit data
σ_m	Systematic vol	0.10	0.11	equity volatility
ω	Primary market issuance cost	0.01		Chen (2010)
m	Average maturity intensity	0.2		Chen, Xu, and Yang (2015)
Δ	Treasury liquidity premium	15bps	40bps	repo-Treasury spread
λ	Meeting intensity	50	20	anecdotal evidence
ξ	Liquidity shock intensity	0.7	1.0	bond turnover rate (TRACE)
β	Investor's bargaining power	0.05		Feldhütter (2012)
α_H	Recovery rate of H type	58.71%	32.56%	bid prices for defaulted bonds and bid-ask spreads
α_L	Recovery rate of L type	57.49%	30.50%	bid prices for defaulted bonds and bid-ask spreads
		B. Calibrated parameters		
σ_f	Idiosyncratic vol	0.25		Baa default rates
N	Holding cost intercept	115		Investment grade bid-ask spreads (G and B)
χ	Holding cost slope	0.06	0.11	Superior bid-ask spread in state G

Table 2: **Default Probabilities and Credit Spreads Across Credit Ratings.** Default probabilities are cumulative default probabilities over 1920-2011 from Moody's investors service (2012), and credit spreads are from FISD and TRACE transaction data over 1994-2010. We report the time series mean, with the standard deviation (reported underneath) being calculated using Newey-West procedure with 15 lags.

	Maturity = 5 years				Maturity = 10 years			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
	Panel A. Default probability (%)							
data	0.7	1.3	3.1	9.8	2.1	3.4	7.0	19.0
model	0.2	0.8	2.4	7.5	1.6	3.9	7.9	15.8
	Panel B. Credit spreads (bps)							
	State G							
data	56	86	149	315	61	90	150	303
	(4)	(7)	(15)	(34)	(4)	(6)	(13)	(23)
model	57	72	112	234	81	120	182	303
	State B							
data	107	171	275	542	106	159	262	454
	(6)	(10)	(24)	(30)	(7)	(14)	(29)	(44)
model	111	134	189	340	130	182	261	406

Table 3: **Bid-ask Spreads and Bond-CDS Spreads Across Credit Ratings.** In Panel A, the normal time bid-ask spreads in the data are taken from Edwards, Harris, and Piwowar (2007) for median sized trades. The numbers in recession are normal time numbers multiplied by the empirical ratio of bid-ask spread implied by Roll's measure of illiquidity (following Bao, Pan, and Wang (2011)) in recession time to normal time. The model implied bid-ask spread are computed for a bond with time-to-maturity of 8 years, which is the mean time-to-maturity of frequently traded bonds (where we can compute a Roll (1984) measure) in the TRACE sample. The Bond-CDS spreads in Panel B are for 10-year bonds.

	Panel A. Bid-Ask spreads (bps)					
	State G			State B		
	Superior	Investment	Junk	Superior	Investment	Junk
data	40	50	70	77	125	218
model	40	47	61	114	137	186

	Panel B. Bond-CDS spreads (bps)							
	State G				State B			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
data	23	37	58	68	72	104	162	191
model	48	61	69	92	53	60	79	107

Table 4: **Comparative statics and comparison to alternative models for 10 year bonds.** The "benchmark" case is our benchmark calibration. The "hc_s" case is when holding costs depend on aggregate state only (we calibrate $hc_G = 1.38$ and $hc_B = 2.32$ to match investment-grade bid-ask spreads). The "$\xi = 1.4, 2$" case is when we double the liquidity shock intensities in both states. The "$\lambda = 75, 30$" case is when we increase the meeting intensity in both states from $(50, 20)$ to $(75, 30)$. The "$m = 1/3$" case is when we lower the firm's average debt maturity from 5 ($m = 0.2$) to 3 years.

Panel A. Credit spreads (bps)

	State G				State B			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
benchmark	86	122	182	301	136	185	261	404
hc_s	95	126	176	278	146	185	245	359
$\xi = 1.4, 2$	106	137	192	313	165	209	282	432
$\lambda = 75, 30$	65	87	130	232	108	139	193	313
$m = 1/3$	96	141	210	332	148	209	296	445

Panel B. Bid-Ask Spreads (bps)

	State G			State B		
	Superior	Investment	Junk	Superior	Investment	Junk
benchmark	40	47	61	114	137	186
hc_s	45	47	49	124	129	138
$\xi = 1.4, 2$	34	40	52	87	104	142
$\lambda = 75, 30$	26	29	37	80	93	123
$m = 1/3$	41	52	69	117	147	203

Panel C. Bond-CDS Spreads (bps)

	State G				State B			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
benchmark	48	53	61	60	69	79	92	107
hc_s	60	61	60	46	84	85	86	79
$\xi = 1.4, 2$	73	81	96	111	104	118	138	171
$\lambda = 75, 30$	35	38	44	47	50	56	65	78
$m = 1/3$	49	54	59	48	71	82	94	102

Table 5: **Structural Decomposition for 10-Year Bonds Across Ratings.** We perform the structural liquidity-default decomposition for a 10-year bond following Section 4.1, given rating and aggregate state, and then aggregate over the empirical leverage distribution in Compustat. The reported credit spreads are relative to the risk-free rate.

