Website Morphing 2.0: Switching Costs, Partial Exposure, Random Exit, and When to Morph

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Abstract

Website morphing infers latent customer segments from clickstreams then changes websites’ look and feel to maximize revenue. The established algorithm infers latent segments from a pre-set number of clicks and then selects the best “morph” using Expected Gittins’ Indices. Switching costs, potential website exit, and all clicks prior to morphing are ignored.

We model switching costs, potential website exit, and the (potentially differential) impact of all clicks to determine when to morph for each customer. Morphing earlier means more customer clicks are based on the optimal morph; morphing later reveals more about the customer’s latent segment. We couple this within-customer optimization to between-customer Expected Gittins’ Index optimization to determine when and to which website look and feel to morph. We evaluate the improved algorithm with synthetic data and with a proof-of-feasibility application to Japanese bank card-loans. The proposed algorithm generalizes the established algorithm, is feasible in real time, performs substantially better when tuning parameters are identified from calibration data, and is reasonably robust to misspecification.

Keywords: Automated marketing, Bayesian methods, clickstream analysis, dynamic programming, Internet marketing, optimization, switching costs, website design, website morphing.
1. Website Morphing and Its Limitations

Website morphing customizes the look and feel of a website to customers’ cognitive styles. The basic algorithm, developed by Hauser, Urban, Liberali and Braun (2009, hereafter HULB), combines Bayesian inference of latent cognitive-style segments with dynamic programming optimization to match website designs to customers. Bayesian inference on a customer’s clickstream infers probabilities that the customer belongs to the latent segments. Using these probabilities and data from past purchases, the dynamic program automatically selects the best look and feel for the website for each customer. The “morph” assignment is (near) optimal in the sense that it balances learning about the best assignment for a segment with the profit that can be obtained by exploiting current knowledge of morph-to-segment purchase probabilities. HULB use data from a “calibration study” to simulate what would had happened had the BT Group implemented morphing on its broadband-sales website. HULB estimate that morphing would have increased revenue by 21%, with little marginal cost.

Subsequently, Urban, Liberali, MacDonald, Bordley, and Hauser (2013) [hereafter Urban, et al.] adapted morphing to banner advertising. The only modification in HULB’s algorithm was to account for multiple customer visits to the same website. In a field test with over 100,000 customers viewing over 450,000 banners on a CNET website, they report that banner morphing almost doubled click-through rates relative to a random assignment of banners. They also conducted a laboratory test on an automotive information-and-recommendation website to test the basic concept of morph-to-segment matching. The experiment replaced the automated algorithm with direct measurement in a 4-5 week longitudinal study. Urban, et al. found that click-through rates on banners, as well as consideration and preference for Chevrolet-branded vehicles, increased significantly when morphs were matched to customer segments. The banner morphing application expanded the definitions of customer segments to include the stage of the automotive buying process (collecting data, comparing vehicles, committing to a purchase).

The behavioral model underlying the BT Group, CNET, and Chevrolet tests assumes that the customer’s initial clicks on the website do not affect the probability of purchase (or, in the case of banners, click-through rates). The algorithm assumes further that a switch in the look and feel of a website has no effect on the probability of purchase. And, finally, the algorithm does not take into account that customers might exit the website, sometimes after a relatively few clicks.

This paper proposes an improved algorithm that accounts for the three phenomena: the impact of all clicks customer experience on the website, switching costs, and random exit. We do so by transforming an inherently hard switching-cost problem, with a curse-of-dimensionality computational explosion, into a formulation that accounts for the phenomena and can run between clicks on a real website. We demonstrate feasibility with an application to a Japanese bank website offering information on “card loans.” (Details in §6.)
The improved algorithm nests HULB’s algorithm in the sense that it reduces to the HULB algorithm for specific (and we feel extreme) values of key parameters. When parameters can be identified from a calibration study, the algorithm does at least as well as HULB’s algorithm. We show with synthetic data that there exist many cases, with reasonable parameter values, where the improved algorithm outperforms HULB’s algorithm substantially. We examine whether performance degrades if parameter values are chosen incorrectly and address how the parameter values might be obtained in the field.

We begin by reviewing HULB’s algorithm, then present the behavioral model, and propose the improved algorithm. Subsequent sections describe the synthetic-data analyses and the proof-of-feasibility empirical test. We close with a discussion of unsolved problems.

2. Brief Review of HULB’s Website Morphing Algorithm

This section presents an abridged version of HULB’s website morphing algorithm. To stay consistent, we adopt the notation used by both HULB and Urban, et al.. Appendix 1 summarizes all notation. We seek to provide sufficient detail to reproduce the algorithm. Greater details on derivations are available in HULB. All code is available from the authors. Non-proprietary data from the proof-of-concept empirical application is also available.

2.1. The Constructs of Morphing: Morphs, Segments, Clickstream, Calibration

We first provide a qualitative overview of the constructs used in morphing. Detailed examples are available in both HULB and Urban, et al. and questionnaires are available from the authors.

2.1.1. Morphs. Morphs refer to alternative implementations of the overall look and feel of a website (HULB) or banners (Urban, et al.) For simplicity of notation, this paper focuses on the look and feel of websites. Creating alternative morphs imposes a fixed cost on website of banner design—Urban, et al. report an estimated cost of $250,000 for the Chevrolet banners. This fixed cost (and sample size issues discussed later) means that most applications will use a moderate number of morphs. HULB use eight.

2.1.2. Customer segments. Customers are classified into a set of mutually exclusive and collectively exhaustive segments under the hypothesis that different segments respond differently to the morphs. The BT Group, CNET, and Chevrolet applications classified customers by cognitive styles; in the Chevrolet application customers were also classified by the stage of their buying process. Typically, in a calibration study prior to implementing the morphing algorithm, segment membership can be observed with intrusive measurement. During day-to-day implementation on a website, segments cannot be observed directly and must be inferred (probabilistically) from the customer’s clickstream. Because inference is probabilistic in day-to-day website morphing, we call the segments latent. As with morphs, practical considerations require that the number of segments be moderate—16 for the BT Group application, four for the CNET application, and 12 for the Chevrolet application.
2.1.3. **Clickstream.** As a customer explores a website in day-to-day operations we observe the customer’s clickstream. In particular, on any webpage the customer must choose among a number of links. Some links are pictures, some text, some take the customer to another webpage, some open a comparison tool, etc. We call these click alternatives and we observe the customer’s sequential choice of click alternatives. So that we might use the customer’s sequential choices to infer the customer’s segment probabilities, we code each click alternative by a series of click characteristics. For example, HULB code each click alternative by eleven characteristics, some of which are based on evaluation by independent judges (e.g., graphs or text), some of which code specific targets (e.g., an analytic tool), and some of which code actions (e.g., post a comment).

2.1.4. **Outcomes.** Our goal is to maximize an outcome such as sales (BT Group), click-through (CNET), or consideration and preference (Chevrolet). The outcome is observed at the end of the customer’s visit to the website. For example, a BT Group customer might either purchase broadband service or leave the website. Typically, we code a customer as leaving the website if there is no activity in 30 minutes. The BT Group application considered only one visit to the website, but the CNET application used cookies to track multiple visits. In CNET a successful outcome occurred if the customer clicked through on at least one of the visits. Morphing seeks to assign morphs to customers in order to maximize the time discounted value of successful outcomes from the current customer and all future customers.

2.1.5. **Calibration Study.** Prior to implementing the morphing algorithm in day-to-day operations, some parameters must be estimated. To obtain data to estimate these parameters, we recruit a sample of customers to answer questions before and after visiting the website. During the calibration study morphs are assigned randomly to respondents. These calibration-study questions enable us to assign calibration customers to segments and estimate a model that, conditioned on segment membership, predicts customers’ click-alternative choices as a function of the characteristics of the click alternatives. Because it is not feasible to ask such calibration-study questions to day-to-day visitors, we use the model to estimate day-to-day customers’ latent segment probabilities. We refer to customers in the calibration study as respondents to avoid confusion.

### 2.2. Posterior Probabilities of (Latent) Segment Membership

#### 2.2.1. Notation.** Let \( n \) index customers, \( r \) index segments, \( m \) index morphs, and \( t \) index clicks for each customer. Capital letters, \( R, M, \) and \( T_n \) denote totals. (We do not need \( N \) for customers. We use \( N \) later for a different construct.) Let \( c_{tn} \) denote the \( t^{th} \) click by \( n^{th} \) customer and \( \tilde{c}_{tn} = \{c_{1n}, c_{2n}, \ldots, c_{tn}\} \) denote the vector of clicks up to an including the \( t^{th} \) click. At each click choice, the customer faces \( J_t \) click alternatives as denoted by \( c_{tnj} \) where \( j \) indexes click alternatives. We let \( c_{tnj} = 1 \) if customer \( n \) clicks the \( j^{th} \) click alternative on the \( t^{th} \) click and \( c_{tnj} = 0 \) otherwise. Let \( x_{tnj} \) de-
note the characteristics for click-alternative $j$ faced by customer $n$ on the $t^{th}$ click. Let $\tilde{x}_{tn}$ be the set of the $\tilde{x}_{tn,j}$ up to an including the $t^{th}$ click for all $j = 1 \text{ to } J_{tn}$. Let $\tilde{u}_{tn,j}$ be the utility that customer $n$ obtains from clicking on the $j^{th}$ click alternative on the $t^{th}$ click. Let $\tilde{\omega}_{r}$ be a vector of click-alternative-characteristic preferences for the $r^{th}$ customer segment and $\tilde{\epsilon}_{tn,j}$ be an extreme value error such that 

$$\tilde{u}_{tn,j} = \tilde{x}_{tn,j}^t \tilde{\omega}_{r} + \tilde{\epsilon}_{tn,j}.$$ 
Let $\Omega$ be the matrix of the $\tilde{\omega}_{r}$’s.

### 2.2.2. Calibration of preferences for click-alternative-characteristics

In the calibration study we observe the customer’s segment, $r$, the customer’s click-alternative choices, $\tilde{c}_{r,n}$, and the click-alternative characteristics the customer faced at each click-alternative choice, $\tilde{x}_{tn,j}$’s. The extreme value error gives us the standard logit model with the following likelihood for the $n^{th}$ respondent in the calibration study.

$$Pr\left(\tilde{c}_{r,n} \mid r = r, \tilde{\Omega}, \tilde{x}_{tn} \right) = \Pr\left(\tilde{c}_{r,n} \mid r = r \right) = \prod_{t=1}^{T_{n}} \prod_{j=1}^{J_{tn}} \left( \frac{\exp[\tilde{x}_{tn,j}^t \tilde{\omega}_{r}]}{\sum_{j=1}^{J_{tn}} \exp[\tilde{x}_{tn,j}^t \tilde{\omega}_{r}]} \right)^{c_{ntj}}$$

The likelihood over all respondents in the calibration study is simply the product of the individual-respondent likelihoods and we obtain estimates (maximum-likelihood methods) or Bayesian posteriors (MCMC sampling) of $\tilde{\Omega}$. (HULB use Bayesian posteriors; Urban, et al. use maximum-likelihood methods.) Denote these estimates (or mean of the posterior distributions) by $\tilde{\Omega}$. Because morph assignments are made in real time, it is not feasible to sample from the full posterior distribution.

### 2.2.3. Probabilities that a Customer Belongs to Each (Latent) Segment

In day-to-day website operations we treat $\tilde{\Omega}$ as known. We observe the customer’s clickstream, $\tilde{c}_{r,n}$, up to the $r^{th}$ click and we observe the relevant click-alternative characteristics, the $\tilde{x}_{r,n}$. We seek to estimate the probability that the $n^{th}$ customer bellows to segment $r$ for all $r = 1 \text{ to } R$. Denote this probability, after the $r^{th}$ click, by $q_{rnto}$. From the calibration study we know the unconditional prior probabilities, $Pr_0(r_n = r)$, that the $n^{th}$ customer belongs to segment $r$. (Some websites might recalibrate these priors periodically.) Given the observed clickstream, clickstream characteristics, and $\tilde{\Omega}$, we use Equation 1 to obtain $Pr(\tilde{c}_{r,n} \mid r_n = r, \tilde{\Omega}, \tilde{x}_{tn})$. Bayes Theorem then provides:

$$q_{rnto}(\tilde{c}_{r,n}, \tilde{\Omega}, \tilde{x}_{r,n}) = \frac{Pr(\tilde{c}_{r,n} \mid r_n = r, \tilde{\Omega}, \tilde{x}_{r,n}) Pr_0(r_n = r)}{\sum_{s=1}^{R} Pr(\tilde{c}_{r,n} \mid r_n = s, \tilde{\Omega}, \tilde{x}_{r,n}) Pr_0(r_n = s)}$$

Equation 2 (and the imbedded Equation 1) runs sufficiently fast on most computers that we obtain the $q_{rnto}(\tilde{c}_{r,n}, \tilde{\Omega}, \tilde{x}_{r,n})$ quickly.
2.3. Choosing the Optimal Morph If We K new the Customer’s Cognitive Style

For ease of exposition only, assume we know customer $n$’s true segment. We relax this assumption in §2.4. Let $p_{rmn}$ be the probability that customer $n$ in segment $r$, who experienced morph $m$, will make a purchase (or other success criterion). We represent our knowledge, prior to customer 1, about $p_{rm1}$ with a beta distribution which has parameters $\alpha_{rm1}$ and $\beta_{rm1}$. Specifically,

$$f_1(p_{rm1}|\alpha_{rm1}, \beta_{rm1}) \sim p_{rm1}^{\alpha_{rm1}-1}(1 - p_{rm1})^{\beta_{rm1}-1}.$$  

Define $p_{rmn}$, $\alpha_{rmn}$, and $\beta_{rmn}$ accordingly. After each customer we update to the posterior distribution, $f_\delta(p_{rmn}|\alpha_{rmn}, \beta_{rmn}, \delta_{rmn})$, where $\alpha_{rmn}$, and $\beta_{rmn}$ are sufficient to summarize data from customer 1 to $n - 1$. Note that $\alpha_{rmn}$ and $\beta_{rmn}$ are used to make decisions about customer $n$, hence they summarize observations up to and including customer $n - 1$.

