Online Appendix for

"Rare Disasters and Risk Sharing with Heterogeneous Beliefs"

A Securities' prices and portfolio positions

In this appendix we compute the prices of the claim on aggregate endowment (stock), the claim on individual agents' consumption streams (agents' personal wealth), disaster insurance, and the equilibrium portfolio positions. We begin with the general setting of time-varying disaster intensity. To concentrate on the effects of heterogeneous beliefs, we assume that the two agents have the same relative risk aversion γ.

A.1 Aggregate and individual consumption claim prices: general setting

The price of the aggregate endowment claim is

$$P_t = \int_0^\infty E_t^A \left[\frac{M_{t+T}^A}{M_t^A} C_{t+T} \right] dT \,, \tag{O.1}$$

where M_t^A is the stochastic discount factor

$$M_t^A = e^{-\rho t} C_t^{-\gamma} \left(1 + \left(\zeta_0 e^{\log \eta_t}\right)^{\frac{1}{\gamma}} \right)^{\gamma} . \tag{O.2}$$

This price can be viewed as a portfolio of zero coupon aggregate consumption claims

$$\begin{aligned} M_t^A P_t^{t+T} &= E_t^A [M_{t+T}^A C_{t+T}] \\ &= e^{-\rho(t+T)} e^{T[\bar{g}_A(1-\gamma)+\frac{1}{2}\sigma_c^2(1-\gamma)^2]} e^{(1-\gamma)c_t} \times E_t^A \left[e^{(1-\gamma)c_{t+T}^d} \left(1 + \left(\zeta_0 e^{\log \eta_{t+T}}\right)^{\frac{1}{\gamma}} \right)^{\gamma} \right] . \end{aligned}$$

Under our assumption of integer γ, the final term will be a sum of expectations of the form

$$E_t^A [e^{(1-\gamma)c_{t+T}^d + \beta_i \log \eta_{t+T})}] = e^{A_i(T)+(1-\gamma)c_t^d + \beta_i \log \eta_t + B_i(T)\lambda_t}, \tag{O.3}$$

where (A_i, B_i) satisfy a simplified version of the familiar Riccati differential equations

$$\dot{B}_i = -\frac{\bar{\lambda}^B}{\bar{\lambda}^A}\beta_i - \kappa B_i + \frac{\sigma_\lambda^2}{2}B_i^2 + (\phi(\langle 1-\gamma, \beta_i\rangle) - 1)\ , \quad B_0(0) = 0\,, \tag{O.4a}$$

$$\dot{A}_i = \kappa\theta B_i\ , \quad A_i(0) = 0\,, \tag{O.4b}$$

where ϕ is the moment generating function of jumps in $\langle c_t^d, a_t\rangle$.

It follows that price/consumption ratio of the zero-coupon equity varies only with the stochastic weight $\tilde{\zeta}_t$ and the disaster intensity:

$$P_t^{t+T} = C_t h^T(\lambda_t, \tilde{\zeta}_t)\,. \tag{O.5}$$

Next, agent A's wealth $P_t^A = \int_0^\infty E_t^A\left[\frac{M_{t+T}^A}{M_t^A}C_{t+T}^A\right]dT$ at time t is a portfolio of her zero coupon consumption claims

$$\begin{aligned} M_t^A P_t^{A,t+T} &= E_t^A[M_{t+T}^A C_{t+T}^A] \\ &= e^{-\rho(t+T)}e^{T[\bar{g}_A(1-\gamma)+\frac{1}{2}\sigma_c^2(1-\gamma)^2]}e^{(1-\gamma)c_t} \times E_t^A\left[e^{(1-\gamma)c_{t+T}^d}\left(1+(\zeta_0 e^{\log\eta_{t+T}})^{\frac{1}{\gamma}}\right)^{\gamma-1}\right]. \end{aligned}$$

We can compute agent A's wealth process by making a similar binomial expansion as in the case of P_t, and then computing the expectation concerning the same affine jump diffusion process. Finally, the wealth process of agent B is simply $P_t^B = P_t - P_t^A$.