Rating	State	Credit Spread	Structural Decomposition			
			Pure Def	*Liq* $\rightarrow$ *Def*	*Pure Liq*	*Def* $\rightarrow$ *Liq*
Aaa/Aa	G	71	20	2	45	5
	(%)		(27)	(3)	(63)	(7)
	B	96	22	2	63	9
	(%)		(23)	(2)	(66)	(9)
A	G	107	46	4	45	12
	(%)		(43)	(4)	(42)	(11)
	B	145	55	4	63	22
	(%)		(38)	(3)	(44)	(15)
Baa	G	167	93	7	45	22
	(%)		(56)	(4)	(27)	(13)
	B	221	109	9	63	41
	(%)		(49)	(4)	(29)	(18)
Ba	G	286	192	13	45	38
	(%)		(67)	(4)	(16)	(13)
	B	364	215	16	63	70
	(%)		(59)	(5)	(17)	(19)

Table 6: **Effect of Liquidity Provision Policy on 10-Year Bonds Across Ratings.** We consider a policy experiment that improves the liquidity environment (χ and λ) in the B state to be as good as G state (i.e., $\chi_B = 0.06$ and $\lambda_B = 50$). We fix the distribution of cash flow levels y at the values that deliver the observed market leverage distribution in Compustat (excluding financial and utility firms) for the corresponding state in our baseline calibration. We then report the average credit spreads (relative to the risk-free rate) under the policy for each state together with credit spread without policy. We perform the structural liquidity-default decomposition to examine the channels that are responsible for the reduced borrowing cost. We report the percentage contribution of each component to the credit spread change.

Rating	State	Credit Spread		Contribution of Each Component		
		w/o. policy	w. policy	pure LIQ (%)	LIQ→DEF (%)	DEF→LIQ (%)
Aaa/Aa	G	71.1	41.9	83	5	12
	B	96.0	44.4	83	3	14
A	G	107	71.9	69	8	23
	B	145	82.1	68	5	27
Baa	G	167	125	57	10	33
	B	221	143	55	7	38
Ba	G	286	234	46	12	42
	B	364	262	42	9	49

Table A1: **Structural Decomposition of the Price Gap between a 10-Year Defaultable Bond and Default-free Bond Without Liquidity Frictions.** The default-free Bond without liquidity frictions has its value equal to the face value of the bond, which is $100.

Rating	State	Value Lost	Default-Liquidity Decomposition			
			Pure Def	$Liq \rightarrow Def$	*Pure Liq*	$Def \rightarrow Liq$
Aaa/Aa	G	5.4	1.5	0.1	3.4	0.3
	(%)		(28)	(3)	(64)	(6)
	B	7.2	1.7	0.2	4.8	0.5
	(%)		(23)	(2)	(67)	(7)
A	G	8.0	3.6	0.3	3.4	0.7
	(%)		(45)	(4)	(43)	(8)
	B	10.7	4.2	0.3	4.8	1.3
	(%)		(40)	(3)	(45)	(12)
Baa	G	12.2	7.0	0.5	3.4	1.2
	(%)		(58)	(4)	(28)	(10)
	B	15.8	8.2	0.6	4.8	2.2
	(%)		(52)	(4)	(31)	(14)
Ba	G	19.8	13.8	0.8	3.4	1.7
	(%)		(70)	(4)	(17)	(9)
	B	24.4	15.3	1.0	4.8	3.2
	(%)		(63)	(4)	(20)	(13)

Table A2: **Effect of Liquidity Provision Policy on Bond Valuation for 10-Year Bonds.** We consider a policy experiment that improves the liquidity environment (χ and λ) in the B state to be as good as G state (i.e., $\chi_B = 0.06$ and $\lambda_B = 50$). We fix the distribution of cash flow levels y at the values that deliver the observed market leverage distribution in Compustat (excluding financial and utility firms) for the corresponding state in our baseline calibration. We then report the average dollar value for each state without and with the policy. We also perform the structural liquidity-default decomposition to examine the channels that are responsible for the increase in bond value. The default-free Bond without liquidity frictions has its value equal to the face value of the bond, which is $100.

Rating	State	Dollar Value			Contribution of Each Component		
		w/o. policy	w. policy	Increase (%)	pure LIQ (%)	LIQ→DEF (%)	DEF→LIQ (%)
Aaa/Aa	G	94.59	96.78	2.3	85	6	9
	B	92.79	96.59	4.1	85	4	11
A	G	92.00	94.53	2.7	73	10	17
	B	89.31	93.79	5.0	73	5	23
Baa	G	87.81	90.71	3.3	64	10	26
	B	84.25	89.45	6.2	62	5	32
Ba	G	80.18	83.40	4.0	57	9	34
	B	75.63	81.70	8.0	54	6	41