Let $\delta_{rmn} = 1$ if the $n^{th}$ customer in segment $r$ makes a purchase after seeing morph $m$; $\delta_{rmn} = 0$ otherwise. (We temporarily add the $r$ subscript to $\delta_{rmn}$ to indicate that the segment, $r$, is known.) Then, because the binomial outcomes are naturally conjugate to beta priors, Appendix 2 shows that $\alpha_{rm,n+1} = \alpha_{rmn} + \delta_{rmn}$ and $\beta_{rm,n+1} = \beta_{rmn} + (1 - \delta_{rmn})$. Normalizing the value of a purchase to 1.0, implies that the immediate reward we expect from the $n^{th}$ customer is $E[p_{rmn}|\alpha_{rmn}, \beta_{rmn}] = \alpha_{rmn}/(\alpha_{rmn} + \beta_{rmn})$.

Assigning the optimal morph to the $n^{th}$ customer is more complicated than simply maximizing the immediate reward. Whenever we assign a morph $m$ to a customer in segment $r$ and observe an outcome, we update the posterior distribution for $p_{rmn}$. The updated distribution enables us to make better decisions in the future. The dynamic decision problem must balance immediate rewards with the knowledge gained that enables better decisions in the future. (In website morphing, the automated algorithm chooses the morph; not the customer.)

HULB demonstrate that this dynamic decision problem, for known $r_n$, is the classic multi-armed bandit problem studied by Gittins (1979). Gittins proved that the problem is indexable and that the optimal solution is to compute an index for each arm and choose the arm with the largest index. Because the detailed derivations are available in Gittins (1979), summarized in HULB, and otherwise widely known, we do not repeat them here. We repeat only the basic equations.

Let $G_{rmn}$ be the Gittins’ index for the $m^{th}$ morph assuming the customer is in segment $r$ and we have updated $\alpha_{rmn}$ and $\beta_{rmn}$ based on all customer purchases up to but not including the $n^{th}$ customer. If $\alpha \leq 1$ is the discount rate from one customer to the next and if $V_{Gittins}(\alpha_{rmn}, \beta_{rmn}, \alpha)$ is the value of continuing with parameters $\alpha$, $\alpha_{rmn}$, and $\beta_{rmn}$, then $G_{rmn}$ solves the following Bellman equation. (Intuitively, the right-hand side chooses the maximum of a fixed “arm” and an uncertain “arm.” The fixed “arm” pays the expected value Gittins’ index over all future $n$. The uncertain arm pays a unit reward if the
outcome is a success and nothing if it is not. A success occurs with an expected probability of \( \alpha_{r,mn} / (\alpha_{r,mn} + \beta_{r,mn}) \). The uncertain arm also pays the value of continuing to explore optimally for future \( n \), but discounted by \( a \). The continuation values account for updating the beta distribution.

\[
V_{\text{Gittins}}(\alpha_{r,mn}, \beta_{r,mn}, a) = \max \left\{ \frac{\alpha_{r,mn}}{1 - a}, \frac{\alpha_{r,mn}}{\alpha_{r,mn} + \beta_{r,mn}} \left[ 1 + aV_{\text{Gittins}}(\alpha_{r,mn} + 1, \beta_{r,mn}, a) \right] + \frac{\beta_{r,mn}}{\alpha_{r,mn} + \beta_{r,mn}} aV_{\text{Gittins}}(\alpha_{r,mn}, \beta_{r,mn} + 1, a) \right\}
\]

Equation 3 does not have an analytic solution, but we can readily compute Gittins’ indices with a simple iterative numeric algorithm. (We reuse code developed by Gittins and can make it available upon request.) For a given \( a \), we table \( C_{r,mn} \) as a function of \( \alpha_{r,mn} \) and \( \beta_{r,mn} \). If the segment, \( r \), were known, HULB’s algorithm would simply look up the Gittins’ indices for all \( m \) and choose the morph with the largest index. This strategy would lead to the maximum long-term profit taking both exploration and exploitation into account. In the special case where we are limited to one website or banner for everyone \( (R = 1) \), we can use the Gittins’-index algorithm to choose the optimal website or banner for each \( n \).

2.4. Choosing a Morph when Cognitive Styles are Partially Observable

2.4.1. POMDP. When a customer’s segment is not known with certainty, the dynamic program changes. Because the segment is only partially observable the optimization problem requires we solve a partially-observable Markov decision process (POMDP). The state space is Markov because the full history is summarized by the \( \alpha_{r,mn} \)’s, the \( \beta_{r,mn} \)’s, and the latent segments.

2.4.2. Expected Gittins’ Index. Krishnamurthy and Michova (1999) prove that the POMDP is an indexable decision process. (Indexability has a technical definition, but in lay terms, an “arm” of a multi-armed bandit is indexable if, when an index increases, the set of states for which choosing the arm is optimal does not decrease. Indexability implies an index strategy is feasible, although it does not guarantee an index strategy is optimal. Very often an index strategy is near optimal.) Krishnamurthy and Michova establish further that that if we compute the expectation of the Gittins’ index over our uncertainty about the customer’s segment and choose the morph is the largest Expected Gittins’ Index \([EGI]\), then, in many cases, the EGI policy will lead to 99% of optimality. More critically, because the EGI is easy to compute, we can select a (near) optimal morph in real time (between clicks on the website). HULB provide simulation evidence that the EGI strategy works extremely well for website morphing.

Specifically, the EGI algorithm replaces Gittins’ index with the Expected Gittins’ Index, \( EG_{r,mn} \), and chooses the morph with the largest \( EG_{r,mn} \) where:

\[
EG_{r,mn} = \sum_{r=1}^{R} q_{r,mn}(\hat{c}_{r,\tau_0}, \hat{X}, \hat{\Omega}) C_{r,mn}(\alpha_{r,mn}, \beta_{r,mn}, a)
\]
HULB use $\tau_o = 10$ and Urban, et al. use $\tau_o = 5$. Equation 4, with minor modification, also enables us to choose the best starting morph for each customer. We simply replace $q_{rm,0}(\tilde{c}_{rn,0}, \tilde{\bar{\Omega}}, \tilde{X}_{rn,0})$ with $q_{r,n0}$, our beliefs about the customer’s segment prior to observing any clicks. HULB use $Pr_o(r_n = r)$, but it would also be feasible to use periodic updates.

**2.4.3. Updating Beliefs when Segments are Partially Observable.** Updating beliefs when the customer’s segment is known is particularly easy because the binomial distribution and the beta distribution are naturally conjugate. When we need to take all uncertainty about the segments into account, Appendix 2 demonstrates that updating is no longer naturally conjugate. However, we can still update if we consider “fractional observations.” That is, if we observe a success, $\delta_{mn}$, conditioned on the customer having seen morph $m$, we consider this as a fractional success for each latent customer segment, $r$. (Note that the outcome is no longer conditioned on $r$.) The fractional success is $q_{r,m,n}(\tilde{c}_{rn,n}, \tilde{\bar{\Omega}}, \tilde{X}_{rn,n})\delta_{mn}$ for each $r = 1$ to $R$. The binomial distribution is well-defined for fractional observations and naturally conjugate to the beta distribution. (A binomial distribution with continuous outcomes is sometimes called a Pólya distribution.) HULB update via:

$$
\alpha_{rm,n+1} = \alpha_{rm,n} + q_{r,m,n}(\tilde{c}_{rn,n}, \tilde{\bar{\Omega}}, \tilde{X}_{rn,n})\delta_{mn}
$$

$$
\beta_{rm,n+1} = \beta_{r,m,n} + q_{r,m,n}(\tilde{c}_{rn,n}, \tilde{\bar{\Omega}}, \tilde{X}_{rn,n})(1 - \delta_{mn})
$$

To maximize information, we use all clicks, $\tilde{c}_{rn,n}$, rather than just the clicks prior to a morph. Appendix 2 provides complete derivations as well as arguments that these updating formulae cause the posterior distribution to converge to a mass point at the true values of the $p_{r,m,true}$’s as $n \to \infty$. HULB provide simulation evidence that the fractional updating formulae lead to effective and profitable morph-to-segment assignments. Such simple formulae are necessary to enable the morphing algorithm to run in real time between a customer’s clicks on the website.

**2.5. Experience with HULB’s Algorithm**

Website morphing imposes a high initial fixed cost. The estimated $80 million in incremental revenue for the BT Group, the observed banner click-through lifts of 80-100% for CNET’s context-matched banners, and the observed 30% lift in Chevrolet brand consideration justify the initial fixed cost for these high-traffic websites. Morphing also requires high traffic because of its convergence properties.

A sale by the BT Group to the $n + 1^{st}$ customer is worth almost as much as a sale to the $n^{th}$ customer. HULB’s example with 100,000 visitors per annum implies a discount rate of $\alpha = 0.999999$ [HULB, p. 208]. When $\alpha$ is close to 1.0, the optimal strategy includes substantial exploration. With known segments, HULB report that their algorithm explored heavily for the first 1,000 customers; it did
not stop exploring until the 3,000th customer. With 16 equal-sized segments, the HULB algorithm would likely stabilize around the 50,000th customer.

Success probabilities for banners are typically much lower than success probabilities for sales on a morphing website—the order of 0.003 in Urban, et al. versus 0.38 in HULB. Gittins’ indices are likely to take longer to stabilize in banner morphing than in website morphing. Nonetheless, Urban, et al.’s experience is informative. In their application with 100,000 customers, the algorithm stabilized for the largest segments, but it was still exploring for the smallest segment which had approximately 9,000 customers. Urban, et al. also discuss how to handle customers who leave and return to websites.

The experience by HULB and Urban, et al. suggest that we should evaluate performance up to approximately 10,000 customers per segment. Thus, the four-segment synthetic data experiments in this paper examine performance from 1 to 40,000 customers per segment—10,000 customers per segment.

### 3. Behavioral Theory Improvements: Key Assumptions

HULB switch to the best morph (could be the current morph) after the 10th click; Urban, et al. switch after the 5th click. Both applications assumed that the morph seen by customers up to the $\tau_i$ click had negligible impact on the customer’s probability of a successful outcome (sales or click through). HULB justified this decision because the average respondent in their calibration study clicked more than 10 times and 10% of the respondents clicked more than 30 times. The implications of these approximations were never tested.

More critically, HULB assumed that there were no costs to the customer in switching between morphs. They needed this assumption because the Gittins’ solution requires that switching costs are zero (Banks and Sundaram 1994). Without special structure, switching costs cause the optimization problem to become NP complete (Jun 2004, p. 526; Ny and Feron 2006, Theorem 1).

Before we generalize HULB to allow the algorithm to choose when to morph, we must specify a theory of customer behavior. To date no such theory exists with respect to morphing. We seek a theory that is parsimonious, captures the essence of the phenomena we seek to model, and is sufficiently “conjugate” that a when-to-morph algorithm is feasible in real time between clicks on the website. To the extent that these assumptions can be improved, our tests are conservative. Future elaborations might improve website morphing further.

#### 3.1 Assumption 1. Switching Costs

An extensive literature in psychology and marketing studies the cognitive cost of switching. Papers in psychology, beginning with Jersild (1927), generally study the cognitive cost of completing a task, with switching costs higher when the tasks require like stimuli (Spector and Biederman 1976). For example, Meiran (2000) establishes that switching among methods of response in a computer task imposes...
cognitive loads on respondents. In marketing, switching costs are well-established in many sales contexts (e.g., Weiss and Anderson 1992) and appear to apply as well for cognitive costs (Jones, Mothersbaugh, and Beatty 2000, 2002). In their research, cognitive switching costs affect purchase intention. For websites Balabanis, Reynolds and Simintiras (2006, p. 217) suggest that “cognitive search costs in online shopping environments can be significant” and Johnson, Bellman and Lohse (2003, p. 63) report that “perceived switching costs … create a cognitive ‘lock-in’” for websites. It is likely that the cognitive costs of switching morphs reduce the benefit of matching a website’s look and feel to a customer segment.

The cost of switching is most salient with an hypothetical example. Suppose that after the $t^{th}$ click, posterior probabilities suggest that the best morph for the $n^{th}$ customer is graphical and focused. The customer learns to search based on this look and feel. Now suppose that subsequent clicks in the customer’s clickstream suggest instead that the best morph is verbal and general. If we were to switch morphs the customer might become confused and have to relearn his/her search strategy. Although the verbal-general morph might have been best for the customer had the website had those characteristics from the beginning, it may not be best after the customer has learned to use a graphical-focused website. In general, such path dependence makes switching-cost optimization problems NP complete.

Additive switching cost are common in the literature and algorithms exist (e.g., Banks and Sundaram 1994; Dushochet and Hongler 2003; Jun 2004), but additive switching costs impose path dependence and make it infeasible to determine when to morph in real time. On the other hand, a multiplicative switching cost can be factored out in a Bellman equation. As we demonstrate below, we can solve a problem with multiplicative switching costs in real time. That is, we assume that a switch lowers the customer’s purchase probability when the switch is made. It lowers the purchase probability by a factor of $\gamma$ where $\gamma \leq 1$. HULB’s algorithm assumes the special case of $\gamma = 1$.

Fortunately, multiplicative switching costs have descriptive advantages. First, because predicted outcomes, $p_{rmn}$’s, are bounded between 0 and 1, a multiplicative factor does not violate that bound whereas an additive factor might. Second, although untested, we expect that the amount by which a low probability is lowered by a switching cost would be less than the amount by which a high probability is lowered by a switching cost. For example, suppose a switch lowers $p_{rmn}$ from 0.900 to 0.810. Comparable proportional cost would lower $p_{rmn}$ of 0.090 to 0.081 while a comparable additive cost would lower $p_{rmn}$ from 0.090 to 0.000.

In theory we might generalize $\gamma$ to assume that $\gamma_{mm}$ is a function of the morphs from which and to which the customer switches. We leave that generalization to future research because (1) it would introduce severe state dependence that would make the dynamic program infeasible, (2) it would introduce a substantial measurement burden requiring a large number of parameters—56 parameters in the BT
Group application and 132 parameters in the Chevrolet application, and (3) HULB’s algorithm treats morphs as independent. Nonetheless, morph-specific switching costs are an interesting and challenging extension.