A.2 Special case: constant disaster risk

Closed form expressions can now be obtained in the special case of constant disaster intensity and constant disaster size. Let's denote $\tilde{\zeta}_t \equiv \zeta_0 e^{\log\eta_t}$. Again by expanding the binomial for the cases with integer γ,

$$\begin{aligned} E_t^A\left[M_{t+T}^A C_{t+T}\right] &= e^{-\rho(t+T)}E_t^A\left[\left(1+(\tilde{\zeta}_{t+T})^{1/\gamma}\right)^\gamma C_{t+T}^{1-\gamma}\right] \\ &= e^{-\rho(t+T)}C_t^{1-\gamma}\sum_{k=0}^{\gamma}\binom{\gamma}{k}E_t^A\left[\frac{(\tilde{\zeta}_{t+T})^{k/\gamma}C_{t+T}^{1-\gamma}}{C_t^{1-\gamma}}\right]. \end{aligned}$$

Plugging in the explicit expressions for aggregate consumption C_t, the stochastic discount factor M_t^A, and performing the simple affine jump diffusion expectation we obtain

$$P_t^{t+T} = C_t \sum_{k=0}^{\gamma} \alpha_{k,t} e^{-\beta_k T} \,, \tag{O.6}$$

with

$$\alpha_{k,t} \equiv \begin{pmatrix} \gamma \\ k \end{pmatrix} \frac{(\tilde{\zeta}_t)^{k/\gamma}}{(1+(\tilde{\zeta}_t)^{1/\gamma})^{\gamma}} \,, \tag{O.7a}$$

$$\beta_k \equiv \rho + (\gamma-1)\bar{g} - \frac{1}{2}\sigma_c^2(\gamma-1)^2 - \bar{\lambda}(e^{(\gamma-1)\bar{d}+\frac{k\Delta a}{\gamma}} - 1) + \frac{\bar{\lambda}k}{\gamma}(e^{\Delta a} - 1) \,, \tag{O.7b}$$

where Δa is given in (5).

Finally, integrating over time T yields the explicit price of aggregate endowment claim

$$P_t = \int_0^\infty P_t^{t+T} dT = C_t \sum_{k=0}^{\gamma} \frac{\alpha_{k,t}}{\beta_k} \,. \tag{O.8}$$

The restriction $\beta_k^A > 0$ is needed to ensure finite value for P_t. We will come back to this type of restriction below.

By identical approach, we obtain the price of agent A's consumption claim (i.e. her wealth process)

$$P_t^A = \int_0^\infty P_t^{A,t+T} dT = C_t \sum_{k=0}^{\gamma-1} \frac{\alpha_{k,t}^A}{\beta_k} \,, \tag{O.9}$$

where β_k remains the same as above and

$$\alpha_{k,t}^A \equiv \begin{pmatrix} \gamma-1 \\ k \end{pmatrix} \frac{(\tilde{\zeta}_t)^{k/\gamma}}{(1+(\tilde{\zeta}_t)^{1/\gamma})^{\gamma}} \,. \tag{O.10}$$

Price of disaster insurance

Let $P_{t,t+T}^{DI}$ denotes the price of disaster insurance which pays \$1 at maturity time $t+T$ if there was at least one disaster taking place in the time interval $(t, t+T)$. In the main text we consider disaster insurance P_t^{DI} of maturity $T = 1$ in particular.

$$
\begin{aligned}
P_{t,t+T}^{DI} &= E_t^A \left[\frac{M_{t+T}^A}{M_t^A} \mathbf{1}_{(N_{t+T} > N_t)} \right] \\
&= \frac{e^{-\rho T}}{(C_t^A)^{-\gamma}} E_t^A \left[(C_T^A)^{-\gamma} \mathbf{1}_{(N_{t+T} > N_t)} \right] \\
&= \frac{e^{(-\rho - \gamma \bar{g} + \frac{1}{2}\gamma^2 \sigma_c^2)T}}{(1 + (\tilde{\zeta}_t)^{1/\gamma})^\gamma} E_t^A [e^{\gamma \bar{d} \Delta N_T} (1 + (\tilde{\zeta}_{t+T})^{1/\gamma} e^{(\Delta a \Delta N_T - \bar{\lambda} T (e^{\Delta a} - 1))/\gamma})^\gamma \mathbf{1}_{(\Delta N_T > 0)}] \\
&= \frac{e^{(-\rho - \gamma \bar{g} + \frac{1}{2}\gamma^2 \sigma_c^2)T}}{(1 + (\tilde{\zeta}_t)^{1/\gamma})^\gamma} \{ E_t^A [e^{\gamma \bar{d} \Delta N_T} (1 + (\tilde{\zeta}_{t+T})^{1/\gamma} e^{(\Delta a \Delta N_T - \bar{\lambda} T (e^{\Delta a} - 1))/\gamma})^\gamma] \\
&- (1 + (\tilde{\zeta}_t)^{1/\gamma} e^{-\bar{\lambda} T (e^{\Delta a} - 1)/\gamma})^\gamma \mathbb{P}_A(\Delta N_T = 0) \} \,,
\end{aligned}
$$

