Probabilistic Modeling of Runway Inter-departure Times

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I. Introduction

The Erlang distribution has been used to model runway service process in queuing models of air traffic operations [1–3], since its use was first proposed by Hengsbach and Odoni [4]. It has been shown to offer certain computational advantages, because it can be viewed as a sum of exponential distributions. However, there have been few prior efforts to validate Erlang distribution assumptions using operational data, and even those have only been performed informally with aggregate or low-fidelity data [5].

This paper uses high-fidelity surface surveillance data to model the probability distributions of runway service times, and examine the goodness of fit of the Erlang distribution. For this, probabilistic models for the runway service times are derived from empirical departure throughput distributions and empirical departure time distributions. The results are compared in terms of bias, goodness of fit and computational advantages. The analysis has implications to both the modeling of airport operations, and to the estimation of airport capacities.

II. Data sources

The Aviation System Performance Metrics (ASPM) database maintained by the Federal Aviation Administration (FAA) records the wheels-off and wheels-on times of all domestic flights in the United States (US) [6]. These reports are obtained automatically through a system called ACARS for the major carriers, and are inferred for the others [7]. While a valuable data source, ASPM presents several challenges for validating runway service time distributions. Aircraft takeoff times are rounded to the nearest minute [7] and due to this quantization, aircraft taking off less than one minute apart from a runway can appear to be taking off with zero inter-departure time. In addition, the ACARS

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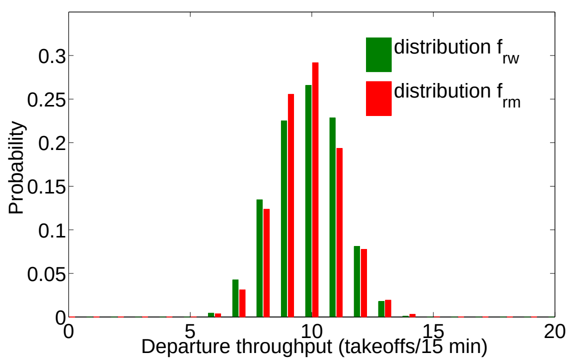
data is not of sufficient fidelity to identify instances when there was at least one aircraft waiting at the departure runway threshold [8].

The Airport Surface Detection Equipment – Model X (ASDE-X) system combines data from surface radar tracks, multilateration and ADS-B where available to present a more detailed view of airport surface operations [9]. Aircraft wheels-off times are typically captured with a precision of seconds. In addition, ASDE-X data can be used to measure the precise number of aircraft that are physically present in the queuing area at the departure runway threshold [10,11]. As a result, runway inter-departure times can be estimated conditioned on the state of the departure queue. For these reasons, ASDE-X data provides way to accurately model runway service times. This paper presents these techniques for the case of Boston Logan International Airport (BOS) for the 22L, 27 | 22L, 22R runway configuration in 2011.

III. Estimation of service time distributions from the departure throughput

The first step in the analysis is the extraction of instances in which there was persistent departure demand, and to fit an Erlang distribution to support the throughput seen during these instances. Persistent demand in this case is identified by 15 min periods when there were more than 22 aircraft taxiing out [8]. The empirical throughput distribution over these instances is shown in red in Figure 1 (left).



Distribution	f_{rw}	f_{rm}
Mean (aircraft/15 min)	9.81	9.81
Standard deviation (aircraft/15 min)	1.38	1.34

Figure 1. (Left) Empirical (f_{rw}) and modeled (f_{rm}) probability distributions of the departure throughput of 22L, 27 | 22L, 22R at BOS; (Right) First and second moments of f_{rw} and f_{rm}.