3.2. Assumption 2. The Impact of Being Exposed to Multiple Morphs

HULB assume that only the last morph affects purchase probabilities. However, suppose that the system morphs after the 10th click and the customer makes a purchase decision after the 20th click. There is no reason to assume that the first 10 clicks have less impact than the last 10 clicks. The last 10 clicks may have more impact (recency), less impact (primacy), or equal impact. To allow various assumptions, we assign weights, \( w_t \), to clicks to account for the differential impact of early vs. late morphs. Let \( \bar{w} \) be the vector of these weights. (§3.5 discusses other models.) Setting \( w_t = w \) for all \( t \) implies equal impact. Setting \( w_t \) equal to an increasing (decreasing) function of \( t \) assigns greater impact to later (earlier) morphs. HULB’s algorithm assumes a special case for \( \bar{w} \). In particular, HULB’s algorithm implicitly sets \( w_t \) equal to zero for \( t \leq 10 \) and \( w_t = w \) for \( t > 10 \).

In equation form, if customer \( n \) sees morph \( m_{tn} \) at the \( t^{th} \) click (or the \( t^{th} \) observation period), we assume the expected purchase probability at the end of the visit (prior to switching-cost adjustment) is:

\[
p_{rn} = \sum_t w_t p_{rtn}
\]

When convenient we normalize the impact weights so that they sum to 1.0 over clicks (or observation periods). To keep the number of \( w_t \)’s small, we allow \( t \) to index observation periods that may be one click or more than one click.

3.3. Assumption 3. Variation in the Number of Clicks for Each Customer’s Visit

Some customers find the information they need quickly, make a purchase decision, and exit the website. Other customers visit many areas of the website gather extensive information and leave later. We cannot assume, nor do the data support, an assumption that all customers stay for all observation periods. The number of clicks does not seem to be particularly correlated with either the morph given or the purchase probability, but rather reflects the (unobservable) information needs of the customer.

Let \( \psi_{tn} \) be the probability that customer \( n \) leaves on the \( t^{th} \) click given that customer \( n \) has already made \( t - 1 \) clicks. For example, before we observe the first click we expect the customer to leave after that click with probability \( \psi_{1n} \), to leave after the second click with probability \((1 - \psi_{1n})\psi_{2n}\), to leave after the third click with probability \((1 - \psi_{1n})(1 - \psi_{2n})\psi_{3n}\), and so on. For each possibility, we normalize the effective impact weights, \( w_t' \), to account for the number of clicks before exit. This model generalizes HULB’s implicit assumptions that \( \psi_{10,n} = 0 \) and \( \psi_{T_n,n} = 1 \) for some \( T_n \gg 10 \).

The only datum we observe for each customer is the click at which he or she leaves, thus we need
a reasonably parsimonious model that balances non-stationarity over $t$ with heterogeneity over $n$. In §6.6 we demonstrate how to estimate the parameters of the model from the calibration study. In particular, we obtain maximum-likelihood estimates based on the observed times of exit.

3.3.1. Model 1. Heterogeneity over customers. This model assumes that $\psi_{tn}$ is beta distributed over customers with parameters $\alpha_{\psi}$ and $\beta_{\psi}$ but independent of $t$: $f_{\psi}(\psi_{tn}|\alpha_{\psi}, \beta_{\psi}) \sim \psi_{tn}^{\alpha_{\psi}-1}(1 - \psi_{tn})^{\beta_{\psi}-1}$. Given this assumption we can readily calculate the probability that a randomly chosen customer will leave after any given click. We obtain the predicted probability of leaving by integrating out the heterogeneity. For example, simple calculus provides the probability of leaving after the first click as $\alpha_{\psi}/(\alpha_{\psi} + \beta_{\psi})$. The probability of leaving after the second click is $\alpha_{\psi}\beta_{\psi}/[(\alpha_{\psi} + \beta_{\psi} + 1)(\alpha_{\psi} + \beta_{\psi})]$, after the third click $\alpha_{\psi}\beta_{\psi}(\beta_{\psi} + 1)/[(\alpha_{\psi} + \beta_{\psi} + 2)(\alpha_{\psi} + \beta_{\psi} + 1)(\alpha_{\psi} + \beta_{\psi})]$, etc. Using these values we calculate the probability that a customer remains on the website through the $t$th click.

3.3.2. Model 2. Non-stationarity over clicks. Assume $\psi_{tn} = \psi_t$ for all $n$.

3.3.3. Model 3. Hybrid model. Respondents might remain on the website for some initial clicks, but, after those initial clicks, revert to a different $\psi_{tn}$ that is heterogeneous in $n$ (as in Model 1).

3.4. Selecting Values of the Tuning Parameters

The switching discount ($\gamma$), the impact weights ($\vec{w}$), and the parameters of a model for exit probabilities ($\psi_{tn}$’s) are tuning parameters in the more generalized algorithm. They must be selected before the algorithm is used to morph a website (in day-to-day operations). The time of exit is easy to observe in the calibration study. We estimate the parameters of the chosen model using standard methods. The other parameters, $\gamma$ and $\vec{w}$, require either managerial judgment or experiments during the calibration study. In a calibration study, segment membership is measured directly, therefore the true $r_n$ is known among calibration respondents. Applications to date vary morphs randomly over customers to estimate the priors, $p_{rm1}$. A more complex experimental design might vary morphs randomly over customers and randomly switch morphs within customers. With a sufficient sample size, estimates of the tuning parameters, $\gamma$, $\vec{w}$, and the $\hat{p}_{rm1}$’s, are all identified from observed customer outcomes (sales or click-throughs). For example, if there are 8 morphs, 4 segments, and 4 observation periods, we need to estimate 37 parameters ($32 \hat{p}_{rm1}$’s, 4 $\vec{w}_t$’s, and $\hat{\gamma}$). For each segment some customers see one morph for all periods. Some customers see two morphs with switches varied over $t = 1, 2, or 3$. A full factorial would be $8 \times 4$ (single morph combinations) plus $8 \times 7 \times 4 \times 3$ (two morph combinations) for a total of 704 combinations—more than enough to identify 37 parameters. If a feasible algorithm could be developed to account for different $\gamma_{mm}$’s, they, too, would be identified, albeit with a large sample-size requirement.

We might also update the tuning parameters after initial experience on the website, say after observing 50,000 customers. Although the optimal morph allocations are based on the assumed values of the
tuning parameters, we can still identify $\tilde{y}$ and $\tilde{w}$ based on the likelihood principle. See related discussions in Hauser and Toubia (2005) and Liu, Otter, and Allenby (2007). Future algorithms might also explore optimal experimentation to learn these parameters while learning about the $p_{rmi}$’s.

3.5. Alternative Assumptions

Making the decision on when (whether, how often) to morph is a challenging optimization problem. We must solve the problem optimally or near optimally, and we must do so in real time between customer clicks. Modeling switching costs, impact weights, and exit probabilities with separable functions enables rapid solutions. We believe our assumptions are simple, realistic, matched well to the data available in website-morphing applications, reasonable first-order approximations, and (for impact weights) flexible. However, other assumptions are possible. For example, we might make $\gamma$ an increasing function of the clicks before a switch and a decreasing function of the clicks after a switch; or we might allow $\gamma$ to depend upon the number of prior switches. Equation 6 might be replaced by a non-linear function.

All website morphing applications to date used clicks rather than clock time because clock time adds unobserved variance due to network speed, distractions while browsing, and variations in reaction time. Although clicks are correlated with clock time in our field experiment ($\rho = 0.57, p < 0.01$), future applications might model $\gamma$, $\tilde{w}$, and $\psi_{tn}$ as functions of clock time rather than clicks. With experience and new research we might find one or more of these extensions feasible and profitable. For now, we believe our assumptions are reasonable, robust, and generalize website morphing.


HULB’s algorithm identifies the best morph to give to each customer based on a fixed number of customer clicks. We can do better with an algorithm that determines endogenously the number of clicks to observe before morphing. The algorithm also allows, but does not require, more than one change in morphs. In this section, $t$ indexes observation periods that may be more than one click.

4.1 Dynamic Decision Problem

Figure 1 illustrates the when-to-morph decision problem for the case where the customer makes a purchase (or leaves the website) after four observation periods. (The theory applies when the number of observation periods is a random variable; four morphs is just an illustration.) Specifically, during observation period $t$ the website displays morph $m_{tn}$. The respondent makes clicks, $c_{tn}$, while exploring the website and we update our beliefs about the customer’s segment, $q_{rtn}(\tilde{c}_{tn}, \hat{X}_{tn})$. Using the new information, and anticipating more information from subsequent decision periods, we decide which morph, $m_{t+1,n}$, to display in the next decision period. To keep track of morph changes, we define $\Delta_{m_{tn}'}$ as an indicator variable such that $\Delta_{m_{tn}'} = 1$ if we change to morph $m_{tn}'$ for customer $n$ in period $t$. With this
notation, the total number of morph changes for customer $n$ is $N_{tn} = N_{t-1,n} + \sum_{m_t} \Delta_{m_t,tn}$. We represent the purchase decision by $\delta_n = 1$ if the customer makes a purchase and $\delta_n = 0$ if the customer does not make a purchase. We dropped the $m$ subscript from HULB’s $\delta_{mn}$ to allow for the fact that more than a single morph may have affected outcomes for customer $n$.

Figure 1 illustrates the basic dilemma. The longer we wait to morph, the more clicks we observe. More clicks enable us to identify better customer $n$’s cognitive-style segment and, hence, the best morph for customer $n$. However, when the impact weights imply that early-morph experience affects purchase probabilities or if the customer is likely to leave early, we want to get to that best morph as rapidly as feasible. The problem is compounded because switches are costly.

4.2. Formulating a Feasible Bellman Equation

Optimizing morph decisions for the problem in Figure 1 is challenging. It is even more challenging when embedded within a dynamic program to learn the best morph to assign for each customer segment (as in HULB). For example, as Asawa and Teneketzis (1996, p. 329) caution: “inclusion of a switching penalty drastically changes the nature of the bandit problem. … the optimal policy is not given by an index rule anymore.” Although heuristics exist using multiple indices (e.g., Dusonchet and Hongler 2006), the number of indices would explode exponentially in a morphing problem were there are, potentially, thousands of costly switches (one or more switches for each customer).

The problem is further compounded when Bayesian updating is used to identify customer segments. Each switch presents new click opportunities which customers choose probabilistically based on their (latent) customer segment and the morph decision. We might anticipate how our decisions in observation period $t$ affect the observations which update $q_{rtn}(\hat{c}_{tn}, \hat{\Omega}_t, \hat{x}_{tn})$ for $\tau > t$. This problem quickly becomes intractable. We need to finesse the explosion in the number of potential click paths. Finally, any solution must take into account that customers leave stochastically between observation periods.

We must solve these additional challenges with an algorithm that runs sufficiently fast—between a customer’s clicks on the website. Our solution exploits the ability of Gittins’ indices to summarize the rewards from future stochastic processes. For the current customer we summarize decisions on future customers, anticipated outcomes, and Bayesian posterior distributions of $p_{rtn}$ by the Gittins’ indices, $G_{rmn}$. We continue to update the $G_{rtn}$ when we observe $\delta_n$ at the end of customer $n$’s visit to the website. This heuristic strategy is analogous (but not identical) to optimal solutions in the branching bandit literature. (We argue in §4.2.5 why it is reasonable to decouple the optimization in this manner.)

To make the algorithm feasible between clicks, we exploit the likelihood principle. Specifically, at $t$ the best Bayesian estimate of customer $n$’s purchase probabilities after all observation periods ($\hat{c}_{tn}$)
is based only on clicks up to and including the $t^{th}$ observation period ($c_{tn}$). The likelihood principle enables us to finesse the explosion in click opportunities that is dependent upon our decisions for customer $n$ after observation period $t$. This when-to-morph dynamic program is then feasible because the separability assumptions on switching costs ($\gamma$) and impact weights ($\mu$) lead to a computationally fast Bellman equation. To formulate the Bellman equation, we consider carefully what we know about customer $n$’s segment, when we know it, and how this affects the future.

4.2.1. Immediate reward. The first simplification comes from linear impact weights which enables us to separate outcomes by observation period. The second simplification comes from the multiplicative nature of switching costs which enable us to factor them out. The third simplification comes when we recognize that, independently of future clicks, our best estimate of the terminal probabilities, $q_{rn}(\tilde{c}_{t,n}, \hat{\Omega}, \hat{X}_{t-1,n})$, is $q_{rn}(\tilde{c}_{t-1,n}, \hat{\Omega}, \hat{X}_{t-1,n})$. In other words, the $q_{rn}(\tilde{c}_{t-1,n}, \hat{\Omega}, \hat{X}_{t-1,n})$ represent our expectations over all future clicks. Notice that switches have the greatest impact when $w_{tn}$ is large. With these simplifications we write the expected immediate reward, $EIR(m_{tn}, m_{t-1}, \tilde{c}_{t-1,n}, \hat{\Omega}, \hat{X}_{t-1,n})$, as:

$$EIR(m_{tn}, m_{t-1}, \tilde{c}_{t-1,n}, \hat{\Omega}, \hat{X}_{t-1,n}) = \gamma^{A_{mtn}}W_{tn} \sum_{r} E[c_{t,n}c_{t+1,n}, ..., c_{rn}]q_{rn}(\tilde{c}_{t,n}, \hat{\Omega}, \hat{X}_{t,n})G_{rn}$$

$$= \gamma^{A_{mtn}}W_{t} \sum_{r} q_{rn}(\tilde{c}_{t-1,n}, \hat{\Omega}, \hat{X}_{t-1,n})G_{rn}$$

4.2.2. Value of continuing optimally. To formulate the value of continuing optimally, we recognize that the evolution of $q_{rn}(\tilde{c}_{t,n}, \hat{\Omega}, \hat{X}_{t,n})$ depends upon the true customer segment. However, at period $t$ (and even at period $T$) we are uncertain about the true segment. Thus, we have an expectation over $r_{true,n}$ for the purchase probabilities, but we also have an expectation over $r_{true,n}$ for the evolution of $q_{rn}(\tilde{c}_{t,n}, \hat{\Omega}, \hat{X}_{t,n})$ for $\tau \geq t$.