where $\Delta N_T \equiv N_{t+T} - N_t$ is number of disasters taking place in $[t, t+T]$, and $\mathbb{P}_A(\Delta N_T = 0) = e^{-\bar{\lambda} T}$ is the probability that no such disaster did happen. Again by expanding the binomial $(1 + (\tilde{\zeta}_{t+T})^{1/\gamma} e^{(\Delta a \Delta N_T - \bar{\lambda} T (e^{\Delta a} - 1))/\gamma})^\gamma$, and then computing the expectation of each resulting term, we obtain

$$
P_{t,t+T}^{DI} = \frac{a_T}{(1 + (\tilde{\zeta}_t)^{1/\gamma})^\gamma} \{ [\sum_{k=0}^{\gamma} b_{k,T} (\tilde{\zeta}_t)^{k/\gamma}] - e^{-\bar{\lambda} T} (1 + (\tilde{\zeta}_t)^{1/\gamma} e^{-\bar{\lambda} T (e^{\Delta a} - 1)/\gamma})^\gamma \} \,, \tag{O.11}
$$

where

$$
a_T = e^{(-\rho - \gamma \bar{g} + \frac{1}{2}\gamma^2 \sigma_c^2)T} \,, \tag{O.12a}
$$

$$
b_{k,T} = \begin{pmatrix} \gamma \\ k \end{pmatrix} e^{-\bar{\lambda} k T (e^{\Delta a} - 1)/\gamma} e^{\bar{\lambda} T [e^{(\gamma d + \frac{\Delta a k}{\gamma})} - 1]} \,. \tag{O.12b}
$$

B Equilibrium portfolio positions

In the current case of constant jump size with two dimensions of uncertainties (Brownian motion and disaster jump), the market is complete when agents are allowed to trade contingent claims on aggregate consumption (stock) P_t, money market account RFB_t and disaster insurance P_t^{DI}. We can use generalized Ito lemma on jump-diffusion (see, for example, Protter (2003)) to determine the price processes for each asset. Portfolio positions are then determined by equating the exposures to the Brownian and jump risks of each agents consumption claim to a portfolio of the aggregate claim and disaster insurance, which are then financed with the risk free bond.

C Boundedness of prices

This appendix discusses the boundedness of securities prices in general heterogeneous-agent economy. As claimed in the main text, as long as agents have different but equivalent beliefs, necessary and sufficient condition for finite price of a security in heterogeneous-agent economy is that this price be finite under each agent's beliefs in a single-agent economy. This is easy to see since

$$\max(f^{\gamma}_{A,0}, f^{\gamma}_{B,0}\eta_t) \leq M^A_t \leq (2f^{\gamma}_{A,0}) + (2f^{\gamma}_{B,0})\eta_t \tag{O.13}$$

Conditions for the finiteness of prices in the single agent economy can be found by studying the fixed points of the equations (O.4a). Setting $dB/dt = 0$, we find the fixed point of this differential equation is

$$B^* = \frac{\kappa - \sqrt{\kappa^2 + 2\sigma^2_\lambda(1 - \phi^{\mathbb{P}_i}(1 - \gamma^i))}}{\sigma^2_\lambda}\,, \tag{O.14}$$

provided that (12a) holds. Otherwise there is no fixed point and $B \to \infty$ implying infinite prices. Furthermore, it is easily seen that the initial condition $B(0) = 0$ is in the domain of attraction. For equity price to be finite, it is easy to see that the limiting exponent in (O.3) must be negative, or

$$-\rho + (1 - \gamma^i)\bar{g} + \frac{1}{2}(\gamma^i - 1)^2\sigma^2_c + \kappa\bar{\lambda}^i B^* < 0\,, \tag{O.15}$$

for both $i = 1, 2$. This is (12b) after we plug in the above expression for B^*.

D Proofs from Section 3.2

In this section, we provide the proofs for the results in Section 3.2. It is useful to rewrite expression for the consumption fractions in terms of the initial consumption sharing rule

(f_0^A, f_0^B) and the Radon-Nikodym derivative (η_t). In these terms,

$$f_t^A = \frac{f_0^A}{f_0^A + f_0^B \eta_t^{\frac{1}{\gamma}}}, \tag{O.16}$$

$$M_t^A / M_0^A = \left(f_0^A + f_0^B \eta_t^{\frac{1}{\gamma}} \right)^{\gamma} C_t^{-\gamma} / C_0^{-\gamma}, \tag{O.17}$$

$$\lambda_t^{\mathbb{Q}} = \lambda_A e^{-\gamma d} \left(f_t^A + f_t^B \left(\frac{\lambda_B}{\lambda_A} \right)^{\frac{1}{\gamma}} \right)^{\gamma}. \tag{O.18}$$

Additionally, for ease of notation, we set $N_0 = 0$ and $C_0 = 1$ which results in the expressions being fractions of the initial endowment.

