NON-INTEGER TOPOLOGICAL INVARIANT FOR THIN-WALLED PRIMITIVES

M. Shpitalni and H. Lipson

CMSR Laboratory for Computer Graphics and CAD
Faculty of Mechanical Engineering
Technion, Haifa 32000, Israel

***Abstract* -** This paper proposes the use of a thin-walled primitive for modeling the geometry of inherently thin objects. The authors suggest a topological invariant supporting both manifold and thin-walled non-manifold objects based on this primitive. The paper establishes a general topological invariant $s + e - v - g_{nm} + m - f = 0$ regarding the number of components, edges, vertices, holes, volumes, and faces, respectively. Corresponding Euler operators are derived, providing a basis for a modeling system for thin-walled objects. The validity of the proposed invariant constitutes a necessary condition for the validity of a geometrical representation of thin-walled products from a topological point of view. The paper also discusses merging the proposed general formula with standard manifold topology. It specifically proposes the use of non-integer values in the standard Euler-Poincaré formula for representing non-manifold components, thus permitting the use of thin-walled primitives with topological coherence to traditional solid geometry schemes.

1. INTRODUCTION

Since Requicha formally introduced regularized *r*-sets [1], solid modeling has developed rapidly, especially for manifold geometries. Various data structures for solid modeling exist; most are based internally on manifold topology and manifold operators. One example is the geometrical representation proposed by Mantyla [2]. However, many real solids do not comply with manifold topology. One such case is that of a thin-walled object based on the fundamental thin-walled primitive, as illustrated in Fig. 1. This paper discusses the topological properties of this type of primitive and proposes a topological invariant which can be used as a basis for modeling thin-walled constructions in combination with the standard constructive solid geometry paradigm.

MOTIVATION

In contrast to boundary representation, thin-walled primitives are used to describe objects which are inherently thin in reality and not as an alternative means of representing boundaries of full solids. Thin-walled objects are prevalent in many engineering disciplines, such as sheet metal, composite materials and injection molding. Although geometrical models of such products can be represented by full solid primitives, in many cases a thin-wall representation is more suitable. Thin-walled representations more closely communicate the essence of the object geometry as perceived by designers. Dedicated CAD systems often use a thin-walled representation to describe the main geometry of thin objects, with a thickness parameter to represent the third dimension as an additional attribute varying along the main geometry. Shpitalni [3] and Lee [4] have shown that systems based on such a representation are in general more efficient both for computation and for use.

Most systems support the definition and manipulation of thin-walled objects by means of surfaces and boundary representation. These are combined with CSG primitives in an internal hybrid representation. However, a unification of geometrical representations could be achieved by using a skeletal scheme coherent with standard topology. The purpose of this paper is to identify this fundamental representation and propose a topological invariant as a basis for this unification. This invariant can then be used as a basis for modeling, verifying and classifying geometry of both manifold and thin-wall type.

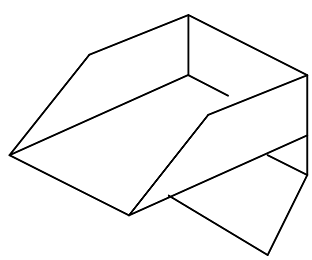


Fig 1. A schematic thin-walled object

The paper first outlines previous work in the area of modeling non-standard geometries. Next, it introduces the concept of a basic thin-wall primitive and discusses the relationships among such primitives within a complex construction. From these relationships, a topological invariant is derived that is applicable to a general thin-walled object. The invariant relationship is demonstrated on some examples. Finally, the paper discusses how this representation can be united with standard manifold topology using the concept of fractions rather than integers in the Euler-Poincaré formula.

The paper aims to propose the following:

- A thin-walled primitive for representing thin-walled objects according to the CSG paradigm.
- A topological invariant for all thin-walled objects which may be used as a necessary condition for verifying topological validity and for reasoning about topological configurations.
- A set of topological 'Euler operators' which may be used as basic building blocks for constructing and manipulating a thin-walled model representation.
- A unified basis for merging thin-walled representations with standard manifold objects, using non-integer topological enumerators.

