MATH 100 SAMPLE FINAL SOLUTIONS

(1) (10 pts) The graph of $g(x)$ is shown above. Sketch the graph of $h(x) = g(x-2)$.

To get the graph of $h(x)$, shift the graph of $g(x)$ to the right by 2 units. Check your work by confirming that $g(0) = -1$ and $h(2) = g(2-2) = g(0) = -1$.

(2) (10 pts) The function $g(x)$ graphed above is a polynomial function. Is the degree of $g(x)$ odd or even?

Since $g(x)$ increases without bound as x becomes large and positive and as x becomes large and negative, $g(x)$ must have even degree. Check your work by remembering the shapes of the (even degree) polynomial function $y = x^2$ and (odd degree) $y = x^3$.

(3) (15 pts) Does the function $g(x)$ whose graph is shown above have an inverse $g^{-1}(x)$? If so, sketch the graph of $g^{-1}(x)$. If not, explain why not.

The horizontal line $y = 0$ (for example) intersects the graph in more than one point, so the graph fails the horizontal line test. $g^{-1}(x)$ cannot exist because each input to the function would not have a unique output.

(4) (10 pts) What is the equation of the line that passes through the points $(1,3)$ and $(2,5)$?

The slope of the line is $m = \frac{\text{rise}}{\text{run}} = \frac{y_2-y_1}{x_2-x_1} = \frac{5-3}{2-1} = 2$.
The point-slope formula for the equation of a line tells us that

$$y - 3 = 2(x-1),$$

or $y = 2x+1$. Check your work by plugging in the points: $3 = 2(1)+1$ and $5 = 2(2)+1$.

(5) (10 pts) Simplify: $\frac{\frac{2x}{x+1}}{\frac{x+2}{x^2-1}}$.

To divide one fraction by another, invert the denominator and multiply it by the numerator.
$\frac{\frac{2x}{x+1}}{\frac{x+2}{x^2-1}} = \frac{2x}{x+1}\frac{x^2-1}{x+2} = \frac{2x(x^2-1)}{(x+1)(x+2)} = \frac{2x(x+1)(x-1)}{(x+1)(x+2)} = \frac{2x(x-1)}{x+2}$

(6) (15 pts) Find all zeros (real and complex) of the function $f(x) = x^3 - 4x^2 + 5x$.

$f(x) = x(x^2-4x+5)$ is zero when $x = 0$ or when $x^2-4x+5 = 0$. The Quadratic Formula tells us that $x^2 - 4x + 5 = 0$ when $x = \frac{4\pm\sqrt{16-4\cdot 5}}{2\cdot 1} = 2 \pm i$. So the zeros of $f(x)$ are $0, 2-i$ and $2+i$. To check your work, you could compute that $(x-0)(x-(2-i))(x-(2+i)) = f(x)$.

(7) (15 pts) Give the equation of a function with zeros at $x = 1$ and $x = 2$ whose graph has a vertical asymptote at $x = 0$ and slant asymptote $y = 2x$.

A rational function has zeros where its numerator is zero, vertical asymptotes where its denominator is zero, and slant asymptote approximately equal to the ratio of the highest degree terms in the numerator and denominator.

Guess: $f(x) = \frac{(x-1)(x-2)}{(x-0)} = \frac{x^2-3x+2}{x}$.

This function has the approximate slant asymptote $y = \frac{x^2}{x} = x$, so we'd need to multiply by 2 to get tye asymptote $y = 2x$.

Final answer: $f(x)\ \frac{2(x-1)(x-2)}{x} = \frac{2x^2-6x+4}{x}$

If you have a graphing calculator, you can check your work by graphing. Otherwise, check your work by finding the asymptotes and zeros of your final function.

(8) (15 pts) Solve for x: $2ln(x) + ln(x + 1) = 0$.

First, use the fact that $2ln(x) = ln(x^2)$.

$$ln(x^2) + ln(x + 1) = 0$$

Next, use the fact that $ln(a) + ln(b) = ln(a \cdot b)$.

$$ln((x^2)(x + 1)) = 0$$

We can now get rid of the natural log function by composing each side with the inverse exponential function.

$$e^{ln((x^2)(x+1))} = e^0$$

$$(x^2)(x + 1) = 1$$

$$x^3 + x^2 - 1 = 0$$

Because of an error in writing the question, this is difficult to solve algebraically. Graphing the function reveals that $x = .7549$ is an approximate solution. You can check your work by using your calculator to compute that $2ln(.7549) + ln(1.7549) \cong 0.00007189$ is very close to 0.

A better question might be to solve $2ln(x) - ln(x) = 0$ for x; the solution to this equation is $x = 1$.

Bonus (5 pts) Give the equation of a circle whose center is at the intersection of the parabola $y = x^2$ with the line $y = 2x - 1$ and whose radius is 2.

The parabola and line intersect at the single point $(1, 1)$; you can find this by setting $x^2 = 2x - 1$ and solving for x, then plugging $x = 1$ back into either equation to find that $y = 1$.

The equation of a circle with center $(1, 1)$ and radius 2 is $(x - 1)^2 + (y - 1)^2 = 4$.