Taking derivatives, we find

$$\frac{\partial \lambda^{\mathbb{Q}}}{\partial f_0^A} = \lambda_A e^{-\gamma d} \gamma \left(f_0^A + f_0^B \left(\frac{\lambda_B}{\lambda_A} \right)^{\frac{1}{\gamma}} \right)^{\gamma - 1} \left(1 - \left(\frac{\lambda_B}{\lambda_A} \right)^{\frac{1}{\gamma}} \right). \tag{O.19}$$

Setting $f_0^A = 1$ and taking the limit $\lambda^B \to 0^+$, we obtain (18).

In order to compute the derivative of the wealth fraction of agent B with respect to f_0^B, we first compute the derivative of the value of his claim, call it P^B, with respect to f_0^B. Since

$$P^B = \int_0^\infty E_0^A[(f_0^A + f_0^B \eta_t^{\frac{1}{\gamma}})^{\gamma-1} f_0^B \eta_t^{\frac{1}{\gamma}} C_t^{1-\gamma}] e^{-\rho t} dt, \tag{O.20}$$

we have that

$$\begin{aligned} \frac{\partial P^B}{\partial f_0^A} = & \int_0^\infty (\gamma - 1) E_0^A[(f_0^A + f_0^B \eta_t^{\frac{1}{\gamma}})^{\gamma-2} (1 - \eta_t^{1/\gamma}) f_0^B \eta_t^{\frac{1}{\gamma}} C_t^{1-\gamma}] e^{-\rho t} dt \\ & - \int_0^\infty E_0^A[(f_0^A + f_0^B \eta_t^{\frac{1}{\gamma}})^{\gamma-1} \eta_t^{\frac{1}{\gamma}} C_t^{1-\gamma}] e^{-\rho t} dt. \end{aligned} \tag{O.21}$$

From which it follows

$$\begin{aligned} \left. \frac{\partial P^B}{\partial f_0^A} \right|_{f_0^A = 1} & = - \int_0^\infty E_0^A[\eta_t^{1/\gamma} C_t^{1-\gamma}] e^{-\rho t} dt \\ & = - \frac{1}{\rho + (\gamma - 1)\bar{g}_A - \frac{1}{2}\sigma_c^2 (1-\gamma)^2 + \frac{1}{\gamma}(\lambda_B - \lambda_A) - \lambda_A (e^{(1-\gamma)\bar{d} + \frac{1}{\gamma} \log\left(\frac{\lambda_B}{\lambda_A}\right)} - 1)}. \end{aligned} \tag{O.22}$$

And so

$$\left.\frac{\partial P^B}{\partial f_0^A}\right|_{f_0^A=1} \to -\frac{1}{\rho + (\gamma-1)\bar{g}_A - \frac{1}{2}\sigma_c^2(1-\gamma)^2 + \frac{\gamma-1}{\gamma}\lambda_A} \quad \text{as } \lambda_B \to 0^+. \tag{O.23}$$

Now, it is easy to see that the derivative of the value of the claim to the entire endowment is bounded and since $P_B = 0$ when $f_0^A = 1$, the derivative $\partial w_0^B/\partial f_0^A$ is simply $\frac{\partial P^B}{\partial f_0^A}$ divided by the value of the claim to the entire endowment. This proves (**??**).

E General valuation of disaster states

In Section 3.2, we demonstrated that within a simple calibration a large fraction of the the value of the endowment claim arises from the disaster states, even though these states are very rare. Here we demonstrate that in fact this property is a feature of a broad class of models. Specifically, suppose that the model is such that the dynamics of aggregate consumption under the actual measure, as well as the risk-neutral measure, follow the dynamics in 1 and that the risk-free rate is constant. This is true in our model with CRRA preferences and remains true with Epstein-Zin preferences (cf. Wachter (2011).) In particular, this reduced form setting removes the link between risk aversion and elasticity of intertemporal substitution.