To keep track of these expectations, we temporarily write the evolution of the clicks as conditioned on $r_{true,n}$. If $m_{t+1,n}^{*}(r_{true})$ is the morph we would choose at $t + 1$ if the true segment for customer $n$ was $r_{true,n}$, then the continuation value is:

$$V_{t+1}(m_{t+1,n}^{*}(r_{true,n}), m_{tn}, \tilde{c}_{t-1,n}, \hat{\Omega}, \hat{X}_{t-1,n}|r_{true,n}) = \max_{m_{t+1,n}} E[c_{t,n}c_{t+1,n}, ..., c_{rn}]V_{t+1}(m_{t+1,n}, m_{tn}, \tilde{c}_{t-1,n}, ..., m_{rn}, \tilde{c}_{t,n}, \hat{\Omega}, \hat{X}_{t,n}|r_{true,n})$$

But $r_{true,n}$ is not known when we are making the decision at $t$, so we account for this dependence for $\tau \geq t$ by taking an expectation over this unknown variable at $t$. That is, we take an expectation over the true segments when we compute the continuation value for $\tau \geq t$. Going forward to any observation period, $\tau$, under the assumption that the true segment is $r_{true,n}$, we would like to anticipate how the
$q_{rn}(c_{tn}, \Omega, \tilde{X}_{tn})$ will evolve for any future sequence of morphs. However, in practical situations, the state space is so large that we cannot anticipate the future clickstream for $\tau \geq t$. At the start of the $\tau^{th}$ decision period, our best estimate of customer $n$’s segment probabilities over all future clickstream paths remains $q_{rn}(c_{t-1,n}, \Omega, \tilde{X}_{t-1,n})$. In other words, $V_t$ depends on $c_{t-1,n}$ for $\tau \geq t$. Backward induction does the rest.

Naturally, we keep track of the $\Delta_{m_{tn}n}$’s for $\tau \geq t$ when we compute the conditional values, $V_t(m^*_{tn}, m_{t-1,n}, c_{t-1,n}, \Omega, \tilde{X}_{t-1,n}|r_{true,n} = s)$. Putting it all together enables us to optimize the current period’s morph, $m_{tn}$ by backward induction using a computationally-rapid Bellman equation.

$$V_t(m^*_{tn}, m_{t-1,n}, c_{t-1,n}, \Omega, \tilde{X}_{t-1,n})$$

$$= \max_{m_{tn}} \left\{ \gamma^{A_{mtn}}w_t \sum_r q_{rn}(c_{t-1,n}, \Omega, \tilde{X}_{t-1,n})G_{rm_{tn}} + \right\}$$

$$\sum_s \left[ q_{sn}(c_{t-1,n}, \Omega, \tilde{X}_{t-1,n})V_{t+1}(m^*_{t+1,n}, m_{tn}, c_{t-1,n}, \Omega, \tilde{X}_{t-1,n}|r_{true,n} = s) \right]$$

**4.2.3. Variation in the number of periods for each customer’s visit.** Purely for ease of exposition we derived when-to-morph Bellman equation assuming a fixed number of observation periods. It is relatively simple to generalize the Bellman equation for variation in the number of periods for each customer’s visit. The assumption of §3.3 provides the separability necessary. Let $\Psi_n(S|t-1)$ be the probability that customer $n$ is still at the website at observation period $S$ given that customer $n$ was at the website at observation period $t-1$. $\bar{\Psi}(S|t-1)$ is the expectation over customers because we do not know customer $n$’s exit probability prior to observing an exit. Using any of the random exit models in §3.3 we calculate this probability via $\bar{\Psi}(S|t-1) = E_n[\prod_{S=t}^{t-1}(1 - \psi_{ns})]$. The random exit (r.e.) Bellman equation is then:

$$V_t(m_{tn}, m_{t-1,n}, c_{t-1,n}, \Omega, \tilde{X}_{t-1,n} | \text{random exit})$$

$$= \max_{m_{tn}} \left\{ \gamma^{A_{mtn}}w_t \sum_r q_{rn}(c_{t-1,n}, \Omega, \tilde{X}_{t-1,n})G_{rm_{tn}} \bar{\Psi}(t|t-1) + \right\}$$

$$\sum_s \left[ q_{sn}(c_{t-1,n}, \Omega, \tilde{X}_{t-1,n})V_{t+1}(m^*_{t+1,n}, m_{tn}, c_{t-1,n}, \Omega, \tilde{X}_{t-1,n}, r.e.|S) \right] \bar{\Psi}(t+1|t-1)$$

**4.2.4. Modified Bayesian Updating.** HULB’s algorithm assumes that only the last morph seen by customer $n$ affects the probability of a successful outcome for customer $n$. For real-time computation, HULB treated the observed outcome, $\delta_{rn}$, as a series of fractional outcomes as determined by the (latent) segment probabilities, $q_{rn}(c_{tn}, \Omega, \tilde{X}_{rn})$, based on the first $\tau$ observation periods. The when-to-morph algorithm is based on a more-general behavioral assumption that allows impact weights for all observation periods. We therefore generalize the updating procedure (Equation 5, §2.4.3). As demonstrated in Append-
dix 2, we can generalize HULB and treat each combination of a period and a latent segment as a fractional observation. If we do so, we obtain a computationally rapid updating equation that converges to the true values of the outcome probabilities.

Let \( \eta_{mnt} = 1 \) if customer \( n \) saw morph \( m \) during the \( t^{th} \) observation period, and \( \eta_{mnt} = 0 \) otherwise. The fractional observation for period \( t \), morph \( m \), and segment \( r \) is based on the \( \eta_{mnt} \), the impact weights, \( w_t \), the switch penalty, and \( q_r(\tilde{c}_{Tn}, \tilde{\Omega}, \tilde{X}_{Tn}) \). Appendix 2 motivates the following updating equations. In the synthetic data experiments in §5 (next) these updating equations converge to a tight distribution around the (known) true values of the \( p_{rm, true} \)’s.

\[
\begin{align*}
\alpha_{rm,n+1} &= \alpha_{rmn} + q_r(\tilde{c}_{Tn}, \tilde{\Omega}, \tilde{X}_{Tn}) \left( \sum_{t=1}^{T} \gamma^{\Delta mnt} \eta_{mnt} w_t \right) \delta_n \\
\beta_{rm,n+1} &= \beta_{rmn} + q_r(\tilde{c}_{Tn}, \tilde{\Omega}, \tilde{X}_{Tn}) \left( \sum_{t=1}^{T} \gamma^{\Delta mnt} \eta_{mnt} w_t \right) (1 - \delta_n)
\end{align*}
\]

### 4.2.5. Motivation for Expected Gittins’ decoupling.

The when-to-morph optimization (Equations 7 and 8) is feasible because we decoupled the decision on when to morph for a respondent from optimal experimentation to balance immediate and long-term rewards while learning about morph-to-segment success probabilities (\( p_{rmn} \)’s). We cannot prove that decoupling retains the near optimality of the Expected Gittins’ Index, but there are many reasons to believe that decoupling will be near optimal.

First, we must wait until \( T_n \) to update the \( p_{rmn} \)’s by observing whether customer \( n \) purchases or leaves the website. Thus, the observed \( \delta_n \) cannot affect the decision of when to morph. Second, decoupling is analogous to the “branching bandits” literature where researchers have proven that it is often optimal to replace the uncertain outcome of an indexable decision process with its Gittins’ index (Bertsimas and Niño-Mora 1996; Tsitsiklis 1994). For example, Weber (1992) analyzes a “super process” in which an initial bandit process has rewards that are themselves bandit processes. To solve this dynamic program optimally, Weber replaces the outcome of each secondary bandit process with its Gittins’ index. He then uses those indices as rewards when computing the indices for the arms in the primary bandit process. Third, learning through Gittins’ indices happens many orders of magnitudes slower than decisions of when to morph. In §2.5 we presented evidence that, for website or banner morphing, the Gittins’ indices stabilized after 40,000-100,000 customers depending on the number of morphs and segments. On the other hand, decisions about when to morph depend upon clickstream observations on the order of fractions of a customer’s website visit.

At minimum, we expect the improved algorithm will provide substantial improvements relative to HULB. We examine the magnitude of such improvements in §5.

### 4.2.6. An illustrative example.

The most difficult conceptual challenge when using the Bell-
man equation is keeping track of what is known and unknown at each period as we choose the optimal path. This is illustrated best with an abstracted problem where the number of decision periods is fixed ($\psi_{nt} = 1$ for $t = 4$, $\psi_{nt} = 0$ for $t < 4$). The customer’s segment probabilities evolve in a known manner that can be anticipated dependent only on segment membership. (That is, we abstract away the computations that give the segment probabilities based on the anticipated clickstream.) The illustrative problem in Table 1 has four potential morphs and four customer segments. The first panel gives the morph-dependent purchase probabilities at $t = T_n$; the second panel gives the expected evolution in the segment probabilities, $q_{r,n}(\tilde{c}_{t-1,n}, \tilde{\Omega}_t, X_{t-1,n} | r = r_{true,n})$, for $r_{true,n} = 1$. In this illustration the segment probability evolutions for other true segments are symmetric (and not shown in Table 1); the priors remain the same. For this illustrative problem, it is relatively easy to represent the Bellman equation in a spreadsheet (available from the authors). Improvements vary from 0% to 50%, depending upon the chosen switching costs, impact weights, and other assumptions.

[Insert Table 1 about here.]

Prior to any observations, the priors and purchase probabilities slightly favor segment $r_n = 2$ and its corresponding best morph $m_{r,n} = 2$. Not surprisingly, before observing any clicks, the dynamic program solution begins with $m_{1,n} = 2$. For illustration, suppose that the customer’s true segment is $r_{true,n} = 1$, and suppose that all periods have equal impact on the purchase probabilities. We illustrate the effect of varying switching costs. For this case and with perfect knowledge (and no switching costs), the best morph is $m_{tn} = 1$.

Without perfect knowledge of $r_{true,n}$, the observations, $c_{1,n}$, update the customer $n$’s segment probabilities, $q_{r,n}(\tilde{c}_{1,n}, \tilde{\Omega}_1, X_{1,n})$. In our illustration the posterior probabilities begin to imply that it is more likely that the customer’s true segment is $r_n = 1$. When switching costs are low it is optimal to set $m_{2,n} = 1$. On the other hand, when switching costs are more substantial, it is optimal to wait to learn more about the customer $n$’s segment before switching. With greater switching costs the dynamic program continues with $m_{2,n} = 2$. We next observe $c_{2,n}$. The new observations update our beliefs about segment membership making $r_n = 1$ more probable. Morph 1 becomes more attractive for this customer so the dynamic program switches to $m_{3,n} = 1$ in the third period. However, when switching costs are substantially larger ($\gamma$ smaller) it becomes optimal to wait longer to change morphs. In fact, if switching costs are extremely large, it is never optimal to change morphs. For example, the optimal solution of the illustrative example implies the following optimal paths for the indicated switching costs:

- $\gamma = 0.95 \Rightarrow m_{1,n} = 2, m_{2,n} = 1, m_{3,n} = 1, and m_{4,n} = 1$
- $\gamma = 0.80 \Rightarrow m_{1,n} = 2, m_{2,n} = 2, m_{3,n} = 1, and m_{4,n} = 1$
Website Morphing 2.0

- \( \gamma = 0.60 \Rightarrow m_{1n} = 2, m_{2n} = 2, m_{3n} = 2, \text{and } m_{4n} = 1 \)
- \( \gamma = 0.40 \Rightarrow m_{1n} = 2, m_{2n} = 2, m_{3n} = 2, \text{and } m_{4n} = 2 \)

We can also examine sensitivity to the impact weights. For example, if later periods have larger impact weights, it is optimal to wait longer to change morphs. Changing \( \hat{w} \) to \((0.00, 0.25, 0.25, 0.50)\) from \((0.25, 0.25, 0.25, 0.25)\) makes it optimal to continue in morph 2 one period longer. If the last period dominates, i.e., \( \hat{w} = (0.00, 0.00, 0.05, 0.95) \), we approach the assumptions of HULB and it is optimal to wait until the dominant last period. Other sensitivity analyses suggest that the optimal solution behaves intuitively: (1) with more-rapid learning the dynamic program changes morphs earlier and (2) with increased rewards for continuation beyond period 4 the dynamic program changes morphs later. The optimal solutions quantify the when-to-morph decision and do so with face validity. Because the assumptions in §3 generalize HULB, the solutions in this illustration revert to those in HULB when \( \gamma = 1 \) and \( \hat{w} = (0,0,0,1) \).

4.3. Summary of the Algorithmic Improvements

HULB’s algorithm has proven successful on calibration-study-based simulations representing the BT Group website and for a field experiment morphing banners on CNET. However, the HULB algorithm assumes that only the last morph matters, that there are no switching costs, and that customers do not leave the website randomly. Our proposed algorithm generalizes HULB’s algorithm to address those issues. The algorithm is based on a series of behavioral assumptions that we feel are reasonable and capture the relevant phenomena. Based on these assumptions the improved algorithm can run in real-time between clicks on a website and identify when to morph while retaining the ability to allocate the best morph to each customer. The generalized algorithm requires “tuning” parameters for impact weights, switching costs, and exit probabilities. All tuning parameters can be estimated with a sufficiently large sample in the calibration study. Lacking a calibration study, the parameters can be set by managerial judgment. We now examine the generalized algorithm with synthetic data and, in §6, with a proof-of-feasibility application.

5. Synthetic Data Experiments

If customers are described accurately by the behavioral assumptions, the new algorithm will outperform HULB by the principle of optimality. We use synthetic data experiments to examine the amount of the improvement. We also examine robustness by comparing HULB’s algorithm to the generalized algorithm when the generalized algorithm uses incorrect values of the tuning parameters.

5.1. Re-analysis of HULB’s BT Group Simulations

HULB tested their algorithm with synthetic data chosen to mimic behavior on the BT Group website. In particular, they used the \( \hat{p}_{rm} \) estimated from the calibration data to create \( p_{rm,true} \)’s. If morph
was assigned to customer \( n \) after the first \( \tau \) clicks, then HULB drew a binomial outcome based on \( p_{rm, true} \) and updated accordingly. To simulate Bayesian segment identification, HULB drew customer click-alternative choices with Equation 1 using the \( \hat{\Omega} \) estimated from the calibration data (up to 20 pages and 16 click alternatives per page). The \( q_{rmn}(c_{\tau,n}, \hat{\Omega}, \hat{X}_{\tau,n}) \) where based on Equation 2.