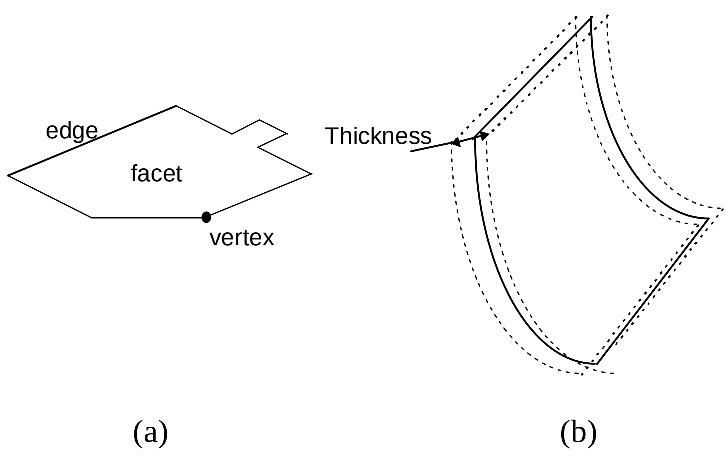
2. PRIOR WORK

Thin-walled objects typically constitute non-manifold objects. A 2-manifold object is defined as a surface on which every point has a neighborhood that is homeomorphic to a 2-disk [2]. A manifold object can be classified as genus zero if it can be deformed continuously into a sphere, that is, if it is *homeomorphic* to a sphere. A manifold object of genus one can be deformed continuously into the shape of a torus, and so on. Hence, objects containing any surface 'forks', as illustrated in Fig. 1, are not 2-manifold and are generally termed *non-manifold* objects. A manifold with a continuous boundary (homeomorphic to a semi-sphere) is referred to as a *manifold with boundary*.

In the last two decades, solid modeling has developed rapidly, especially for manifold geometries. Recently, interest in non-manifold topology has grown. Weiler [5], for example, explored and generalized solid modeling schemes to include wireframes and surfaces. Non-manifold extensions to solid modeling are usually considered in the scope of new *data structures* or more general *invariant formulae,* with both these topics usually yielding new sets of corresponding *topological operators*. Non manifold data structures and operators, first introduced by Baugmart [6], are reviewed in Hoffman *et al* [7] and recently represented in Lopes *et al* [8] using Morse operators. In this paper we focus on a new *invariant property,* rather than data structures. Invariant formulae are typically implemented by using a more general formulation that includes non-manifold elements, such as the *cusps, disks, zones, regions* and *walls* proposed by Gursoz *et al* [9] or the *shells*, *complexes*, *cavities* and *holes* of various types suggested by Masuda *et al* [10] and others. Application of Euler characteristics and topology in design is also discussed by Lear [11] and Lee *et* *al* [4]. A more detailed taxonomy of geometric and topological models is provided by Hoffmann [7], Mantyla [2] and Takala [12]. While works on invariants tend to provide increasingly general formulations, none has concentrated on specific modeling tasks, such as modeling of thin-walled parts. These parts are usually non-manifold and thus comply with general formulae such as those discussed by Gursoz *et al* [9] and Masuda *et al* [10]; nonetheless, they are still confined to a relatively narrow topological domain and may therefore use simpler relationships. For example, they cannot directly include detached ('dangling') edges and vertices. In particular, this relationship can be simplified by use of non-integer topology. Recently, we have investigated the effect of these special conditions on the modeling of sheet metal products [13] in a dedicated modeling environment. In this paper, we attempt to generalize those findings to allow modeling of general thin-walled objects in conjunction with standard CSG primitives on a unified topological basis. This attempt introduces non-integer topological properties.

3. THE FUNDAMENTAL THIN-WALLED PRIMITIVE

A thin-walled object can be considered as consisting of one or more thin elements that are joined together. Hence, the product is composed of basic *thin-wall* primitives. A thin-walled primitive, as illustrated in Fig. 2, can be planar or curved. However, its topology always consists of a single *facet* with a continuous single boundary. The boundary is composed of *edges*, linear or curved, joined at *vertices*, as shown in Fig. 2(a). The primitive also possesses metric attributes, such as thickness (variable), curvature, and size, as illustrated in Fig. 2(b).