Within this setting, let $\bar{g}_A^{\mathbb{Q}}$ denote the growth rate of consumption under the risk neutral measure. The fractional value of consumption in the non-disaster states is then

$$\frac{\int_0^\infty E_0^{\mathbb{Q}}\left[e^{-rt}C_t \times 1_{\{N_t=0\}}\right]}{\int_0^\infty E_0^{\mathbb{Q}}\left[e^{-rt}C_t\right]} = \frac{r - \bar{g}_A^{\mathbb{Q}} - .5\sigma_c^2 - \lambda^{\mathbb{Q}}(e^{\bar{d}}-1)}{r - \bar{g}_A^{\mathbb{Q}} - .5\sigma_c^2 + \lambda^{\mathbb{Q}}} \tag{O.24}$$

The difference between the numerator and denominator is $\lambda^{\mathbb{Q}}e^{\bar{d}}$. In order for disasters to account for a substantial risk premium, this term should be sizeable (it is 6% in the example of Section 3.1.) Moreover, it is reasonable to expect the price-consumption ratio (the inverse of the denominator) should not be too small. Setting these to 4% and 10 gives a fraction 4/14 due to disaster states. Setting them to 6% and 20 give a fraction of 6/11 to the disaster states. In summary, under these very general reduced form assumptions on the endowment and preferences along with the assumptions that (i) disasters account for a significant risk premium and (ii) the price-consumption ratio is not too small, the fraction of wealth due to non-disaster states is significant.[20]

F Extracting Beliefs

In this section we describe how we compute risk-neutral cumulative probabilities from options data and then infer corresponding actual probability intensities for the stochastic intensity model of Section 3.4. First, we use the formula of Breeden and Litzenberger (1978) that (ignoring discounting)

$$\mathbb{Q}_t(S_T/S_t < K) = \partial_K P(K),$$

where $P(K)$ is the put price at time t for a strike of K. In order to help linearize the put price function, we use that $\partial_K P(K) = P(K)\partial_K \log P(K)$. In Figure O.1, we plot for August 2008, a particularly volatile day in our sample the cross-section of midquote put prices (normalized by the index level). We also superimpose a linear we use to approximate $\partial_K \log P(K)$. We compute this line by an OLS regression of available log put prices between 3% and 40% OTM. In the early part of the sample, deep out of the money puts did not have quotes, so in this case we drop the deepest out of the money option (as is likely to have had low liquidity) and then extrapolate from the OLS line.

In order to extract the corresponding model implied disaster intensities, we first make the simplifying assumption that $\mathbb{Q}_t(S_T/S_t < K)$ is approximately equal to the disaster intensity (scaled by $t = 1/12$). We then compute for a given wealth fraction controlled by the optimistic the disaster intensity required to match the computed risk-neutral disaster intensity.[21]

G Sharpe Ratios with Growth Rate Disagreement

In this case, (4) specializes so that the Radon-Nikodym derivative depends only on c_t^c and a deterministic term. As there is no jump risk, the excess return, ER_t, for any asset with

[20] In the CRRA version of this equation, $r = \rho + \gamma \bar{g}_A - .5\sigma_c^2\gamma^2 - (\lambda^{\mathbb{Q}} - \lambda^{\mathbb{P}})$. This causes increasing $\lambda^{\mathbb{P}}$ (and thus $\lambda^{\mathbb{Q}}$) to increase the price-consumption ratio. In the general formula if we fix r and increase $\lambda^{\mathbb{Q}}$ independently this *decreases* P/C so clearly the generic form does not have EIS-risk aversion link problems.

[21] For very extreme disaster intensities, there is a discontinuity in the relationship between λ_t and $\lambda_t^{\mathbb{Q}}$ that arises because for a given wealth level the optimist begins to disproportionately invest in disaster insurance and less in current consumption. This cases occurs only the 6 most extreme days of our sample and in this case we understate the amount of bias. In this regard, Figure 6 can be considered an approximation.