We begin by re-examining HULB’s BT Group simulations. We examine a hypothetical world in which there are four observation periods of five clicks each. For this hypothetical world, data are generated with \( \gamma = 0.95 \) and \( \bar{w} = (0.25, 0.25, 0.25, 0.25) \). We compare the proposed when-to-morph algorithm with HULB’s algorithm, which implicitly assumes \( \gamma = 1 \) and \( \bar{w} = (0, 0, 0, 1) \). We obtain similar results (not shown) with a version of HULB’s algorithm that assumes \( \bar{w} = (0, 0, 0.50, 0.50) \). Our goal is to illustrate that there exist cases where the when-to-morph algorithm substantially improves profits. Because the calculated rewards include switching costs, the rewards in Table 2 are smaller than those in reported by HULB.

We select four customer segments and simulate 10,000 customers from each segment (a total of 40,000 synthetic customers per simulation). Visual inspection of the \( G_{rmn} \) plots indicates that 10,000 synthetic customers per segment are sufficient to observe performance for finite \( n \) and at (or near) convergence. (Relative interpretations do not change when all sixteen segments are simulated.) Although the variance over simulations is small, we simulate each algorithm ten times and take the average.

Table 2 summarizes the relative improvements. We report the net present value of the rewards using the discount factor from HULB, \( \alpha = 0.999999 \). We also report the rewards over the last 400 synthetic customers as an indication of the rewards after the \( G_{rmn} \)’s have converged to the \( p_{rmn} \)’s. The relative improvement in the net present value of rewards due to the when-to-morph algorithm is substantial (51.6% vs. 30.6%). The relative improvement is even larger at convergence (60.0% vs. 33.4%). Table 2 establishes that there exist situations where it is important to model switching costs and impact weights.

5.2. Comparisons for a Wider Range of Tuning Parameters

We now examine whether the when-to-morph algorithm improves outcomes for a wide range of switching costs and impact weights. We vary switching costs from \( \gamma = 0.80 \) to \( \gamma = 1 \). Because HULB report an improvement of approximately 20%, we would expect little, if any, advantage to morphing for \( \gamma < 0.80 \). We vary impact weights from equally valuing all observation periods \( (w_\tau = w \forall \tau) \) to favoring only the last observation period \( (w_\tau = 0 \forall \tau < T_n, w_{T_n} = 1) \). For the observation periods in our synthetic data we vary \( \bar{w} \) over the following values: \((.25, .25, .25, .25)\), \((.10, .15, .25, .50)\), \((.02, .05, .18, .75)\), and \((0, 0, 0, 1)\). We plot performance for \( n = 1 \) to 40,000 to illustrate both small-sample properties and properties near the convergence of the \( G_{rmn} \)’s. We do not test sensitivity to random exit \( (\psi) \) in our full-factorial
experiment because the tuning parameters for random exit are easy to infer from the calibration data (as in §6.6).

Table 3 and Figure 2 compare the net present value of projected revenues, and the values of revenue at convergence, for the when-to-morph algorithm as compared HULB’s algorithm. We report comparisons for all twenty \((5 \times 4)\) combinations of \(\gamma\) and \(\vec{w}\). Projected revenues are averaged over 10 replications—a total of 400 simulations and 16 million synthetic customers.

In Figure 2 the vertical axes are the projected revenue per customer where the revenue for a success is normalized to 1.0. The horizontal axes are synthetic customers reported every 400th customer. In all cases, both algorithms converge smoothly—for some values of the tuning parameters they converge around 20,000 synthetic customers. By 40,000 synthetic convergence is clear, although, for some values of the tuning parameters, profits are still increasing slowly at 40,000. It is unlikely we would gain further insight beyond 40,000 synthetic customers per run. When we examine the case where the when-to-morph algorithm matches HULB’s algorithm, \(\gamma = 1, \vec{w} = (0, 0, 0, 1)\) as shown in the lower right, the algorithms perform identically. In all other cases, the when-to-morph algorithm performs better than HULB’s algorithm—sometimes substantially better. Table 3 quantifies “better.” Improvements in the net present value vary from 19\% \([\gamma = 1, \vec{w} = (.02, .05, .18, .75)]\) to an almost four-fold increase \([\gamma = 0.80, \vec{w} = (0, 0, 0, 1)]\). Increases at convergence are less dramatic, but still substantial (ratios vary from 1.17 to 2.17). Although both algorithms are designed for large \(n\), the proposed algorithm improves profits relative to HULB’s algorithm almost from the beginning.

5.3. Robustness Tests

We now compare projected revenues when the when-to-morph algorithm and/or HULB’s algorithm assume incorrect values of the tuning parameters. Our synthetic data experiments are illustrative. A full-factorial synthetic-data experiment would require \((4 \times 5) \times (4 \times 5) = 400\) cases for a total of 2,000 simulations and 320 million synthetic respondents. Such a large number of simulations would be hard to display and provide little additional insight. Instead we select twelve interesting cases and examine whether the when-to-morph algorithm is reasonably robust to parameter misspecification. We provide our code so that other researchers might run synthetic data experiments for other true and/or assumed values of the tuning parameters. Our ten-replicate analyses are based on an additional 240 simulations and 9.6 million synthetic customers.

Figure 3 displays the results. The first five plots hold \(\vec{w}\) constant and match \(\vec{w}\) to HULB’s implicit assumptions, but vary \(\gamma\). The true \(\gamma = 0.95\). In all cases the when-to-morph algorithm outperforms HULB’s algorithm, even when \(\gamma\) is misspecified. The next four plots hold \(\gamma\) constant and match \(\gamma\) to
HULB’s implicit assumptions, but vary \( \tilde{\omega} \). The true \( \tilde{\omega} = (.10, .15, .25, .50) \). Again the when-to-morph algorithm outperforms HULB’s algorithm. Finally, the last three plots favor HULB’s algorithm. The true \( \gamma \) and the true \( \tilde{\omega} \) match HULB’s assumptions, but the \( \gamma \) and the \( \tilde{\omega} \) in the when-to-morph algorithm are misspecified. HULB’s algorithm does slight better, as anticipated, but not by much. The twelve plots are indicative. The when-to-morph algorithm is reasonably robust to misspecification and can often outperform HULB’s algorithm even if misspecified.

[Insert Figure 3 about here.]

Overall, the synthetic-data experiments suggest that the proposed when-to-morph algorithm has the potential to increase profits substantially. For many values of the tuning parameters, it is robust to misspecification. At minimum, the when-to-morph algorithm has the potential to improve applications. Which applications and for which values of the tuning parameters awaits empirical testing.

6. Proof-of-Feasibility Empirical Application of Website Morphing

The when-to-morph algorithm generalizes HULB’s algorithm, but does so by requiring that we solve a dynamic program between clicks (or observation periods) while customers are actively using a website. In this section, we examine whether it is feasible to apply the when-to-morph algorithm on a real website with respondents drawn to represent real customers. Our sample is small relative to typical applications, but is enough to establish feasibility. Fortunately, even with a small sample, we can examine whether differences in outcomes are trending in the right direction.

Because HULB and Urban, et al. provide extensive discussion and illustration of how to identify cognitive-style segments, we relegate segment definition to a companion online appendix. Instead we concentrate on the context, the implementation, the estimation of random exit tuning parameters, and an algorithmic comparison. We begin with the context.

6.1. Context: Suruga Bank’s Card-Loan Website

In Japan customers prefer “card loans” rather than carrying a balance on their credit cards. (Japanese banks do not allow overdrafts.) The borrower receives a cash card with a balance of ¥3-5 million and pays interest when the funds are withdrawn. The terms of card loans vary among banks and are often confusing. Some banks offer low interest and high limits, but a more-difficult screening process while other banks offer higher interest and lower limits, but an easier screening process. In 2006-2007 the leading card loan banks were Orix, which spent ¥13.6 billion mostly on banner advertising, and Acom, which spent ¥10.9 billion mostly on television advertising ($1 \approx ¥95 in the time period).

Suruga Bank is a Japanese commercial bank in the greater Tokyo area. Unlike most commercial banks, it focused on retail banking for more than twenty years. Suruga began a virtual bank in 1999, one of the first Japanese banks to do so. By 2008 its online presence had grown to ten virtual branches and
eight virtual alliances (Tokoro 2008, p. 7). Suruga is less well-known than other Japanese banks, spending approximately 1/10th that of Acom and Orix on advertising (¥1.4 million, Tokoro 2008, p. 17). As part of an overall strategy to reach more customers, Suruga developed a customer advocacy website on which it presented the best products from all competitors. By using a strategy of openness and honesty Suruga sought to demonstrate that its products (low interest rates, high limits, but careful screening process) would meet the needs of many customers.

Suruga’s managers found website morphing intriguing and authorized a small-scale field experiment. Because we were unable to sell card loans in the experiment, we evaluate the algorithms on Suruga’s goals of enhanced consideration, preference, and purchase likelihood.

6.2. Customer Segments and Estimated Click-Characteristics Preferences

We began with a calibration study to identify cognitive-style segments and estimate customers’ preferences for click-alternative characteristics. In March 2008, 5,454 customers were drawn from a panel of customers maintained by Interface Asia. Of these 3,340 were not interested in card loans or did not meet the age requirements. Thus, 2,114 respondents were offered ¥200 and invited to visit an experimental website and complete a survey. Of these, 502 respondents (23.7%) completed the calibration survey which included a requirement to browse the website for at least 2½ minutes and for at least 10 clicks.

Analysis of answers to questions, that were chosen to measure cognitive styles, identified four segments as defined by two bipolar (ipsative) multi-item scales. The 2 x 2 categorization was impulsive vs. deliberative and holistic vs. analytic. After using their responses to classify customers into segments, we estimated the click-characteristics preferences, \( \hat{\Omega} \), from customers’ observed clickstreams. Table 4 reports \( \hat{\Omega} \) for the 14 click-alternative characteristics that were tracked. The cognitive and cultural characteristic values were based on ratings by six independent judges who were blind to the hypotheses of the research (reliability = 0.84). The functional characteristics and website areas were binary variables. The logit model (Equation 1) was strongly significant (\( p < 0.001 \)) and explained 33.9% of the uncertainty (\( U^2 = 0.339 \)). Details available from the authors. There were many significant and intuitive values for the coefficients. For example, impulsive visitors prefer links to fast solutions, advisors, and forums, but not analytic tools or “content directly addressed to you.”

[Insert Table 4 about here.]

6.3. Website Design

Website designers (native Japanese) developed four morphs that varied on the number of graphs, the amount of technical content, the amount of textual content, the number of options and alternatives presented, the amount of content on popular trends, the amount of “you-directed” content, formal vs. informal Japanese language, and hierarchical vs. egalitarian images. Although the designers did their best to
develop morphs that would be more effective for specific segments, the best morph for each segment is determined automatically by the morphing algorithm. The website designers also developed click alternatives that would help identify segment membership. For example, on the opening page (Figure 4, first panel), customers enter the site by choosing one of two pictures. The pictures are designed to appeal to different customer segments. In another example, customers choose from six ways to obtain information (Figure 4, second panel) with the hope that different segments would choose different paths. To avoid an obvious demand artifact, the website was not identified as a Suruga Bank website.

6.4. Outcome Measures

Suruga Bank evaluated the experiment on three dependent measures: consideration, preference, and purchase likelihood. The implicit hierarchy in the measures, consideration $\rightarrow$ preference $\rightarrow$ purchase likelihood, represents Suruga Bank’s long-term views. If the customer-advocacy website encouraged customers to consider Suruga, then Suruga felt its service features (and other marketing actions) would sell sufficiently many card loans.

Consideration was a binary consider-or-not measure, preference was a 100-point constant sum scale, and purchase likelihood was an 11-point scale as in Juster (1966). Pre-measures were obtained only from those customers who considered a provider. All measures reflect that a customer will not prefer or purchase from Suruga Bank if they do not first consider Suruga Bank. Consistent with Suruga’s nomological hypothesis, preference is significantly correlated with purchase likelihood ($\rho = 0.20, p < 0.01$), but the correlation is not so high that they are the same construct.

6.5. Sample Used in the Suruga Proof-of-Feasibility Application

In November-December 2009 Suruga recruited customers from the Interface Asia panel. Screening and incentives were similar to those in the calibration study—the initial response rate was 22.1%. After screening on interest in card loans and age requirements, 10,182 out of 13,696 potential respondents were declared not eligible for the study. The remaining 3,514 were directed to the card-loan site, of which 1,997 explored the website for at least 2½ minutes and 10 clicks (56.9%). Of these customers, 1,395 completed pre- and post-visit questionnaires providing valid data with which to evaluate the websites (70.1%). This is a net completion rate of 39.7% and an overall completion/response rate of 8.8%. The market research provider, Applied Marketing Science, Inc., reports that such overall rates are typical of complex web-based studies. Of the 1,395 completions, 1,062 respondents experienced website morphing (improved algorithm, test condition) and 333 experienced a static website (control condition). Although there might be some unobserved self-selection toward interest in Internet banking, such bias would apply to both the test and control cells of the proof-of-feasibility experiment and should not bias relative differ-
ences. We chose to assign relatively more respondents to the test site to increase the likelihood that the Gittins’ indices would begin to converge to their long-term values. Nonetheless, we recognize that, even with 1,062 respondents, the generalized algorithm was likely still exploring morph assignments at the end of the proof-of-feasibility trial. (Recall that prior synthetic data experiments and empirical banner-morphing experience suggests that the algorithm will continue to explore to 10,000 or even 40,000 customers.)

6.6. Analysis of the Random Time to Exit (and other Tuning Parameters)

We can use the Suruga experience to examine which random-exit model would have fit the data best. For each respondent we observe the observation period in which the respondent left the website. These data are plotted in Figure 5 as a solid black line. The dashed red line plots the performance of a homogeneous model ($\psi_{tn} = \psi$ for all $t, n$, where $\psi$ is determined by maximum-likelihood estimation). The homogeneous model explains 79.6% of the uncertainty ($U^2 = 0.796$), but visual inspection suggests a kink for the first observation period. We therefore fit a hybrid homogeneous model ($\psi_{tn} = \psi_1$ if $t = 1$, but $\psi_{tn} = \psi$ otherwise) as shown by the green fine-dashed line in Figure 5a. The fit is significantly better ($\chi^2 = 58.0, p < 0.01$) and explains almost all of the uncertainty ($U^2 = 0.968$).