(a) (b)

Fig 2. Thin-walled primitives, (a) topological elements, (b) metric attributes

Thin-wall primitives join along common edges. The edges of the primitives can always be arranged so that the junction occurs along a complete edge, between two distinct vertices, by appropriately splitting or joining edges along the contour.

The following sections establish the existence of a constant relationship among these three elements of a thin-wall primitive (facet, edges and vertices) within a model composed of one or more primitives.

The relationship is developed according to the following steps:

1. A relationship based on Euler-Poincaré formula is derived for a single primitive and a set of separate primitives.
2. The concepts of non-manifold genus and volumes are defined. Corresponding terms are added to the relationship, which retains its validity for a set of single primitives.
3. This relationship is shown to remain valid during construction of a model by assembling primitives and components together through systematic consideration of the possible topological changes.

TOPOLOGICAL PROPERTIES OF THE PRIMITIVE

We proceed to analyze a single thin-wall primitive as a topological graph consisting of edges (corresponding to free edges), vertices, and faces (corresponding to the facets).

Since, by definition, the single primitive contains a single facet surrounded by a single boundary of edges, it obeys the Euler-Poincaré formula for a single component planar graph [2], which asserts that

$$f + v - e = 2 \qquad (1)$$

where f represents the number of faces of the graph (including the exterior face), v the number of vertices, and e the number of edges. In a single primitive, the exterior face does not represent a facet. The corresponding formula for a graph representing a single primitive would therefore take the form

$$f + v - e = 1 \qquad (2)$$

We now consider a set of detached primitives. A more general version of the Euler-Poincaré formula for a multiple component planar graph states that

$$f + v - e = 2\,s \qquad (3)$$

where s is the number of graph components (or *shells*, as often termed in the solid modeling context [2]). A graph is said to have several components if it consists of disconnected subgraphs. Since each component contains an exterior face which is to be ignored for thin-walled components, we arrive at the corresponding relationship:

$$f + v - e = s \tag{4}$$

where s represents the number of disconnected primitives, f the total number of facets in the primitives, v the total number of vertices, and e the total number of edges.

HOLES AND VOLUMES

Assembly of the individual primitives into structures gives rise to *holes* and *volumes.* These holes and volumes must be defined before discussing assembly.

A primitive can contain two types of interior loops, as illustrated in Fig. 3. The first type is a *ring:* an edge loop interior to a primitive facet disconnected from the external boundary of the facet. Traditionally, rings are considered as special topological elements and are counted explicitly in a modified Euler-Poincaré formula for manifold objects [2]. Since a ring is local to its facet and is disconnected from the main component topology, it can either be ignored and modeled as a separate component or "artificially" connected to the external boundary using a pair of free edges; thus, rings can be ignored in the discussion. The second type of opening, one that crosses or touches one or more of the component's edges, is generated when two or more primitives are joined together, leaving a ‘gap’ between them. This type of hole is represented by an interior edge circuit in the graph. The number of such holes is the *genus* of the structure. The original Euler-Poincaré formula uses a definition for genus suitable for manifold objects and is often denoted by the symbol g. To make the following discussion more clear, we label the non-manifold genus with g_{nm}. Later on we will encounter other holes formed as edges join during assembly of the primitives; they will also be counted in g_{nm}.

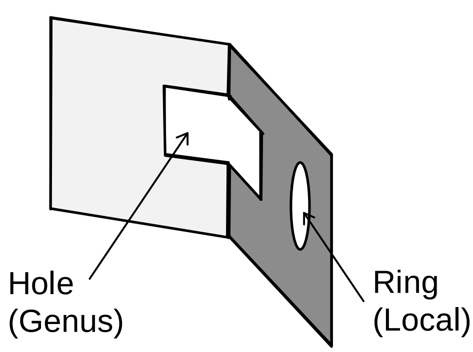


Fig 3. An assembly of two thin-wall primitives with a hole and a ring.