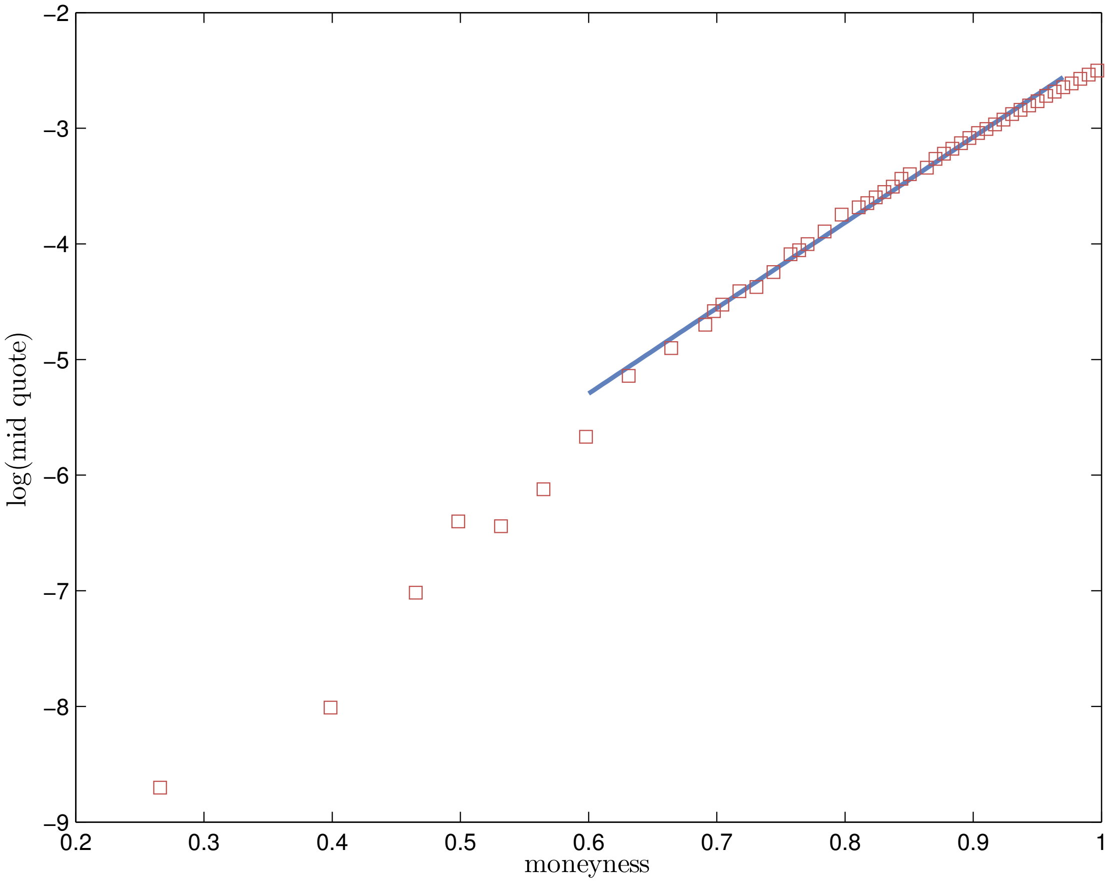

Figure O.1: **Out-of-the-money put prices on October 23, 2008.** This figure plots log misquote put prices (normalized by the index level) with 30 days to expiry on October 23, 2008. Also plotted is the OLS line fit to the available quote between 3% and 40% out of the money.

price process P_t is given by

$$ER_t = -\mathrm{cov}_t\left(\log(P_t), \log(M_t)\right).$$

Since there is a single state variable, c, we can compute the covariance as

$$ER_t = -\partial_c \log P(c)\sigma_c^2 \partial_c \log M(c,t).$$

When there is non-zero sensitivity, the Sharpe ratio, SR_t, is then given by

$$SR_t = -\sigma_c \partial_c \log M(c,t).$$

Now we simply compute that

$$
\begin{aligned}
\partial_c \log M(c,t) &= \gamma - \partial_c \gamma \log(1+\tilde{\zeta}^{\frac{1}{\gamma}}) \\
&= \gamma - \frac{\tilde{\zeta}^{\frac{1}{\gamma}}}{1+\tilde{\zeta}^{\frac{1}{\gamma}}} b \quad \text{by (4)} \\
&= \gamma - f^B \times b \quad \text{by (9)}
\end{aligned}
$$

This gives (20).

H General Forms of Disagreements

The affine heterogeneous beliefs framework in Section 2 can capture other forms of heterogeneous beliefs besides disagreement about disaster intensity. In this section, we first show that disagreement about the size of disasters has similar impact on the risk premium as disagreement about the frequency of disasters. We then provide an example with strong effects of risk sharing even when both agents are pessimistic about disasters.

H.1 Disagreement about the Size of Disasters

For simplicity, let's assume that the drop in aggregate consumption in a disaster follows a binomial distribution, with the possible drops being 10% and 40%. Both agents agree on the intensity of a disaster ($\lambda = 1.7\%$). Agent A (pessimist) assigns a 99% probability to a 40% drop in aggregate consumption, thus having essentially the same beliefs as in the previous example. On the contrary, agent B (optimist) only assigns 1% probability to a 40% drop, but 99% probability to a 10% drop. The rest of the parameter values are the same as in the first example.

Figure O.2 (solid lines) plots the conditional equity premium and jump risk premium under the pessimist's beliefs. When the pessimist has all the wealth, the equity premium is 4.6% (almost the same as in the first example). Again, the equity premium falls rapidly as we starts to shift wealth to the optimist. The premium falls by almost half to 2.4% when the optimist owns just 5% of total wealth, and becomes 1.4% when the optimist's share of total wealth grows to 10%. Similarly, the jump risk premium falls from 7.6 to 4.5 with the optimist's wealth share reaching 10%, which by itself will lower the premium to 2.4%.