Suppose the service times were generated from an Erlang distribution $(k, k\mu)$, where $k \in \mathbb{N}_+$ and $k\mu > 0$ are the shape and rate parameters, respectively. The probability density function of the service times is then given by

$$g_{rm}(t;\ k, k\mu) = \frac{(k\mu)^k t^{k-1} e^{-k\mu t}}{(k-1)!},\ t > 0 \tag{1}$$

The parameters of the Erlang distribution are estimated from the throughput data using an approximation based on the method of moments. Let μ_1 and μ_2 denote the first and second moments of the empirical runway throughput distribution f_{rw}. Assume the runway service times are drawn from the Erlang distribution $(k, k\mu)$. When there are exactly i takeoffs in the time interval Δ, there are at least $(i-1)k+1$ and no more that $(i+1)k-1$ occurrences of a Poisson random variable with rate $k\mu\Delta$. In the first case, the first $k-1$ occurrences corresponding to the ith takeoff occurred in the previous time period, in the latter, the last $k-1$ occurrences correspond to the $(i+1)$th takeoff which takes place in the next time period. Summing over all these possibilities yields the following expressions for the mean and variance of the throughput distributions:

$$\mu_1 = \sum_{i=0}^{\infty} \left(i \cdot \sum_{j=(i-1)k+1}^{(i+1)k-1} \frac{k - |ik - j|}{k} \cdot e^{(-k\mu\cdot\Delta)} \cdot \frac{(k\mu \cdot \Delta)^j}{j!} \right) \tag{2}$$

$$\mu_2 = \sum_{i=0}^{\infty} \left(i^2 \cdot \sum_{j=(i-1)k+1}^{(i+1)k-1} \frac{k - |ik - j|}{k} \cdot e^{(-k\mu\cdot\Delta)} \cdot \frac{(k\mu \cdot \Delta)^j}{j!} \right) \tag{3}$$

The method of moments (MoM) determines the values of the parameters k and μ by matching the above expressions to the empirical data. Since k is constrained to be a natural number, the following approximation is made: The parameter μ is obtained by numerically solving Equation (2) as a function of increasing values of k. For each pair $(\mu, k\mu)$, the error of Equation (3) is calculated. The iterations are stopped when the absolute error increases. Any further increase in k would imply a further decrease in variance and a larger absolute error in the value of the second moment. The empirical, f_{rw}, and fitted, f_{rm}, throughput distributions are shown in Figure 1. The estimated parameters $(k, k\mu)$ of the Erlang distribution, g_{rm}, are (6, 3.92). g_{rm} has an average service time of 1.5 min with variance 0.4 min^2.

IV. Estimation of service time distributions from inter-departure times

If the departure queue has sufficient load that an aircraft takes off as soon as the runway is available, the service time equals the inter-departure time. Such a queue is described as a *queue with pressure*. In order to estimate the condition that implies a queue *with pressure*, the length of the departure queue beyond which inter-departure times do not change significantly with the number of aircraft in the queue is identified. A non-parametric method, namely, the Kruskal-Wallis one-way analysis of variance, is used to compare the distributions of service time distributions when an aircraft takes off with different departure queue sizes behind it. For the case of BOS in the 22L, 27 | 22L, 22R configuration, the analysis suggests a value of 5, as shown in Figure 2. This means that when the departure queue size is 5 or more, the inter-departure time equals the service time.

It is worth noting that the obtained set of service times does not consist of independent samples, since two consecutive takeoffs may be correlated. For example, a Heavy aircraft departure in a 15-minute interval does not significantly impact the departure throughput, because the controllers use the separation behind it to perform runway crossings [12].

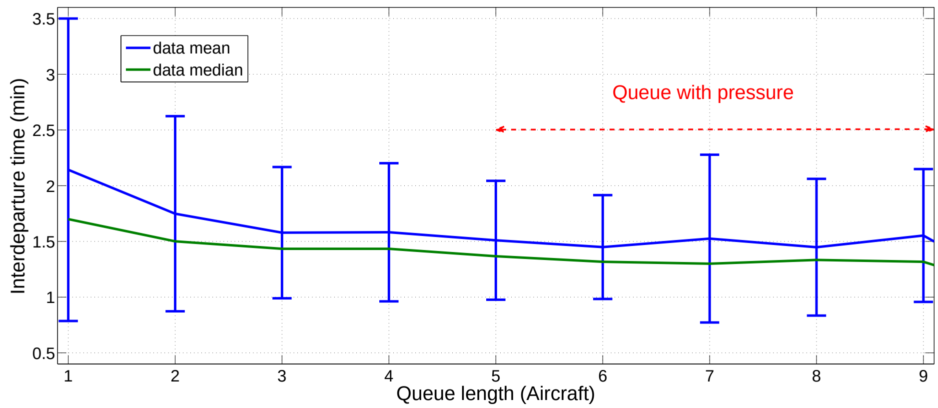


Figure 2. Determination of minimum size of queue with pressure, for 22L, 27 | 22L, 22R at BOS.

