

Compound Interest

If you invest P dollars at the annual interest rate r , then after one year the interest is $I = rP$ dollars, and the total amount is $A = P + I = P(1 + r)$. This is *simple interest*.

For *compound interest*, the year is divided into k equal time periods and the interest is calculated and added to the account at the end of each period. So at the end of the first period, $A = P(1 + r(\frac{1}{k}))$; this is the new amount for the second period, at the end of which $A = P(1 + r(\frac{1}{k}))(1 + r(\frac{1}{k}))$, and continuing this way, at the end of the year the amount is

$$A = P \left(1 + \frac{r}{k}\right)^k.$$

The compound interest rate r thus earns the same in a year as the simple interest rate of

$$\left(1 + \frac{r}{k}\right)^k - 1;$$

this equivalent simple interest rate is in bank jargon the “annual percentage rate” or APR.¹

1. Compute the APR of 5% compounded monthly and daily.²
2. As in part (a), compute the APR of 10% compounded monthly, biweekly ($k = 26$), and daily. (We have thrown in the biweekly rate because loans can be paid off biweekly.)

Solution

1. Compute the APR of 5% compounded monthly and daily.

You’ve been given the formula:

$$\text{APR} = \left(1 + \frac{r}{k}\right)^k - 1.$$

All that remains is to determine values for r and k , then evaluate the expression.

In part (a) the interest rate r is 5%. Hence, $r = 0.05$. (If you accidentally let $r = 5$ you solved the problem for a 500% interest rate.) Months divide the year into 12 equal time periods, so to calculate the APR of

¹Banks are required to reveal this so-called APR when they offer loans. The APR also takes into account certain bank fees known as points. Unfortunately, not all fees are included in it, and the true costs are higher if the loan is paid off early.

²For daily compounding assume that the year has 365 days, not 365.25. Banks are quite careful about these subtle differences. If you look at official tables of rates from precalculus days you will find that they are off by small amounts because U.S. regulations permitted banks to pretend that a year has 360 days.

5% interest compounded monthly you use $k = 12$. In order to obtain a numerical estimate of the APR you will want to use a calculator.

$$\begin{aligned}
 \text{APR of 5\% interest compounded monthly} &= \left(1 + \frac{r}{k}\right)^k - 1 \\
 &= \left(1 + \frac{0.05}{12}\right)^{12} - 1 \\
 &\approx 1.0512 - 1 \\
 &\approx 0.0512 \\
 &\approx 5.12\%
 \end{aligned}$$

To calculate the APR of 5% interest compounded daily, we evaluate the expression at the values $r = 0.05$ (the interest rate remains the same) and $k = 365$.

$$\begin{aligned}
 \text{APR of 5\% interest compounded daily} &= \left(1 + \frac{0.05}{365}\right)^{365} - 1 \\
 &\approx 1.0513 - 1 \\
 &\approx 0.0513 \\
 &\approx 5.13\%
 \end{aligned}$$

We chose to write our final answer with three significant figures for two reasons. First, we need at least three digits to detect a difference between compounding monthly and compounding daily. Second, our estimate of the number of days in a year has three significant figures.

2. As in part (a), compute the APR of 10% compounded monthly, biweekly ($k = 26$), and daily.

In part (b), $r = 0.1$ through the whole solution and k takes on the values of 12, 26 and 365.

$$\begin{aligned}
 \text{APR of 10\% interest compounded monthly} &= \left(1 + \frac{0.1}{12}\right)^{12} - 1 \\
 &\approx 0.10471\% \\
 &\approx 10.47\%
 \end{aligned}$$

$$\begin{aligned}
 \text{APR of 10\% interest compounded biweekly} &= \left(1 + \frac{0.1}{26}\right)^{26} - 1 \\
 &\approx 0.10496 \\
 &\approx 10.50\%
 \end{aligned}$$

$$\begin{aligned}\text{APR of 10\% interest compounded daily} &= \left(1 + \frac{0.1}{365}\right)^{365} - 1 \\ &\approx 0.105156 \\ &\approx 10.52\%\end{aligned}$$

Notice that once again the difference in the APR of the different methods of compounding is only a few hundredths of a percent. The difference between compound interest and simple interest is more significant.