These results show that, in terms of asset pricing, introducing an agent who disagrees

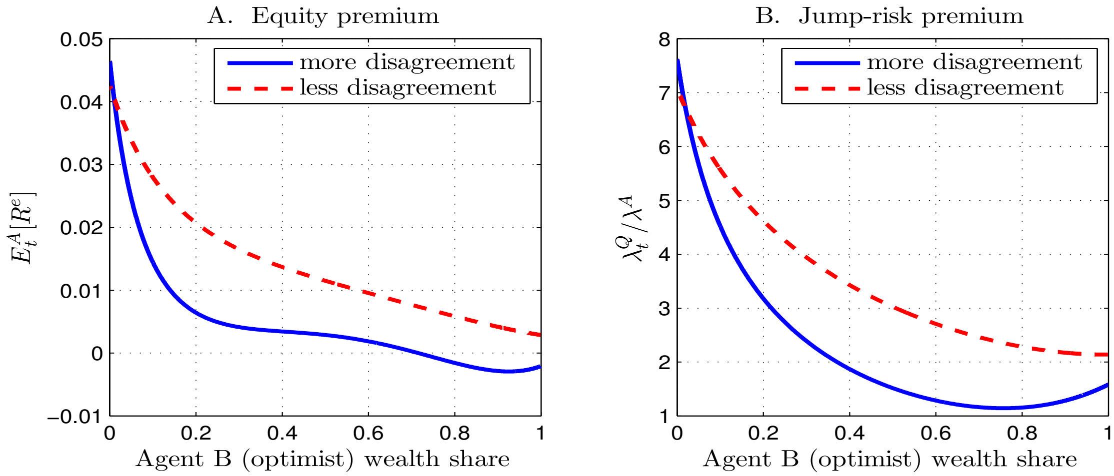

Figure O.2: **Disagreement about the size of disasters.** The left panel plots the equity premium under the pessimist's beliefs. The right panel plots the jump risk premium for the pessimist. In the case with "more disagreement", the pessimist (optimist) assigns 99% probability to the big (small) disaster, conditional on a disaster occurring. With "less disagreement", the probability assigned to big (small) disaster drops to 90%.

about the severity of disasters is similar to having one who disagrees about the frequency of disasters. Even though the two agents agree on the intensity of disasters in general, they actually strongly disagree about the intensity of disasters of a specific magnitude. For example, under A's beliefs, the intensity of a big disaster is $1.7\% \times 99\% = 1.68\%$, which is 99 times the intensity of such a disaster under B's beliefs. The opposite is true for small disasters. Thus, B will aggressively insure A against big disasters, while A insures B against small disasters. For agent A, the effect of the reduction in consumption loss in a big disaster dominates that of the increased loss in a small disaster, which drives down the equity premium exponentially. Such trading can also become speculative when B has most of the wealth: agent A will take on so much loss in a small disaster that the jump risk premium rises up again.

Naturally, we expect that the agents will be less aggressive in trading disaster insurances when there is less disagreement on the size of disasters, and that the effect of risk sharing on the risk premium will become smaller. The case of "less disagreement" in Figure O.2 confirms this intuition. In this case, we assume that the two agents assign 90% probability (as opposed to 99%) to one of the two disaster sizes. While the equity premium still falls rapidly near the left boundary, the pace is slower than in the previous case. Similarly, we see a slower decline in the jump risk premium.

H.2 When Two Pessimists Meet

The examples we have considered so far have one common feature: the new agent we are bringing into the economy has more optimistic beliefs about disaster risk, in the sense that the distribution of consumption growth under her beliefs first-order stochastically dominates that of the other's, and that the equity premium is significantly lower when she owns all the wealth. However, the key to generating aggressive risk sharing is not that the new agent demands a lower equity premium, but that she is willing to insure the majority wealth holders against the types of disasters that they fear most.

In order to highlight this insight, we consider the following example, which combines disagreements about disaster intensity as well as disaster size. Both agents believe that disaster risk accounts for the majority of the equity premium. The key difference in their beliefs is that one agent believes that disasters are rare but big, while the other thinks disasters are more frequent but less severe. Specifically, we assume that disasters can cause aggregate consumption drops of a 30% or 40%. Agent A believes that $\lambda^A = 1.7\%$, and assigns 99% probability to the bigger disaster. B believes that $\lambda^B = 4.2\%$, and assigns 99% probability to the smaller disaster.