Similarly, when there are no Heavy departures in the queue, a controller might perform a sequence of non-Heavy departures followed by a sequence of runway crossings. Such correlations between consecutive inter-departure times motivates the definition of capacity over a long time period (for instance, the *saturation capacity*, the *practical hourly capacity* and the *sustained capacity* are all defined over an hour) [13]. In order to get independent samples of service time distributions, sets of service times that are 15 min apart are randomly sampled. The 15 min value is also consistent with throughput estimates, and has been found to provide a good compromise between length and persistent demand through the duration of the time period [3, 12, 14, 15].

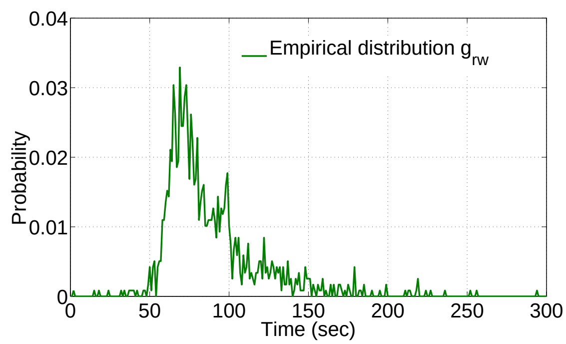


Figure 3. Empirical service time probability distribution for departures of 22L, 27 | 22L, 22R at BOS.

The empirical distribution for the service times of departures for which the departure queue size is 5 or more and which takeoff at least 15 min apart, is shown in Figure 3. The figure shows that the service time distributions have a long tail despite having a queue with pressure. The support of the distribution is seen to start around 50 sec, and not 60 sec as would be expected. The reason for this difference is that inter-departure times are measured at wheels-off, and not at the start of the takeoff roll, where separation is applied. The mode of the distribution is found to be 68 sec. The distribution exhibits also a second distinct peak at around 100 sec, which is attributed to Heavy aircraft departures.

V. Probabilistic modeling of service times

Four potential fits to the empirical service time distributions are compared:

1. The maximum likelihood estimate (MLE) Gamma distribution (g_{gl}), which estimates the the maximum likelihood parameters of a Gamma distribution to fit the empirical distribution in Figure 3.

2. The displaced exponential distribution fit (g_{de}), given by

$$g_{de}(x;\phi,d) = \begin{cases} \phi \cdot e^{-\phi(x-d)} & \text{if } x \geq d \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

   The displaced exponential distribution is often used in traffic engineering applications because it assumes that there is a minimum headway (d) between vehicles, in addition to a probabilistic quantity [16]. It could potentially be a good model for the runway service time distribution, since it reflects the minimum required separation between successive departures. The parameters (ϕ, d) of the displaced exponential distribution are estimated using the Method of Moments (MoM):

$$d + \frac{1}{\phi} = \mathbb{E}[S]; \quad \frac{1}{\phi^2} = \text{var}(S) \tag{5}$$