The following definition for non-manifold genus is proposed:

Definition. *The non-manifold genus g_{nm} of a single component is the maximum number of non-intersecting closed curves that can be drawn on the part surface before partitioning its surface into two previously connected disconnected regions.*

For a multi-component part, the genus is the sum of the genuses of the individual components. Note that for the purpose of this definition, a surface is considered two-sided, and the sides meet at free edges. For example, a flat plate has one surface that spans both sides of the plate, since the two sides are connected along the free edges. If it has no holes, then no closed curve may be drawn on it without partitioning the surface into two disconnected regions. Its genus is therefore zero. A hollow sphere, on the other hand, has two surfaces, internal and external, which are separate because they have no common edge. Consequently, in the case of the sphere as well, no non-partitioning curve can be drawn, and hence its non-manifold genus g_{nm} is zero as well. By the above definition, the genus of an object corresponds to the *connectivity* of its surface, where connectivity is a topological quality which measures the number of topologically different paths connecting any two regions on a surface. Using this criterion, it is easy to verify that for a torus, g_{nm} would be 2, because one closed curve could be drawn on the external surface and another on the internal surface and still no previously connected surface points would be partitioned.

Any hole is a face of the graph but does not represent a facet of the part; therefore, it is not counted as a facet, and hence

$$f = s + e - v - g_{nm} \qquad (5)$$

When thin-wall primitives are joined, some volumes may be sealed off. We define a volume as follows:

Definition. *A volume corresponds to a closed surface from which no curve can be cast to the exterior (infinity or another interior surface).*

Volumes are counted using the term *m*.

Initially, in the set of detached primitives, there are no weld holes, i.e. $g_{nm} \equiv 0$, and no volumes, i.e. $m \equiv 0$. Hence, Eq. (5) can be modified to include volumes and Eq. (6) holds true for a flat pattern

$$s + e - v - g_{nm} + m - f = 0 \qquad (6)$$

ASSEMBLING THIN-WALLED PRIMITIVES TOGETHER

Our analysis so far has applied to a set of detached primitives. We now proceed to join the primitives into a full structure, following [13] to show that Eq. (6), including terms for holes and volumes, still holds true.

As the primitives are assembled, some edges and vertices may join together and unite in various ways, causing the topology to change. We denote the changes in f, s, e, v, g_{nm} and m by •Δf, Δ•s, •Δe, •Δv, •Δg_{nm} and •Δm, respectively. Any assembly operation can be decomposed into a sequential set of operations in which exactly two edges at a time are joined together.

1. During the joining operation, two lines merge into one. The joining effect is that •$\Delta e = -1$ always.

2. The two joining lines may have originally had no common vertices, shared one vertex, or shared both vertices. We shall consider each case separately (refer to Fig. 4).

2.1 **No common vertices:** If the lines shared no common vertices, then merging the two lines causes two vertices to vanish, thus $\Delta v = -2$. The two original lines may have belonged either to one component or to two separate components.

2.1.1 If the two lines belonged to two separate components (Fig 4(a)), then the merge united them, so that $\Delta s = -1$. Any partitioning curve across the joint will partition the component back into two parts and therefore cannot contribute to the genus; thus, $\Delta g_{nm} = 0$.

2.1.2 If the two lines belong to the same component (Fig 4(b)), then $\Delta s=0$ but the genus is increased because, according to the definition of g_{nm}, a single closed partitioning-curve can now be traced around the two-sided merged line without partitioning the component; thus $\Delta g_{nm} = +1$.

The number of volumes is left unmodified in this case, because any ray cast out between the two original lines can still be cast out between their adjacent continuations; thus $\Delta m = 0$. Collectively, $\Delta s\text{-}\Delta v\text{-}\Delta g_{nm}+\Delta m = +1$.

2.2. **One common vertex:** If the lines shared one common vertex (Fig 4(c)), then merging the two lines caused one vertex to vanish; thus $\Delta v = -1$. The two original lines must have originated from the same component because they shared a vertex; therefore $\Delta s=0$. Also, the genus does not change because connectivity is not modified, i.e. no existing paths were removed or new paths formed; therefore $\Delta g_{nm} = 0$. The number of volumes is also left unmodified in this case because any ray cast out between the two original lines can still be cast out between their adjacent continuations on the side of the vanishing vertex; thus $\Delta m = 0$. Collectively, $\Delta s\text{-}\Delta v\text{-}\Delta g_{nm}+\Delta m = +1$.

