

MATH 142
SAMPLE FINAL EXAM

Please put your name on your blue book, show your work and give justifications for your answers. You may use a calculator, the table of integrals you are given, and both sides of an 8 1/2 by 11 sheet of notes on the test; you may not use your cell phone. Try not to spend too much time on any one problem. If you get stuck on a problem, leave a partial answer and move on to the next problem. **There are exam questions on both sides of this page.**

- (1) (10 pts) The graph of a function $f(x)$ is shown above. Write an equation for its antiderivative $F(x)$.
- (2) (5 pts) True or False: The fundamental theorem of calculus says that if $f(x) = F'(x)$, then $\int_a^b F(x)dx = f(b) - f(a)$.
- (3) (10 pts) Find an antiderivative of the function $f(x) = \ln(x)$.
- (4) (15 pts) Is the area of the region to the right of the line $x = 1$ and between the graph of $\frac{2}{x^3}$ and the x -axis finite or infinite? Show your work.

- (5) (15 pts) The Lorentz curve for the distribution of salaries of lawyers in a certain country is described by the function

$$f(x) = \frac{9}{11}x^4 + \frac{2}{11}x.$$

- a) What percent of the total salaries paid to lawyers in that country is paid to the “poorest” 50% of lawyers?
- b) What is the coefficient of inequality for this curve?

- (6) (15 pts) Suppose that for some random variable x , $1 \leq x \leq \infty$, the function $f(x) = \frac{3}{x^4}$ is a probability density function.

- a) What is the probability that $x > 10$?
- b) What is the expected value of x ?

- (7) (20 pts) Consider the function $f(x, y) = x^2 - y^2$.

- a) What is $f(2, 1)$?
- b) What is $\frac{\partial f}{\partial x} = f_x(x, y)$?
- c) What are the critical points of $f(x, y)$?
- d) What is $f_{xx}(x, y)$?
- e) For each critical point, state whether it is a relative maximum, a relative minimum, or neither.

- (8) (10 pts) Find the volume bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 5$, $z = 0$ and the surface $f(x, y) = x^2 + y^2$.