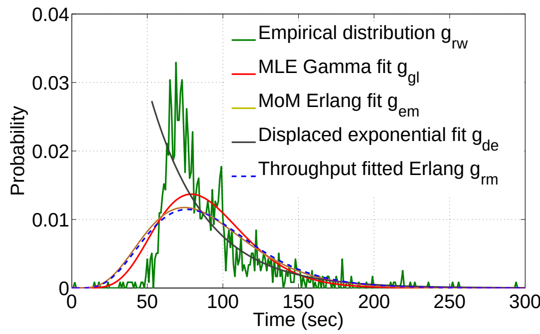
3. The Erlang distribution fit (g_{em}) from applying an approximate MoM to fit an Erlang distribution to the observed service times. The resulting Erlang distribution has a mean $\mathbb{E}[L_k]$ and variance $\sigma^2_{L_k}$ such that

$$\mathbb{E}[L_k] = \frac{\hat{k}}{\hat{\lambda}} = \mathbb{E}[S]; \quad \sigma^2_{L_k} = \frac{\hat{k}}{\hat{\lambda}^2} = \frac{\mathbb{E}[S]^2}{\lfloor \frac{(\mathbb{E}[S])^2}{\text{var}(S)} + 0.5 \rfloor} \approx \text{var}(S) \tag{6}$$

4. The Erlang distribution fit, g_{rm}, obtained from the empirical throughput, as seen in Figure 1. The parameters in the current case were estimated to be (6, 3.92). While f_{rw} comprises all departure throughput observations in saturation (more than 22 departures taxiing out), the service time distribution shown in Figure 3 comprises independent samples of inter-departure times given a queue with pressure. The Erlang distributions g_{rm} and g_{em} model the same quantity, but are estimated differently. The two distributions are expected to differ because g_{rm} alone was obtained through random sampling.

VI. Results

Figure 4 (top) shows the results of applying the four different fitting procedures to ASDE-X data from BOS, for the 22L, 27 | 22L, 22R configuration in 2011. The estimated parameters are shown in Figure 4 (bottom).



Distribution	Parameter 1 (Shape/ Displacement)	Parameter 2 (Rate)	Mean	Variance
Empirical	–	–	1.49	0.30
g_{gl}	8.54	5.72	1.49	0.26
g_{de}	0.88	0.62	1.49	0.30
g_{em}	6	4.02	1.49	0.37
g_{rw}	6	3.92	1.53	0.39

Figure 4. (Top) Service time probability distribution fits for departures of runway configuration 22L, 27 | 22L, 22R at BOS; (Bottom) Distribution parameters.

The plots suggest that the displaced exponential fit best matches the empirical distribution. d is estimated to be 53 sec, suggesting that it accurately captures the minimum separation requirement. The displaced exponential fit matches the tail of the empirical distribution well, but does not accurately predict the mode of the distribution.

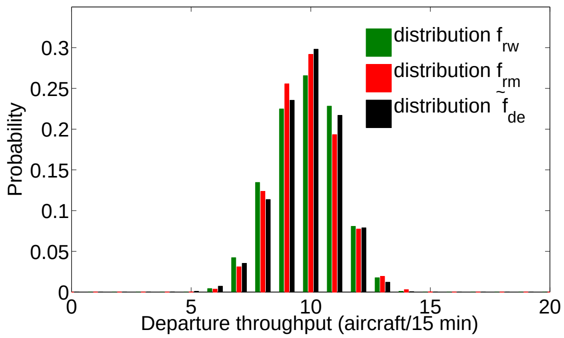
The Gamma and Erlang fits also fail to predict the mode of the distribution, and they overestimate the density of the distribution for values lower than 60 sec. However, they predict the tails of the empirical distribution as well as the displaced exponential fit does. The Gamma and Erlang distributions are different, as can be verified from their parameters in Figure 4 (bottom). The discrepancy is not the result of rounding off $\hat{k}$ (it is rounded up to 6 from 5.98), but due to the different estimation methods applied (MLE versus MoM). Finally, distributions g_{em} and g_{rm} are seen to be very similar despite being derived differently. Their similarity demonstrates that the estimated departure throughput under persistent demand and the inter-departure times given a queue with pressure are consistent with each other.