2.3. **Two common vertices:** If the lines shared two common vertices, then they originally formed a hole. Merging the two lines causes no vertex to vanish; thus $\Delta v = 0$. The two original lines must have originated from the same component because they shared vertices; therefore $\Delta s=0$. Any partitioning curves on the surface can be arranged so that not more than one passes through the hole created by the two lines. Two cases must be distinguished:

2.3.1 If a closed partitioning curve on the surface passed between the lines once (Fig 4(d)), then joining the lines would eliminate that curve; therefore by the definition of non-manifold genus, $\Delta g_{nm} = -1$. In this case, the number of volumes does not change because if a ray is cast through, it can still be cast by following the curve continuation and $\Delta m = 0$.

2.3.2 If, on the other hand, no closed curve on the surface could pass once between the lines (Fig. 4(e)) then the two lines must have formed a single entry into a volume. Therefore, joining the lines did not change the genus, $\Delta g_{nm} = 0$, but did create a volume, $\Delta m = +1$.

Collectively, $\Delta s\text{-}\Delta v\text{-}\Delta g_{nm}+\Delta m = +1$.

3. The number of facets does not change by joining the lines; therefore $\Delta f = 0$.

Combining $\Delta f = 0$ with $\Delta e = -1$ and $\Delta s\text{-}\Delta v\text{-}\Delta g_{nm}+\Delta m = 1$ yields no overall change in the value of Eq. (6).

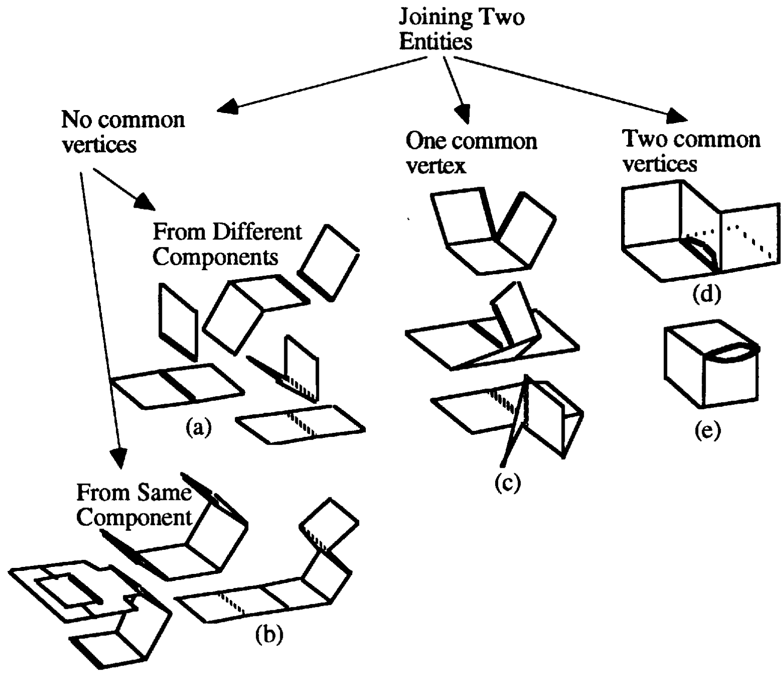


Fig 4. Alternatives in joining two lines.

It can therefore be concluded that Eq. (6) holds true both for final assembly and intermediate stages. Moreover, from the preceding analysis it is evident that a collection of objects composed of facets, edges, vertices and volumes can form a valid schematic model if and only if the number of these elements satisfies Eq. (6). Hence, Eq. (6) can be considered as a necessary integrity criterion for such schematic models.

EXAMPLES

First, consider the product illustrated in Fig. 1. It has 12 vertices, 17 edges and 6 faces. It has one component, so that $s=1$, no holes, so that $g_{nm}=0$, and no closed volumes, so $m=0$. Substituting into Eq. (6), we obtain

$$s + e - v - g_{nm} + m - f = 1 + 17 - 12 - 0 + 0 - 6 = 0$$

thus, Eq. (6) holds true for this product.