By themselves, the two agents both demand high equity premium. We have chosen λ^B so that, under the beliefs of agent A, the equity premium is 4.6% whether A or B has all the wealth. However, they have significant disagreement on the exact magnitude of the disaster. Such disagreement generates a lot of demand for risk sharing. As we see in Panel A of Figure O.3, the conditional equity premium falls rapidly as the wealth share of agent B moves away from the two boundaries. In fact, the premium will be below 2% when B owns between 9% and 99% of total wealth. In Panel B, the jump risk premium also falls by half from 7.6 and 10 on the two boundaries when B's wealth share moves from 0% to 25% and from 100% to 91%, respectively.

To get more information on the risk sharing mechanism, in Panel C and D we examine the equilibrium consumption changes for the individual agents during a small or big disaster. Since agent A assigns a low probability to the small disaster, she insures agent B against this type of disasters. As a result, her consumption loss in such a disaster exceeds that of the aggregate endowment (-30%), and it increases with the wealth share of agent B. When B has almost all the wealth in the economy, agent A sells so much small disaster insurance

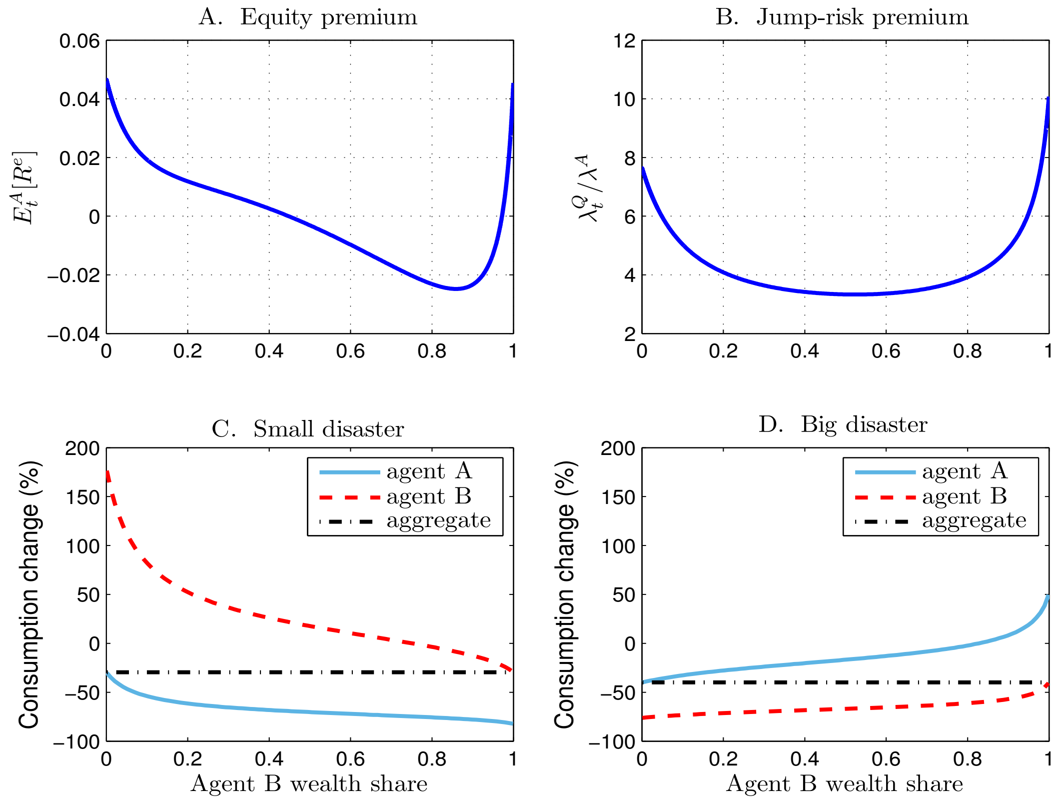

Figure O.3: **When Two Pessimists Meet.** Panel A and B plot the equity premium and jump risk premium under agent A's beliefs. Panel C and D plot the individual consumption changes in small and big disasters.

to B that her own consumption can fall by as much as 82% when such a disaster occurs. As a result, agent B is able to reduce her risk exposure to small disasters significantly. In fact, her consumption actually jumps up in a small disaster when she owns less than 75% of total wealth, sometimes by over 100% (when her wealth share is small).

The opposite is true in Panel D. As agent B insures A against big disasters, she experiences bigger consumption losses in such a disaster than the aggregate endowment (-40%). The equilibrium consumption changes of the two agents are less extreme compared to the case of small disasters, which is due to two reasons. First, the relative disagreement on big disasters is smaller than on small disasters. Second, the insurance against larger disasters is more expensive, so that agent A's ability to purchase disaster insurance is more constrained by her wealth.