We next examine the heterogeneous model described in §3.3.1. Using the functional invariance property of maximum-likelihood estimation, we re-parameterized the model to estimate $E_n[\psi] = \alpha_\psi / (\alpha_\psi + \beta_\psi)$ and $Z_\psi \equiv (\alpha_\psi + \beta_\psi)$. The maximum-likelihood estimate of $E_n[\hat{\psi}]$ is 0.346, however, the maximum-likelihood estimate of $Z_\psi$ appears to diverge. The likelihood was still increasing at $Z_\psi > 50,000$. The resulting heterogeneous model is indistinguishable from a homogeneous model ($\chi^2 = 0, p > .90$). We also fit a hybrid heterogeneous model with $\hat{\psi}_1 = 0.3027, E_n[\hat{\psi}] = 0.425$, and $Z_\psi > 50,000$ for $t > 1$. The hybrid heterogeneous model was significantly different than the heterogeneous model ($\chi^2 = 58.0, p < 0.01$) and indistinguishable from the hybrid homogeneous model ($\chi^2 = 0$). The heterogeneous models are plotted in Figure 5b.

In the Suruga data, a hybrid homogeneous model of random exit appears to fit the data best. A fully time-varying model ($\psi_{tn} = \psi_t$) is significantly better than a hybrid homogeneous model ($\chi^2 = 11.0, p = 0.03$), but the slight increase in $U^2$ on a sample of 1,395 respondents may not be worth the sacrifice in parsimony. We estimated these models a posteriori to demonstrate that future applications could readily fit descriptive models using only the calibration data. The other tuning parameters were set conservatively by managerial judgment ($\hat{p} = 0.99, \hat{\omega}_t = w, \forall t$). In future applications they could be set using calibration data as discussed in §3.4.
6.7. Descriptive Statistics for the Time to a Morph Change

If initial morphs were assigned randomly, then we would expect that, for roughly 25% of the respondents, the optimal morph would be the initial morph. However, the morphing algorithm (both HULB’s version and the generalized version), assigns the initial morph based on the Expected Gittins’ Indices and the prior probabilities that customer n belongs to each segment. If the algorithm works well, we expect the initial morphs to be the optimal morphs for more than 25% of the respondents, especially when one segment is larger than the others. (For Suruga, the largest segment was approximately 43.2% of the respondents.) Furthermore, to avoid switching costs, the algorithm may decide not to change morphs for a small change in the value of the Expected Gittins’ Index. Empirically, the algorithm stayed with the initial morph roughly half of the time. Of those respondents who experienced morph changes, most experienced a single change; very few experienced three or more changes (Figure 6). Figure 6 also suggests that, empirically, the generalized algorithm was able to identify the morph changes rapidly. The majority of morph changes were made after the first period with decreasing numbers in subsequent periods.

6.9. Results of the Comparison Between the Algorithm and a Static Website

Table 5 summarizes the outcome measures at the end of the proof-of-feasibility application. All measures increased on the morphing website (test) relative to the static website (control). We can also correct for pre-measures, but, because Suruga Bank was not known for card loans at the time of the application, only 70 respondents considered Suruga Bank in the pre-measures. Although no individual differences are significant, all double differences, test (post – pre) vs. control (post – pre), favor the morphing website. Using a sign test, the probability that all six measures (post only and pre-vs.-post) improve by chance is $p = 0.02$. Because we rely on a sign test, we are cautious in any statistical conclusions.

Examining Orix and Acom provides further confidence that the outcome measures were trending in the right direction. While the sponsor of the website was blinded to avoid demand artifacts, we expected that the website would have a smaller impact on Orix and Acom than on Suruga. Competitive information favors Suruga and it is harder to change consideration, preference, and purchase likelihood for banks that are already well-known in the market. Consistent with these hypotheses, the corresponding sign tests for Orix and Acom were not significant ($p = 0.66$ and $p = 0.11$, respectively).

The Suruga application was successful in demonstrating that the generalized algorithm can be applied on a realistic website with respondents drawn from real bank customers. We are encouraged by the directional results, but recognize the need for an in vivo test of website morphing with sample sizes comparable to those in Urban, et al. Such a test might compare the generalized algorithm directly to HULB’s
algorithm and both to a control.

7. Summary and Challenges

This paper advances website morphing in four ways: a more-realistic customer-behavior model, a feasible generalized algorithm, synthetic-data experiments to examine its performance, and the first proof-of-feasibility test-vs.-control application of website morphing. Results are promising. Theoretically, we were able to develop models of switching costs, multiple-morph impact, and random exit that are reasonable and conjugate to the endogenous optimization of when to morph. By thinking carefully about when information becomes available, we were able to develop an algorithm that captures path evolution in our knowledge of segment probabilities. The algorithm runs in real time between clicks. Realistic synthetic data suggest reasonable scenarios where the generalized algorithm leads to a substantial increase in revenue relative to HULB’s algorithm. Synthetic-data experiments suggest that the algorithm matches HULB’s algorithm when the tuning parameters match HULB’s implicit assumptions. The proposed algorithm improves outcomes in all other cases. Further, the improved algorithm is reasonably robust with respect to misspecification of its tuning parameters. Finally, the proof-of-feasibility application to a Suruga Bank website (1) demonstrates that the generalized algorithm is feasible in practice, (2) demonstrates how to estimate the random-exit parameters, and (3) suggests that outcome measures improve relative to a static website.

There remain many challenges. (1) Empirical applications are necessary to demonstration that the calibration data can be used to set the values of the switching-cost and impact-weight tuning parameters. (2) Empirical applications might parse the incremental value of the generalized algorithm relative to HULB’s special case. (3) Larger empirical sample sizes might provide more definitive tests. (4) Researchers might investigate whether there exist feasible algorithms that relax assumptions of multiplicative switching costs, additive impact weights, and independent website exit. (5) Researchers might establish optimality properties for the linkage between Expected Gittins’ Index assignments of morphs (learning between customers) and dynamic programming assignment of when to morph (within a customer). (6) Researchers might investigate further “fractional observation” updating of the posterior distributions of the $p_{rmn}$’s. (7) Current algorithms do not consider network externalities such as the ability of customers to share knowledge of a website’s look and feel.
References


Table 1
Illustrative Morph Assignment Problem for the When-to-Morph Dynamic Program

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<table>
<thead>
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<tr>
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<td>$c_{1_n}, c_{2_n}, c_{3_n} \to q_{r_n}(\tilde{c}<em>{3_n} \mid r</em>{true,n} = 1)$</td>
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<td>0.03</td>
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</table>

Table 2
Reanalysis of BT Group Website Morphing (with Switching Costs and Impact Weights)

<table>
<thead>
<tr>
<th></th>
<th>Reward at Convergence</th>
<th>Percent Improvement in Reward</th>
<th>Net Present Value (NPV)</th>
<th>Percent Improvement in NPV</th>
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</thead>
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<td>Baseline for no morphing</td>
<td>0.3171</td>
<td>0.0%</td>
<td>0.3108</td>
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<tr>
<td>Website morphing</td>
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<tr>
<td>Website morphing (HULB)</td>
<td>0.3392</td>
<td>33.4%</td>
<td>0.3306</td>
<td>30.6%</td>
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<tr>
<td>Website morphing with improved algorithm</td>
<td>0.3567</td>
<td>60.0%</td>
<td>0.3442</td>
<td>51.6%</td>
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<tr>
<td>Perfect information ($n \to \infty$)</td>
<td>0.3831</td>
<td>100.0%</td>
<td>0.3754</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

\(a\) For these comparisons, switching costs are $\gamma = 0.95$. Impact weights are $\bar{w} = (0.25, 0.25, 0.25, 0.25)$.

\(b\) Upper bound. Applications do not have perfect information on either customer segments or purchase probabilities.

\(c\) Rewards for the last 400 customers out of 40,000 customers.

\(d\) (reward due to algorithm – reward at no morphing) / (reward due to perfect information – reward at no morphing).

\(e\) Net present value of the rewards divided by the number of periods.

\(f\) (NPV due to algorithm – NPV at no morphing) / (NPV due to perfect information – NPV at no morphing).
Table 3
Proposed When-to-Morph Algorithm (WTM) vs. HULB’s Algorithm as Switching Costs (\(\gamma\)) and Impact Weights (\(\bar{w}\)) Vary

| \(\bar{w}\)'s | \(\gamma = .80\) | \(\gamma = .85\) | \(\gamma = .90\) | \(\gamma = .95\) | \(\gamma = 1\) | \(\gamma = .80\) | \(\gamma = .85\) | \(\gamma = .90\) | \(\gamma = .95\) | \(\gamma = 1\) | \(\gamma = .80\) | \(\gamma = .85\) | \(\gamma = .90\) | \(\gamma = .95\) | \(\gamma = 1\) |
|-------------|----------------|------------|------------|------------|------------|----------------|------------|------------|------------|------------|----------------|------------|------------|------------|------------|------------|
| (.25, .25, .25, .25) | 2.08 | 1.90 | 1.78 | 1.69 | 1.36 | 38% | 43% | 49% | 52% | 51% | 18% | 23% | 27% | 31% | 38% |
| (.10, .15, .25, .50) | 3.03 | 2.67 | 2.02 | 1.66 | 1.28 | 44% | 51% | 56% | 62% | 63% | 14% | 19% | 28% | 37% | 50% |
| (.02, .05, .18, .75) | 3.31 | 2.65 | 2.14 | 1.51 | 1.19 | 51% | 60% | 64% | 66% | 75% | 15% | 23% | 30% | 44% | 63% |
| (0, 0, 0, 1) | 3.89 | 2.48 | 1.82 | 1.36 | 1.00 | 62% | 64% | 68% | 73% | 77% | 16% | 26% | 37% | 54% | 77% |

| \(\bar{w}\)'s | \(\gamma = .80\) | \(\gamma = .85\) | \(\gamma = .90\) | \(\gamma = .95\) | \(\gamma = 1\) | \(\gamma = .80\) | \(\gamma = .85\) | \(\gamma = .90\) | \(\gamma = .95\) | \(\gamma = 1\) | \(\gamma = .80\) | \(\gamma = .85\) | \(\gamma = .90\) | \(\gamma = .95\) | \(\gamma = 1\) |
|-------------|----------------|------------|------------|------------|------|----------------|------------|------------|------------|------|----------------|------------|------------|------------|------|------|
| (.25, .25, .25, .25) | 1.74 | 1.77 | 1.78 | 1.79 | 1.49 | 41% | 47% | 54% | 60% | 62% | 23% | 27% | 30% | 33% | 41% |
| (.10, .15, .25, .50) | 1.85 | 1.97 | 1.78 | 1.56 | 1.30 | 46% | 56% | 62% | 69% | 74% | 25% | 29% | 35% | 44% | 57% |
| (.02, .05, .18, .75) | 1.93 | 1.94 | 1.80 | 1.45 | 1.17 | 55% | 66% | 71% | 73% | 84% | 29% | 34% | 39% | 52% | 72% |
| (0, 0, 0, 1) | 2.17 | 1.82 | 1.61 | 1.28 | 1.00 | 68% | 71% | 77% | 82% | 87% | 31% | 39% | 48% | 64% | 87% |
### Table 4
Estimation of Click-Characteristic Preferences ($\hat{\theta}$) \(^a\)

<table>
<thead>
<tr>
<th>Cognitive and cultural characteristics</th>
<th>Holistic versus Analytic</th>
<th>Impulsive versus Deliberative</th>
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<td>Pictures and graphs</td>
<td>0.88</td>
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</tr>
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<td>Technical, detailed content</td>
<td>-0.66</td>
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</tr>
<tr>
<td>Textual content</td>
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<td>-1.28</td>
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<tr>
<td>Options and alternatives</td>
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<tr>
<td>Popular trends</td>
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<td>'You-directed' language</td>
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<tr>
<td>Hierarchical images</td>
<td>0.24</td>
<td>-0.51</td>
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<th>Functional characteristics</th>
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<tr>
<td>Provide information</td>
<td>-0.53</td>
<td>1.56</td>
</tr>
<tr>
<td>Analytic tool</td>
<td>-0.60</td>
<td>-1.44</td>
</tr>
<tr>
<td>Graphical elements</td>
<td>-0.89</td>
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<table>
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<th></th>
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<tr>
<td>Advisor</td>
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<tr>
<td>Fast solutions</td>
<td>-0.70</td>
<td>4.85</td>
</tr>
<tr>
<td>Learn information</td>
<td>0.11</td>
<td>1.89</td>
</tr>
<tr>
<td>Forum</td>
<td>0.37</td>
<td>2.48</td>
</tr>
</tbody>
</table>

\(^a\) Based on a maximum-likelihood logit analysis of Equation 1. $\chi^2_{42} = 2,505.5$ and $U^2 = 33.9\%$. Constants not shown.

### Table 5
Results of the Suruga Bank Field Experiment

<table>
<thead>
<tr>
<th></th>
<th>Morphing Website</th>
<th>Static Website</th>
<th>Morphing minus Static Website</th>
<th>Correcting for Pre-measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consideration</td>
<td>29.8%</td>
<td>27.0%</td>
<td>2.8%^a</td>
<td>4.5%^a</td>
</tr>
<tr>
<td>Preference</td>
<td>19.3%</td>
<td>16.2%</td>
<td>3.1%^a</td>
<td>3.2%^a,^b</td>
</tr>
<tr>
<td>Purchase Likelihood</td>
<td>24.3%</td>
<td>23.1%</td>
<td>1.2%^a</td>
<td>1.7%^a,^b</td>
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<tr>
<td>Sample Size</td>
<td>1,062</td>
<td>333</td>
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</table>

\(^a\) Individual differences not significant at the 0.05 level. Sign test of all six differences significant at the 0.02 level.