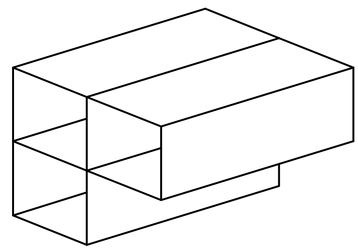


Fig 5. A schematic sheet object with three holes

The object shown in Fig 5 is one unit, i.e. $s=1$, and its non-manifold genus is 3. It has 10 faces, 28 edges, no volumes and 16 vertices. Thus, according to Eq. (6),

$$s + e - v - g_{nm} + m - f = 1 + 28 - 16 - 3 + 0 - 10 = 0$$

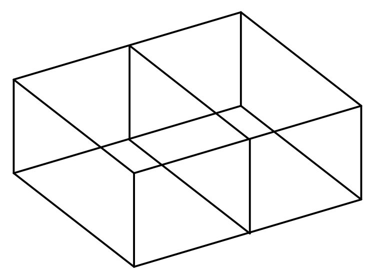


Fig 6 A rectangular box with an internal partition

Finally, consider the assembly of primitives shown in Fig. 6, representing a rectangular box with an internal partition. This box has 11 faces, one component, 20 edges, 12 vertices, genus 0 and 2 volumes. Applying Eq. (6), it can be verified that

$$s + e - v - g_{nm} + m - f = 1 + 20 - 12 - 0 + 2 - 11 = 0$$

4. Euler Operators for Thin-Walled Parts

The existence of a topological invariant for thin-walled structures lays the foundation for defining sets of 'Euler operators'. A necessary condition for a valid topological operation is to maintain the validity of Eq. (6). However, this only ensures that the number of topological entities is correct; their specific configuration must also satisfy additional criteria, such as non-existence of dangling edges, incomplete boundaries and faces without boundaries. In general additional criteria for validity, such as completeness of cycles and various restrictions on operators, as well as metric criteria such as collisions and continuity, depend on the data structure used to describe the model. However, these criteria are often local and cannot be formulated into a global invariant, and are therefore beyond the scope of this paper. See [8] and [14] for a discussion of some of these local topological restrictions.

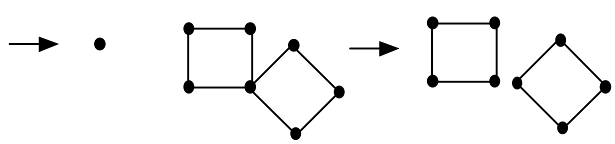
In the original Euler-Poincaré equation for manifold solids, the basic topological manipulations complying with the equation are termed *Euler operators.* They were originally introduced by Baumgart [6] and are discussed in detail by Mantyla [2] and Braid *et al* [15]. The same notion can be carried over to analyze thin-walled parts using Eq. (7).

By historical convention, the operators are denoted by mnemonic names. The key to the names used here is as follows:

M = Make	**V** = Vertex	**G** = Genus (Non-Manifold)	
K = Kill	**E** = Edge	**U** = Volume	*(7)*
	F = Facet		
	C = Component		

For example, the operator MEV is translated as "Make Edge and Vertex". These operators can be implemented on top of a data structure describing a thin-walled part. Numerous valid operators can be established. However, only a few of them are essential in that they are sufficient to allow any manipulation or creation leading to construction of a part. A fundamental set of operators provides the basic tools in an implementation of a thin-walled modeling system in the same way that the original manifold operators provided the basis for solid modeling.

There are many possible sets of operators; the following is a description of one such set. The most basic operation is creation of a primitive. The operation MCV creates a new component comprised of one vertex. Here we adopt a more abstract definition of a thin-walled product, one also allowing for null creations, such as a product with zero facets. Fig.7 (a) shows how a vertex is created by the MCV operator. Fig. 7(b) shows the split operator also realized by the MCV operation. The operator KCV is the reverse operator, undoing any effects of MCV.