Estimating the service time distribution from the throughput distribution appears to accurately calculate not only the mean and the variance of the departure throughput, but also the mean and variance of the inter-departure time given a queue with pressure. On the other hand, Figure 4 suggests that the displaced exponential is a better fit to the empirical service times than an Erlang distribution. This hypothesis is tested by using the estimated parameters $(k, k\mu)$ of the fitted f_{rm} distribution to derive a displaced exponential distribution with the same mean and variance:

$$\tilde{g}_{de}(x; \tilde{\phi}, \tilde{d}) = \begin{cases} \tilde{\phi} \cdot e^{-\tilde{\phi}(x-\tilde{d})} & \text{if } x \geq \tilde{d} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$\text{such that } \tilde{d} + \frac{1}{\tilde{\phi}} = \frac{1}{\mu}; \quad \frac{1}{\tilde{\phi}^2} = \frac{1}{k\mu^2} \tag{8}$$

The parameters are calculated to be (0.91, 0.62), which are, as expected, similar to those of g_{de} (Figure 4 (right)). The corresponding $\tilde{f}_{de}$ is shown in Figure 5 (left), and is a good match to the empirical distribution. Figure 5 (right) shows that $\tilde{f}_{de}$ has a smaller Kullback Leibler (KL) divergence (a measure of the difference between two probability distributions [17]) from the actual throughput distribution, when compared to f_{rm}. It is therefore conjectured that $\tilde{g}_{de}$ accurately represents the service time distribution. It models the minimum service time requirement, the observed tail of the empirical service time distribution, as well as the associated departure throughput distribution.

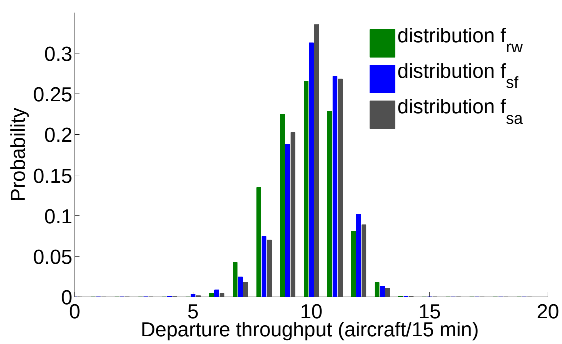


Distribution	Mean	Variance	KL Divergence
f_{rw}	9.81	1.90	–
f_{rm}	9.81	1.80	0.010
$\tilde{f}_{de}$	9.81	1.77	0.008

Figure 5. (Left) Empirical departure throughput distribution, f_{rw}, and fits f_{rm} and $\tilde{f}_{de}$ for 22L, 27 | 22L, 22R at BOS; (Right) Comparison of distributions.

VII. Need for sampling runway service times

Figure 3 showed the empirical distribution g_{rw} of the sampled service times, given a queue with pressure. Suppose this distribution was used to generate the corresponding throughput distribution, f_{sf} in a 15-minute period. Consider the alternative distribution g_{sa} of all service times, given a queue with pressure. The corresponding throughput distribution, f_{sa}, in a 15-minute period can be similarly generated.



Distribution	Mean	Variance	KL Divergence
f_{rw}	9.81	1.90	–
f_{sf}	10.05	1.78	0.0445
f_{sa}	10.06	1.49	0.1053

Figure 6. (Left) Empirical throughput distribution, f_{rw}, and fits f_{sf} and f_{sa} for departures of BOS runway configuration 22L, 27 | 22L, 22R; (Right) Comparison of the distributions.

Figure 6 (left) and (right) compare f_{sf} and f_{sa} to the empirical distribution, f_{rw}. The comparisons show that f_{sa}

has lower variance than f_{rw}, due to its use of dependent observations. Figure 6 (right) shows that f_{sa} has a higher KL-distance from f_{rw} than f_{sf}, illustrating the need to sample inter-departure times.

VIII. Effect of fleet mix on runway service times

Heavy jets are expected to have a runway service time of about 2 minutes (120 sec), on account of the increased wake vortex separation required behind them. Figure 7 shows the empirical service time distributions parametrized by the type of aircraft taking off. Heavy jets are seen to have longer service times (mean of 119 sec and mode of the distribution is at 105 sec) that non-Heavy aircraft (mean service time of 87 sec), as expected given their separation requirement. Since the service time is measured as the difference between successive wheels-off times, it is less on average than the required separation at the start of the takeoff roll, since Heavy aircraft tend to have longer roll times.