\(^b\) We interpret these double-difference measures cautiously. Pre-measures for preference and purchase likelihood were obtained for only those 70 customers who considered Suruga Bank prior to visiting the websites.
When-to-Morph Decision Problem (Example Customer, 4-Period Illustration)

(choose morph \((m_n)\) at each decision period to maximize sales, \(\delta_n\). The \(q_{tn}\) are also a function of \(\hat{\Omega}\) and \(X_{tn}\). Some arguments are suppressed in this figure to avoid complexity.)

\[ t_n = 1 \]
- Leave after period with probability \(\psi_{1n}\).
- Observe clicks \(c_{1n}\) \(\rightarrow\) \(q_{m_n}(c_{1n})'s\).
- Reduce purchase probability by factor of \(\gamma\) if \(m_{2n} \neq m_{1n}\).
- Impact weight, \(w_1\).

\[ t_n = 2 \]
- Leave after period with probability \(\psi_{2n}\).
- Observe clicks \(c_{2n}\) \(\rightarrow\) \(q_{m_n}(c_{1n}c_{2n})'s\).
- Reduce purchase probability by factor of \(\gamma\) if \(m_{3n} \neq m_{2n}\).
- Impact weight, \(w_2\).

\[ t_n = 3 \]
- Leave after period with probability \(\psi_{3n}\).
- Observe clicks \(c_{3n}\) \(\rightarrow\) \(q_{m_n}(c_{1n}c_{2n}c_{3n})'s\).
- Reduce purchase probability by factor of \(\gamma\) if \(m_{4n} \neq m_{3n}\).
- Impact weight, \(w_3\).

\[ t_n = 4 \]
- Leave after period with probability \(\psi_{4n}\).
- Observe clicks \(c_{4n}\) \(\rightarrow\) \(q_{m_n}(c_{1n}c_{2n}c_{3n}c_{4n})'s\).
- Reduce purchase probability by factor of \(\gamma\) if \(m_{4n} \neq m_{3n}\).
- Impact weight, \(w_4\).

Purchase decision, \(\delta_n\), if leave after \(t_n = 4\).
Figure 2
Proposed Algorithm (WTM) vs. HULB as Switching Costs, $\gamma$, and Impact Weights, $\bar{w}$, Vary
Figure 3
Representative Sensitivity Analyses
(Results when true values of switching costs, $\gamma$, and impact weights, $\bar{w}$, do not match assumptions in when-to-morph [WTM] algorithm.)
Figure 4
Example Screens from the Suruga Bank Website

Figure 5
Model Comparison – Random Exit ($\psi_{tn}$)

(a) Homogeneous models
(b) Heterogeneous models

Figure 6
Suruga Bank Experiment: Descriptive Statistics
Appendix 1: Notation Used in this Paper and Appendix 2

\( a \) the amount by which we discount a sale to customer \( n+1 \) relative to customer \( n \)
\( B(\alpha, \beta) \) beta distribution with parameters \( \alpha \) and \( \beta \)
\( c_{tn} \) indicator variable to indicate whether customer \( n \) chooses click alternative \( j \) at \( t \)
\( c_{tn} \) customer \( n \)'s clicks in the \( t^{th} \) observation period; \( n \) may be suppressed
\( \tilde{c}_{tn} \) customer \( n \)'s clicks up to and including the \( t^{th} \) observation period, \( \{c_{1n}, c_{2n}, ..., c_{tn}\} \)
\( EG_{mn} \) expected Gittins' index for \( m^{th} \) morph for customer \( n \)
\( EIR \) expected immediate reward (used in the when-to-morph Bellman equation)
\( f(\cdot|\cdot) \) posterior distribution. Specific distribution when arguments are specific. Sometimes subscripted by \( n \).
\( G_{rmn} \) Gittins' index for \( r^{th} \) segment and \( m^{th} \) morph for customer \( n \); \( n \) may be suppressed
\( HULB \) Hauser, Urban, Liberali and Braun (2009).
\( j \) indexes click alternatives (links)
\( J_{tn} \) the number of click alternatives faced by customer \( n \) on the \( t^{th} \) click
\( m \) indexes morphs
\( m_{tn} \) morph that customer \( n \) sees in observation period \( t \)
\( m_{tn}^* \) optimal morph for customer \( n \) in observation period \( t \)
\( M \) number of morphs available from which to choose
\( n \) indexes customers.
\( n_{ob} \) number of customers in an observation period; used in Appendix 2
\( n_{ob,rm} \) number of consumers in an observation period who were in segment \( r \) and saw morph \( m \)
\( p_{rmn} \) probability that customer \( n \) in segment, \( r \), purchases when shown morph \( m \)
\( Pr_{0}(r_{n} = r) \) prior probability that customer \( n \) belongs to segment \( r \)
\( q_{rn}(\tilde{c}_{t-1,n}, \tilde{X}_{t-1,n}) \) probability that customer \( n \) is in cognitive-style segment \( r \)
\( q_{rn} \) shorthand for \( q_{rn}(\tilde{c}_{t-1,n}, \tilde{X}_{t-1,n}) \); used in Appendix 2.
\( q_{rns}(s) \) shorthand for \( q_{rn}(\tilde{c}_{t-1,n}, \tilde{X}_{t-1,n}|r_{true} = s) \); used in Appendix 2.
\( r \) indexes cognitive-style segments
\( r_{true,n} \) customer \( n \)'s true segment (used derive the when-to-morph dynamic program)
\( S \) used in summations
\( S_{s} \) set of customers in the last \( n_{ob} \) customers whose true segment is \( s \). Used in Appendix 2.
\( t \) indexes observation periods
\( T_{n} \) number of observation periods for customer \( n \)
\( \tilde{u}_{tn,j} \) utility customer \( n \) obtains from clicking on the \( j^{th} \) click alternative on the \( t^{th} \) click
\( V_{t}(m_{tn}^*, m_{t-1,n}, \tilde{c}_{t-1,n}, \tilde{X}_{t-1,n}, N_{t-1,n}) \) continuation value function for when-to-morph decision
\( V_{Gittins}(\alpha_{rmn}, \beta_{rmn}, \alpha_{\psi}) \) continuation value function when calculating Gittins' index
\( w_{t} \) impact weight for the \( t^{th} \) observation period
\( \tilde{w} \) vector of the \( w_{t} \)'s. (\( \tilde{w} \) if estimated from the calibration data.)
\( \text{WTM} \) when-to-morph algorithm; proposed generalization of HULB’s algorithm
\( \tilde{x}_{tn,j} \) vector of characteristics of the \( j^{th} \) click alternative for the \( t^{th} \) click of customer \( n \)
\( \tilde{X}_{tn} \) set of all \( \tilde{x}_{tn,j} \)'s for all clicks up to an including click \( t \), \( \{\tilde{x}_{1n}, \tilde{x}_{2n}, ..., \tilde{x}_{tn}\} \forall j = 1 to J_{tn} \)
\( Z_{\psi} \) equal to \( \alpha_{\psi} + \beta_{\psi} \).
\( \tilde{Z}_{\psi} \) maximum-likelihood estimate of \( Z_{\psi} \).

\( \alpha_{rmn} \) parameter of the beta distribution used to model uncertainty over \( p_{rmn} \)
\( \beta_{rmn} \) parameter of the beta distribution used to model uncertainty over \( p_{rmn} \)
\( \alpha_{\psi} \) parameter of a beta distribution used to model heterogeneity in exit probabilities
Website Morphing 2.0

\[ \beta_\psi \] parameter of a beta distribution used to model heterogeneity in exit probabilities

\[ \gamma \] discount for switching from one morph to another. After a switch, the probability of a purchase is reduced by a factor of \( \gamma \). (\( \hat{\gamma} \) if estimated from the calibration data.)

\[ \delta_n \] indicator variable to indicate whether customer \( n \) makes a purchase; used in the when-to-morph algorithm when multiple morphs (might) affect outcomes

\[ \delta_{rmn} \] indicator variable to indicate whether customer \( n \) makes a purchase given that customer \( n \) saw morph \( m \) after a switch; used in HULB

\[ \delta_{rmn} \] indicator variable to indicate whether customer \( n \) makes a purchase given that customer \( n \) is in known segment \( r \) and saw morph \( m \); used in Appendix 2.

\[ \Delta_{m_{tn}} \] indicator variable to indicate if we change to morph \( m_{tn} \) in period \( t \) for customer \( n \)

\[ E_n[\Psi] \] maximum likelihood estimate of \( \alpha_\psi/(\alpha_\psi + \beta_\psi) \)

\[ \hat{\epsilon}_{nt} \] extreme value error; used to model customer \( n \)’s preference for click alternatives

\[ \eta_{mnt} \] indicator variable to indicate if customer \( n \) saw morph \( m \) in observation period \( t \)

\( \Sigma \) used to indicate summation, not a variable

\( \tau \) used as a variable for summation and for ranges of \( t \)

\( \tau_0 \) in HULB, the number of clicks observed prior to a morph

\[ \psi_{tn} \] probability that customer \( n \) does not continue in the \( t^{th} \) observation given the customer was on the website in the \( t + 1^{st} \) observation period

\[ \Psi_n(S|t-1) \] probability customer \( n \) is still on the website in observation period \( S \) given the customer was on the website in observation \( t - 1 \)

\[ \overline{\Psi}(S|t-1) \] expectation of \( \Psi_n(S|t-1) \) over customers

\[ \bar{\omega}_r \] vector of preference weights for the \( \bar{x}_{tn} \) for the \( r^{th} \) cognitive-style segment

\[ \Omega \] matrix of the \( \bar{\omega}_r \)

\[ \hat{\Omega} \] estimate of \( \Omega \) (either maximum likelihood or Bayesian mean posterior)

\[ \zeta \] used as a variable for summation in Appendix 2 to sum over customers beyond \( n \)

Appendix 2. Full Probability Model and Updating Formulae

A2.1. Review of Updating for Gittins’ Indices

Notation follows the text. If we knew the customer’s segment, \( r \), and the customer saw the same morph, \( m \), for the entire visit, then the relevant posterior probability after customer \( n \) would be \( p_{rmn} \), the probability that customer \( n \) in segment \( r \) who saw morph \( m \) would make a purchase. We assume that the prior distribution is beta as given by:

(A2.1) \[ f_n(p_{rmn}|\alpha_{rmn}, \beta_{rmn}) = B(\alpha_{rmn}, \beta_{rmn})p_{rmn}^{\alpha_{rmn}-1}(1-p_{rmn})^{\beta_{rmn}-1} \]

Where \( \alpha_{rmn} \) and \( \beta_{rmn} \) are parameters of the beta distribution and \( B(\alpha_{rmn}, \beta_{rmn}) \) is the beta function.

We now observe whether or not customer \( n \) makes a purchase. Let \( \delta_{rmn} = 1 \) if customer \( n \) in known segment \( r \) made a purchase and \( \delta_{rmn} = 0 \) otherwise. The data likelihood is then given by a binomial distribution:

(A2.2) \[ f_n(\delta_{rmn}|p_{rmn}) = p_{rmn}^{\delta_{rmn}}(1-p_{rmn})^{1-\delta_{rmn}} \]

Combining Equations A2.1 and A2.2, the posterior distribution for \( p_{rm,n+1} \) is proportional to:
Equation A2.3 is again a beta distribution. Thus we have the updating equation as given by:

\[
\begin{align*}
\alpha_{rm,n+1} &= \alpha_{rmn} + \delta_{rmn} \\
\beta_{rm,n+1} &= \beta_{rmn} + (1 - \delta_{rmn})
\end{align*}
\]  

We easily extend the formal updating to the case where we observe \( n_{ob} \) additional customers before we update (read \( n_{ob} \) as "n observed"). In this case we have a binomial data likelihood and we allow a more general definition of \( \delta_{rmn_{ob}} \) as the number of purchases made by the \( n_{ob} \) customers that follow the \( n^{th} \) customer. Following similar steps, the posterior distribution is also a beta distribution with parameters given by:

\[
\begin{align*}
\alpha_{rm,n+n_{ob}} &= \alpha_{rmn} + \delta_{rmn_{ob}} \\
\beta_{rm,n+n_{ob}} &= \beta_{rmn} + (n_{ob} - \delta_{rmn_{ob}})
\end{align*}
\]

Because updating for the Gittins’ index is naturally conjugate, computations are simple and rapid. It is feasible to update after every customer or observation period. Furthermore, by comparing Equations A2.4 and A2.5 it is obvious that we obtain the same posterior distribution at the \( (n + n_{ob})^{th} \) customer whether we update sequentially using Equation A2.5 or update all \( n_{ob} \) customers at one time.

**A2.2. Review of Updating for HULB’s Hidden Markov Model**

HULB extend Gittins’ updating to a situation where we do not know the customer’s segment, but rather have estimates, \( q_{rmn_n}(\bar{c}_{rmn}, \bar{\Omega}, \bar{X}_{rmn}) \), that customer \( n \) belongs to segment \( r \) for each possible segment. HULB continue to assume that the customer saw a single morph, \( m \), for for a sufficient fraction of the visit that we need only consider the effect of morph \( m \) on customer \( n \)’s purchase. With these assumptions, the probability, \( p_{mn} \), that customer \( n \) who saw morph \( m \) makes a purchase is given by:

\[
p_{mn} = \sum_{r=1}^{R} q_{rmn_n}(\bar{c}_{rmn}, \bar{\Omega}, \bar{X}_{rmn}) p_{rmn}
\]

HULB assume a beta prior distribution as in Equation A2.1. For indexability, HULB assume that each \( r, m \)-arm of the bandit process is independent. Independent arms imply that the prior distributions are independent over \( r \) and \( m \). The joint prior distribution generalizes Equation A2.1 to become Equation A2.7 prior to observing \( \delta_{mn} \). For simplicity of notation, we let \( \mathbf{a}_{mn} \) and \( \mathbf{b}_{mn} \) be the vectors parameters (over \( r \)).
The data likelihood is binomial and generalizes Equation A2.2.

\[ f_n(\delta_{rmn} | prmn) = p_{mn}^\delta_{mn} (1 - p_{mn})^{1 - \delta_{mn}} = \left( \sum_{r=1}^{R} q_{rnT_n}(\vec{c}_{T_n}, \vec{\Omega}, \vec{X}_{T_n})p_{rmn} \right)^{\delta_{mn}} \]

(A2.8)

\[
\cdot \left( 1 - \sum_{r=1}^{R} q_{rnT_n}(\vec{c}_{T_n}, \vec{\Omega}, \vec{X}_{T_n})p_{rmn} \right)^{1 - \delta_{mn}}
\]

We extend the analysis to allow updating after \( n_{ob} \) customers. Unlike in the single-armed case, the analysis does not simplify. Specifically, each customer has a different set of \( q_{rnT_n}(\vec{c}_{T_n}, \vec{\Omega}, \vec{X}_{T_n})'s \) based on that customer's unique clickstream. Thus, the data likelihood is now given by Equation A2.9 where \( \zeta \) is notation for use in the summation so that it does not conflict with \( n \) or \( n_{ob} \).