(a) (b)

Fig 7. The MCV operator (a) vertex from nil, (b) split operation

The next two operators are MEV and MEF (with the corresponding undo's, KEV and KEF). These operators correspond exactly to polygon and vertex splitting operations for plane models [2]. In essence, MEV "splits" a vertex into two vertices joined with an edge. The MEF operator joins two vertices while creating an additional facet. Their effect is demonstrated in Fig. 8.

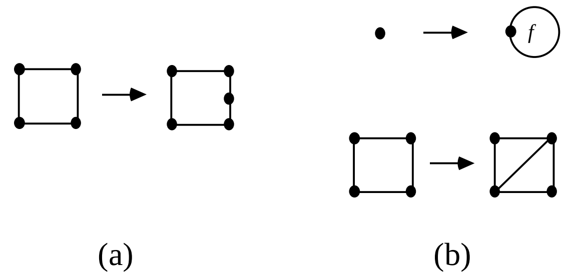


Fig 8. The operator (a) MEV, (b) MEF

The next operator, MGE (KGE), provides a means for manipulating the non-manifold genus of a model and for joining and merging circuits. In Fig. 9, a single component composed of a face with a local ring is transformed to single component with a genus. This operator provides a mechanism for handling local rings and incorporating them into the main topology, as discussed previously.

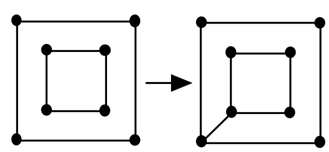


Fig 9. The operator MGB

The MUKE (KUME) operator allows sealing off volumes by merging edges, as illustrated in Fig. 10.

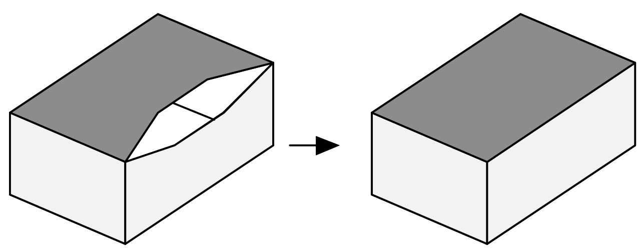


Fig 10. Merging two edges to seal off a volume

It is possible to analyze a given structure and determine the sequence of operators necessary to reach that topology. This is achieved by noting that within a six-dimensional topological space, Eq. (6) defines a five-dimensional hyperplane *E*.

$$E: s + e - v - g_{nm} + m - f = 0 \qquad (6)$$

Each schematic model of a thin-walled part is represented by a point *P* on hyperplane *E*, and the set of operators span a lattice on the plane *E*. Determining a sequence of operators to construct a particular topology is thus equivalent to finding a path on the lattice leading from the origin to the point *P*. This can be done by representing point *P* in terms of the lattice basis, following the technique discussed by Braid *et al* [15].

5. UNIFICATION WITH STANDARD MANIFOLD TOPOLOGY BY NON-INTEGER ENUMERATORS

The general manifold topology used as the underlying basis of geometrical modeling data structures is given by the Euler-Poincaré invariant

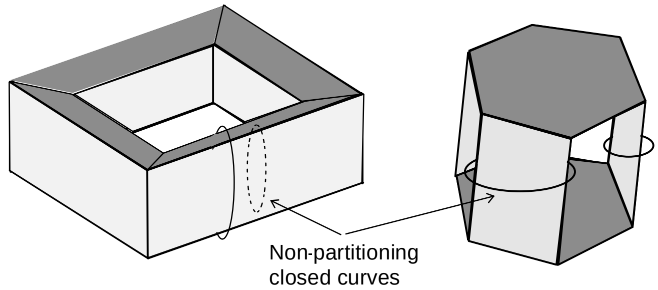
$$v - e + f = 2(s - g) \tag{8}$$

whereas the proposed invariant, for both manifold and non-manifold topologies, is given by Eq.(6)

