MATH 301
SAMPLE FINAL

Please write all answers in your blue book. Where appropriate, prove the correctness of your answers in a clear and organized fashion. (You may use a separate bluebook for scratch work.)

You may use a calculator, your notes, and your textbook on this quiz. You may not use a cell phone or computer.

If you find that you are spending a lot of time on one problem leave it blank and move on to the next. If you have time left over when you complete the exam please use it to check your work.

(1) (20 pts) (Chapter 6 Problem 5) Show that $U(8)$ is isomorphic to $U(12)$.

(2) (20 pts) Is the set of integers $\mathbb{Z}$ an integral domain? Why or why not?

(3) (20 pts) (Chapter 12 Problem 5) Prove that in a ring R with unity 1, multiplicative inverses are unique.

(4) (20 pts) Define addition in S_4 by:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ a & b & c & d \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 & 4 \\ e & f & g & h \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}$$

(here $\{a, b, c, d\} = \{e, f, g, h\} = \{1, 2, 3, 4\}$ but it is not necessarily true that $a = e = 1$.) Prove or disprove: S_4 with operations this addition and the usual multiplication is a ring.

(5) (20 pts) (Chapter 12 Problem 6) Find an integer n and a number a so that in the ring $\mathbb{Z}_n$, $a^2 = a$ and a is not 0 or 1.