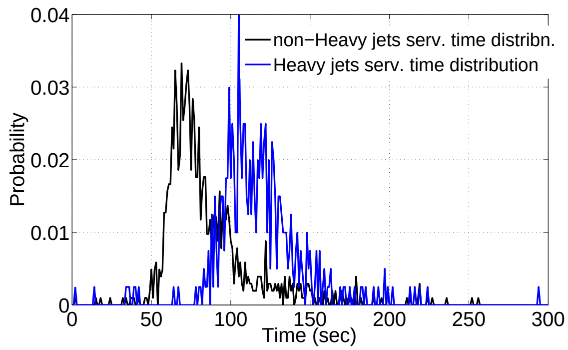


Figure 7. Empirical service time probability distributions for departures of BOS runway configuration 22L, 27 | 22L, 22R for Heavy and non-Heavy jets.

Finally, these findings can be compared to prior estimates of jet departure capacity [18], which concluded that the departure throughput does not depend on Heavy aircraft. By contrast, Figure 7 shows that Heavy aircraft tend to be separated from subsequent departures for longer than non-Heavy aircraft. However, Figure 7 does not show the impact of Heavy departures on the service times of surrounding non-Heavy aircraft. In Figure 7, this would imply that short non-Heavy aircraft service times are correlated with Heavy departures in the surrounding time-window. Similarly, the impact of an arrival bank will not be seen in the inter-departure time of a single aircraft, but will be seen in the 15-min departure throughput.

IX. Applications to other airports

Due to the limited availability of ASDE-X data, service times for departures from runway 17R at Dallas Fort-Worth International Airport (DFW) were analyzed for 11 days from 2009. Runway configuration 17C, 17L, 18R | 17R, 18L, 18R was in use during these periods, and the queue size was at least 4 aircraft. The empirical distribution of service times is shown in Figure 8, and is seen to be qualitatively similar to the service time distributions for BOS (Figure 3). The mode of the distribution is 55 seconds, and it exhibits a long tail extending to more than 200 sec. As more data becomes available, this analysis can be replicated for more runways and airports.

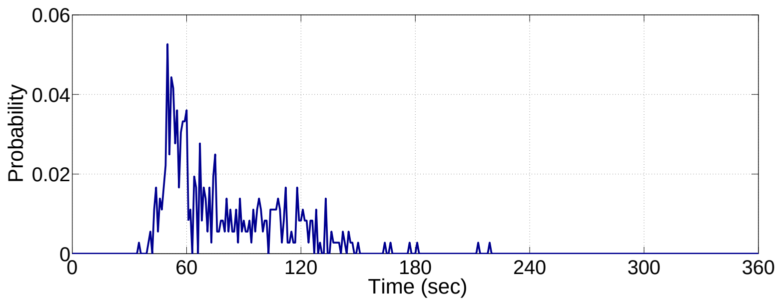


Figure 8. Empirical service time probability distributions for departures of DFW runway configuration 17C, 17L, 18R | 17R, 18L, 18R.

X. Conclusions

This paper determined probabilistic models of runway service times using high-fidelity surface surveillance data. Firstly, a modeling framework is developed for estimating Erlang service times distributions from the empirical departure throughput distributions. Subsequently, it is shown how empirical service time distributions can be derived from high-fidelity surface surveillance data. Three parametric distributions are fitted to the empirical distribution. For the case of the airport of BOS, the analysis also shows that a displaced exponential distribution may be a better match to empirical service time distributions than the Erlang distribution. However, the Erlang distribution is found to accurately model the means and variances of the empirical service time and throughput distributions, as well as the tail of the service time distribution. It is also shown that its parameters can be accurately derived from the empirical departure throughput distribution. These features, combined with its computational benefits, support the Erlang service time distribution assumption for queuing models of airport operations.

A complete representation of the runway service process, incorporating the impact of exogenous variables (arrival crossings, airspace route availability, propellor-driven aircraft procedures, etc.), would require a complex hidden Markov model with both exogenous and endogenous variables.

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