$$v - e + f = s + m - g_{nm} \tag{6}$$

The difference between Eq. (6) and Eq. (8) is in the right hand side of the equality and can be explained as follows. First, in a manifold object, each component corresponds to a single volume, whereas in a non-manifold object, a component can correspond to an arbitrary number of volumes (including 0 volumes). Hence the term $2s$ in Eq. (8) corresponds to $s+m$ in Eq. (8). Second, the terms g and g_{nm} (genus) are defined differently in the two equations. Each genus unit of a manifold object (g) corresponds to two genus units in the non-manifold definition (g_{nm}). For example, a torus has genus 1 under the classical manifold definition of genus. However, the genus of a torus is 2 in the non-manifold (general) sense because two closed curves can be drawn on its surface, one on the internal side and another on the external side, and still no previously connected points will be partitioned. However, an open cylinder has one surface on which only one closed curve can be drawn (in the longitudinal direction). Such situations cannot be captured by the original manifold definition of *genus*. This fundamental difference in the meaning of *genus* is rooted in the concept that a manifold object has an inner side and an outer side, whereas a non-manifold object has no "sides". Therefore for strictly manifold objects, $s+m=2s$ and $g_{nm}=2g$, and Eq. (6) is reduced to the Euler-Poincaré formula.

The following two examples illustrate this point.



(a) (b)

Fig 11. Two general objects

The object illustrated in Fig. 11(a) (a squared torus) consists of 16 vertices, 28 edges, 12 faces, one volume and one component, and a genus of 2 (two non-partitioning curves). For this object, according to Eq. (8),

$$16 - 28 + 12 = 1 + 1 - 2$$

Eq. (8) handles this object correctly because $s=m$, and the traditional meaning of *genus* applies to this object.

Fig. 11(b) illustrates a non-manifold object, an extruded hexagon with three alternating longitudinal facets removed. This object consists of 12 vertices, 18 edges, 5 faces, 1 component but 0 volumes, and a genus g_{nm}=2. Consequently, according to Eq. (16),

$$12 - 18 + 5 = 1 + 0 - 2$$

which is correct. On the other hand, Eq. (8) for manifold objects cannot describe this object correctly, because

$$12 - 18 + 5 \neq 2(1 - g)$$

no matter how g is counted.

Based on these observations, there are two approaches to unifying the topological representations of standard manifold and thin-walled objects. The first approach is simply adopting the more general formula, Eq. (6), and substituting $m \equiv s$ and $g_{nm} \equiv 2g$ for every standard manifold component.

However, a second approach can be taken whereby the number of components and the genus in the standard relationship of Eq. (8) is not restricted to integer numbers. We propose a modified number of components s^* defined to be the average of the number of components and volumes, i.e. $s^* \equiv \frac{1}{2}(s+m)$, and a modified genus g^* defined to be half of the non-manifold genus, i.e., $g^* \equiv \frac{1}{2} g_{nm}$. Using these modified values, non-manifold topology of thin-walled objects complies with the standard

Euler-Poincaré formula for standard solids, because

$$v - e + f = 2(s^* - g^*) = s + m - g_{nm} \qquad (9)$$

Furthermore, this notation provides a more intuitive interpretation for non-manifold objects; for example, a manifold with boundary, topologically equivalent to a semi-sphere, is considered as half a component, since $s^*=\frac{1}{2}(1+0)=\frac{1}{2}$, and similarly, an open cylinder will be ranked with half a genus, $g^*=\frac{1}{2}$, which intuitively corresponds to placing it somewhere between a torus $(g^*=1)$ and a sphere $(g^*=0)$.

6. SUMMARY

In this paper, we have proposed the use of a thin-walled primitive for modeling the geometry of inherently thin objects and provided a topological invariant supporting both manifold and non manifold objects. The validity of the proposed invariant constitutes a necessary condition for the validity of a geometrical representation of thin-walled products from a topological point of view. Based on this invariant, we have defined a reduced basis of Euler operators which can serve as the fundamental tool set required in managing the topological representation of a part in a modeling system. The paper also discussed merging the proposed general formula with standard manifold topology; we specifically propose the use of non-integer values in the standard Euler-Poincaré formula for representing non-manifold components, thus permitting the use of thin-walled primitives in coherence with traditional solid geometry schemes.

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