\[ f_{n+n_{ob}}(\delta_{rmn|ob} | pr_{rm,n+ob}) = \prod_{\zeta = n+1}^{n+n_{ob}} \left\{ \left( \sum_{r=1}^{R} q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta})p_{rm,n+ob} \right)^{\delta_{m\zeta}} \right\} \cdot \left( 1 - \sum_{r=1}^{R} q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta})p_{rm,n+ob} \right)^{1 - \delta_{m\zeta}} \]

Equation A2.9 uses the substitution in Equation A2.6 for \( p_{rm,n+ob} \) for all \( \zeta \) in the summation. This implies that the posterior distribution after observing \( n_{ob} \) additional customers is given by Equation A2.10 (below).

\[ f_{n+n_{ob}}(p_{rm,n+ob} | \vec{d}_{mn}, \vec{\beta}_{mn}, \delta_{rmn}) = \prod_{\eta = n+1}^{n+n_{ob}} \left\{ \left( \sum_{r=1}^{R} q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta})p_{rm,n+ob} \right)^{\delta_{m\zeta}} \right\} \cdot \left( 1 - \sum_{r=1}^{R} q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta})p_{rm,n+ob} \right)^{1 - \delta_{m\zeta}} \]

\[ - \sum_{r=1}^{R} q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta})p_{rm,n+ob} \]}

A2.2.1. The Infeasibility of Using this Posterior Distribution for Website Morphing

If we knew every customer’s segment, the \( q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta}) \) would be either 0 or 1 and Equation A2.10 would become naturally conjugate. When the \( q_{r\zeta T_\zeta}(\vec{c}_{T_\zeta}, \vec{\Omega}, \vec{X}_{T_\zeta}) \) are 0 or 1 we obtain the same updating formulae as in Equation A2.5.
When the segments are not known with certainty, the \( q_{r\xi T'(\xi')} \) are not 0 or 1 and we have the full summation in Equation A2.10. We no longer have naturally conjugate updating, hence \( f_{n+n_{ob}}(\delta_{mn}, \beta_{mn}, \delta_{rmn_{ob}}) \) is not a beta distribution, even when \( n_{ob} = 1 \). To make things worse, the \( p_{rm',n+n_{ob}} \) do not factor out for \( m' \neq m \). If we were only interested in the posterior distribution of \( p_{rm,n+n_{ob}} \) it might be feasible to discretize \( p_{rm,n+n_{ob}} \) and use Equation A2.10 to compute numerically the marginal posterior distribution for \( p_{rm,n+n_{ob}} \) after integrating out the \( p_{rm',n+n_{ob}} 's \) for \( m' \neq m \). Computations will not be rapid between customers on a high traffic website, but it might be feasible to compute the posterior distributions at the end of each day and use the updated distributions for the following day.

However, even if we chose to compute the posterior distribution offline, we must use the posterior distributions to compute the indices (via a dynamic program) in order to assign morphs. HULB exploit the property that the Gittins’ index for beta-binomial updating is particularly simple and can be easily tabled for \( \alpha_{rmn} \) and \( \beta_{rmn} \). They use table lookup for the indices based on updated \( \alpha_{rmn} \) and \( \beta_{rmn} \). It is not feasible to re-solve Gittins’ dynamic program after each customer. Morphing when customer segments follow a hidden Markov model is indexable, as proven by Krishnamurthy and Michova (1999), and indexability does not depend on the fact that the posterior is an analytic distribution. Unfortunately, we know of no way to compute the index sufficiently rapidly when the posterior distribution is defined numerically.

### A2.2.2. Practical Solution: Fractional Observations

To obtain a feasible solution, HULB make an heroic independence assumption and use the concept of fractional observations. We first note that the binomial distribution, a probability mass function, can also be interpreted as a probability density function for fractional observations. This distribution is sometimes known as the Pólya distribution, although definitions vary. We then interpret the outcome variable, \( \delta_{mn} \), as a combination of fractional outcomes for each potential customer segment, \( r \), and treat them as if they were independent over \( r \). In particular, the fractional outcomes becomes \( \delta_{rmn} = q_{r\xi T(n)}(\delta_{T(n)}, \delta_{T(n)}/n)n \) \( \forall r \). Using this interpretation and the shorthand notation \( q_{r\xi n} = q_{r\xi T(n)}(\delta_{T(n)}, \delta_{T(n)}) \), the data likelihood becomes:

\[
(A2.11) \quad f_n(q_{r\xi n} | \delta_{mn}) = p_{r\xi n}^{q_{r\xi n} \delta_{mn}}(1 - p_{r\xi n})^{q_{r\xi n}(1 - \delta_{mn})}
\]

We continue to use Equation A2.1 as the prior distribution which gives Equation A2.12 as the posterior distribution.

---

1 We examine in §2.4 why the updating formulae are still likely to converge and we present evidence of convergence from the synthetic data experiments.
(A2.12) \[ f_{n+1}(p_{rm,n+1} | \alpha_{rmn}, \beta_{rmn}, q_{rn}\delta_{rmn}) \sim p_{rm,n+1}^{\alpha_{rmn}+q_{rn}\delta_{m,n}-1} \cdot (1 - p_{rm,n+1})^{\beta_{rmn}+q_{rn}(1-\delta_{mn})-1} \]

With fractional observations, the posterior distribution is naturally conjugate and the updating equations become (HULB Equation 2, p. 210), using full notation:

\[
\begin{align*}
\alpha_{rm,n+1} &= \alpha_{rmn} + q_{rnT_n}(\tilde{c}_{T_n}, \tilde{y}_{T_n})\delta_{mn} \\
\beta_{rm,n+1} &= \beta_{rmn} + q_{rnT_n}(\tilde{c}_{T_n}, \tilde{y}_{T_n})(1 - \delta_{mn})
\end{align*}
\]

### A2.3. Extending HULB’s Updating Formula to Switching Costs, Impact Weights and Multiple Morphs

Once we accept the concept of fractional observations as a practical means to retain the structure of expected Gittins’ index updating, we can extend Equation A2.13 to account for switching costs, impact weights, and multiple morphs. No new derivations need to be introduced. We simply substitute the new fractional observations into Equations A2.12 through A2.15. The new concepts are that the switching costs, impact weights, and multiple morphs, may affect customer n’s purchase probabilities.

First, customer n does not see morph m for the entire website visit; customer n sees morph m for those periods in which the morphing algorithm selects morph m. The effect on purchasing is proportional to the impact weights for the periods in which morph m was shown. As defined in the text, \( \eta_{mnt} \) if morph m is shown to customer n in observation period t and \( w_t \) is the impact weight for observation period t. The probability is also affected by whether or not a switch in morphs occurs in the period as summarized by \( \Delta_{mnt} \). The new fractional observation becomes \( q_{r}(\tilde{c}_{T_n}, \tilde{y}_{T_n}) \sum_{t=1}^{T} \gamma_{mnt} \eta_{mnt} w_t \).

With the revised definition of a fractional observation, the revised updating formula becomes Equation A2.14 which is Equation 9 in the text. The data likelihood and the posterior distributions generalize Equations A2.11 and A2.12 as expected.

\[
\begin{align*}
\alpha_{rm,n+1} &= \alpha_{rmn} + q_{r}(\tilde{c}_{T_n}, \tilde{y}_{T_n}) \left( \sum_{t=1}^{T} \gamma_{mnt} \eta_{mnt} w_t \right) \delta_{n} \\
\beta_{rm,n+1} &= \beta_{rmn} + q_{r}(\tilde{c}_{T_n}, \tilde{y}_{T_n}) \left( \sum_{t=1}^{T} \gamma_{mnt} \eta_{mnt} w_t \right) (1 - \delta_{n})
\end{align*}
\]

The updating formulae in Equation A2.14 are practical because they maintain a naturally conjugate beta posterior distribution, because it can be computed quickly, and because, using arguments similar to those in §A2.4, it will converge to the true values, \( p_{rm, true} \), when the assumed model of behavior is correct.

### A2.4. Motivation of why Fractional Observations are a Good Approximation

The trick of fractional observations requires that we assume a level of independence that we
know is violated. Despite this assumption, both HULB’s algorithm and the generalized algorithm improve outcomes substantially with synthetic data—HULB’s algorithm relative to a static website and the generalized algorithm relative to HULB’s algorithm. In this section we motivate why that is likely that fractional observations are a good approximation. Our arguments are not a formal proof and they do not rule out the algorithms getting trapped in local minima, rather they suggest why we expect the fractional-observation updating formulae to converge to \( p_{rm, \text{true}} \).

For simplicity of notation consider the HULB’s assumption of \( \gamma = 1 \) and \( \vec{w} = (0, 0, 0, 1) \). We are interested situations where the data overwhelm the priors. That is, assume we observe \( n_{ob, m} \) customers where \( n_{ob, m} \) is sufficiently large: \( n_{ob, m} \gg \sum_{r=1}^{R}(\alpha_{rmo} + \beta_{rmo}) \). Let \( S_{rm} \) be the set of customers whose true segment is \( r \) and who saw morph \( m \). We continue to use the shorthand notation, \( q_{rn} = q_{rn\tau_n}(\vec{c}_{\tau_n}, \vec{X}_{\tau_n}) \), and define \( q_{rn}(s) = q_{rn\tau_n}(\vec{c}_{\tau_n}, \vec{X}_{\tau_n}|r_{true} = s) \). Under the conditions that the data overwhelm the priors, both HULB’s updating formulae (Equation A2.13) and the when-to-morph formulae become (for all \( r \) and \( m \)):

\[
\alpha_{rmn_{ob}} = \sum_{n=1}^{n_{ob, m}} q_{rn} \delta_{mn} = \sum_{s=1}^{R} \sum_{\tau_n \in S_m} q_{rn}(s) \delta_{mn}
\]

(A2.15)

\[
\beta_{rmn_{ob}} = \sum_{n=1}^{n_{ob, m}} q_{rn} (1 - \delta_{mn}) = \sum_{s=1}^{R} \sum_{\tau_n \in S_m} q_{rn}(s) (1 - \delta_{mn})
\]

We now take expected values.

\[
E_n[\alpha_{rmn_{ob}}] = \sum_{s=1}^{R} \sum_{\tau_n \in S_m} q_{rn}(s) p_{sm, \text{true}}
\]

(A2.16)

\[
E_n[\beta_{rmn_{ob}}] = \sum_{s=1}^{R} \sum_{\tau_n \in S_m} q_{rn}(s) (1 - p_{sm, \text{true}})
\]

Under the assumption that customers are homogeneous within true segments such that \( q_{rn}(s) = q_{r}(s) \), we recognize that:

\[
E_n[\alpha_{rmn_{ob, m}}] = \sum_{s=1}^{R} q_{r}(s) n_{ob, sm} p_{sm, \text{true}} = q_{r}(r)n_{ob, rm} p_{rm, \text{true}} + \sum_{s \neq r} q_{r}(s) n_{ob, sm} p_{sm, \text{true}}
\]

(A2.17)

\[
E_n[\beta_{rmn_{ob, m}}] = \sum_{s=1}^{R} q_{r}(s) n_{ob, sm} (1 - p_{sm, \text{true}})
\]

\[
= q_{r}(r)n_{ob, rm} (1 - p_{rm, \text{true}}) + \sum_{s \neq r} q_{r}(s) n_{ob, sm} (1 - p_{sm, \text{true}})
\]
If the first term dominates (the term containing \( q_r(r') \)), the beta posterior will converge to a point mass at \( p_{rm, true} \) as \( n_{ob,m} \) gets large. It will converge to a point mass because both \( E_n[\alpha_{rmn_{ob}}] \) and \( E_n[\beta_{rmn_{ob}}] \) are the order of \( n_{ob,m} \). Furthermore, \( E[\alpha_{rmn_{ob,m}}]/(E[\alpha_{rmn_{ob,m}}] + E[\beta_{rmn_{ob,m}}]) \) with approach \( p_{rm, true} \) when the first term dominates (because \( q_r(r)n_{ob, rm} \) drops out).

There are two forces that make it likely that, as \( n_{ob,m} \) gets large, the first term is much larger than the remaining terms for the morphs that are chosen for segment \( r \). First, if \( \hat{\Omega} \) is well-estimated and the logit model does a good job in predicting customer segments, then \( q_r(r) \gg q_r(s) \forall s \neq r \). Second, as the algorithm gets better at identifying the best morph for segment \( r \) (and if there is sufficient variation in morphs that different morphs are best for different segments), then \( n_{ob, rm} > n_{ob, sm} \) for \( s \neq r \). (\( m_r^* \) is the best morph for segment \( r \).) If the algorithm is indeed close to optimal, \( m_r^* \) will be chosen for segment \( r \). Convergence will be best for optimal segment \( x \) morph matches and, especially, for those morphs that are shown many times. (Because it is likely that \( p_{sm} \leq p_{rm} \), we can also argue that convergence will be from below. Details available from the authors.)

It is an empirical question whether these two forces are sufficiently strong so that fractional updating converges. Fortunately, synthetic-data experiments and empirical experience suggest that HULB’s algorithm and the generalized algorithm, which rely on fractional-observation updating, lead to improved outcomes and the ability to match morphs to segments. For the synthetic-data experiments in §5, we can compare the true purchase probability values, \( p_{rm, true} \)’s, to the posterior means. With \( \gamma = 1 \) and \( \vec{w} = (0, 0, 0, 1) \), we test HULB’s updating formulae in Equation A2.13. For this case, the mean absolute percent error is approximately 0.028 which is 8.7% of the average true probability. As the number of customers gets large, the algorithm tends toward identifying the optimal morph for each segment, hence we gain the most information about optimal segment \( x \) morph matches (\( r \) and \( m_r^* \)). When we look at optimal segment \( x \) morph matches, the error reduces to 5.8% of the average true probability. This reduces to 0.5% when we require a morph to have been shown to customers at least 2,000 times.

We examine Equation A2.14 with \( \gamma = 0.95 \) and \( \vec{w} = (0.25, 0.25, 0.25, 0.25) \). For this case, the mean absolute percent error is approximately 0.0318 which is 10.0% of the average true probability. When we look at the optimal morph for each segment, the error reduces to 7.3% of the average true probability. This reduces to 3.4% when we require a morph to have been shown at least the equivalent of 2,000 times (i.e., \( 2,000T_r \) total periods). The mean absolute error decreases further when morphs are shown even more often. We see similar patterns for \( \gamma = 0.80 \) and \( \vec{w} = (0.25, 0.25, 0.25, 0.25) \).