Analysis of Aircraft Fuel Burn and Emissions in the Landing and Take Off Cycle using Operational Data

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Abstract-An understanding of aircraft fuel burn and emissions is necessary to evaluate their effects on the environment. In this paper, data from Flight Data Recorders (FDRs) have been used to estimate the operational values of times in mode, fuel flow rates, fuel burn, NO_x emission indices, and quantities of emissions produced during the Landing and Take Off (LTO) cycle for 12 aircraft-and-engine combinations. These operational values are statistically compared to those reported in the ICAO Engine Exhaust Emissions Databank. In most cases, the operational values are found to differ from the ICAO databank values in a statistically significant manner. The ICAO databank is found to typically overestimate the values of LTO cycle fuel burn and emissions. The use of FDR archives also enables the characterization of the operational variability in fuel burn and emissions among different instances of the same aircraft-engine combination.

Index Terms—fuel burn; emissions; times in mode; LTO cycle; Flight Data Recorder; ICAO Engine Exhaust Emissions Databank; statistical analysis

I. INTRODUCTION

Aircraft emissions are a growing source of environmental pollution. The Intergovernmental Panel on Climate Change (IPCC) report on Aviation and the Global Atmosphere projects that the total aviation traffic (passenger, freight, and military) in 2050 will be 6.5-15.5 times its value in 1990, whereas the total aviation fuel burn in 2050 will be 1.5-9.5 times that in 1990. As a consequence, carbon dioxide emissions in 2050 are expected to be 1.6-10 times the 1992 values [1].

In order to evaluate techniques to reduce aircraft emissions, one must be able to estimate them accurately. The emissions from a flight depend on the time spent in different phases of flight (times in mode), as well as the fuel flow rates and emission indices in these modes. Landing and Take Off (LTO) cycle emissions have a direct impact on local air quality, and impact the health of the people living in the vicinity of airports. Currently, aircraft emissions in the LTO cycle are estimated using the International Civil Aviation Organization (ICAO) Engine Exhaust Emissions Databank [2] (hereafter referred to as the ICAO databank). The ICAO databank assumes constant values of thrust settings, times in mode, fuel flow rates and emission indices for the different phases of the LTO cycle. The fuel burn and emissions in a particular phase, for a particular engine, are then given by the product of the times in mode, fuel flow rates and emissions indices. The estimates of fuel burn obtained from this approach are known to be approximate, and the values of the emissions indices reported in the ICAO databank have been shown to deviate significantly from measured operational values [3, 4]. For example, the effective hydrocarbon emission index for an aircraft idling on the ground is believed to exceed the ICAO databank values by 40-90% [3].

Several prior studies have focused on developing approaches for an accurate estimation of fuel burn and emissions, in order to overcome the limitations posed by the ICAO databank. These approaches have included the application of neural network methods to data from aircraft flight manual charts to estimate fuel burn [5], as well as the application of energy balance methods and empirical relations to aircraft path profile data [6]. Other efforts have used the physics-based modeling of the engine combustor performance to estimate emission indices [7]. The FAA's System for assessing Aviation's Global Emissions (SAGE) uses publicly available databases (like BADA, ETMS, EDMS and the ICAO databank) to estimate global aviation emissions [8]. However, none of these efforts have used engine performance data from actual operations. As a result, it is not known how the actual emissions of operational aircraft may differ from the values predicted by these models.

There is an opportunity to develop better models of operational engine performance using information from Flight Data Recorders (FDRs). The FDR is the most accurate, onboard repository of operational data from an aircraft. Moreover, it can account for effects of nonphysical factors (like pilot effects, airline operating procedures) on engine performance, and also estimate the variability in performance parameters. There have only been a few studies using FDR data for modeling of engine performance and aircraft emissions, but they have been limited to particular phases of flight, such as cruise [9], arrival/departure paths [10, 11], or taxi [12]. In prior work, we developed a reduced-order model of engine performance and compared it to the FDR reported values [13]. This paper investigates how FDR derived operational engine performance in different phases of flight may differ from values reported in other sources such as the ICAO databank [2].

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II. FLIGHT DATA RECORDER DATASET

A. Aircraft and Engine Types

Our FDR dataset contains records corresponding to 12 distinct aircraft types and 13 engine types, as shown in Table I. The table also reports the number of flights of each aircraft type used in our study.

TABLE I. AIRCRAFT AND	ENGINE TYPES	IN THE FDR	DATASET.
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I	Sr.	Aircraft Type	Engine Type	Number	Number
l	No.			of	of
				Engines	Flights
Ì	1.	Airbus A319-112	CFMI CFM56-5B6	2	130
	2.	Airbus A320-214	CFMI CFM56-5B4	2	169
l	3.	Airbus A321-111	CFMI CFM56-5B1	2	117
l	4.	Airbus A330-202	GE CF6-80E1A4/	2	84
l			PW 4168		
l	5.	Airbus A330-223	GE CF6-80E1A4/	2	179
l			PW 4168		
l	6.	Airbus A330-243	GE CF6-80E1A4/	2	100
			PW 4168		
	7.	Airbus A340-541	RR Trent 556	4	52
	8.	Airbus A340-313	CFMI CFM56-5C4	4	76
l	9.	Boeing B757-	PW 2037/2040	2	150
		200			
l	10.	Boeing 767-300	GE CF6-80C2B6	2	91
l	11.	Boeing 767-300	PW 4060	2	44
l	12.	Boeing 777-	GE 90-115B	2	131
l		3FXER			
l	13.	Avro RJ85/100	Honeywell LF507-	4	153
l			1F		

B. Parameters

The FDR dataset contains records of 103 parameters. Parameters used in our study include the ambient conditions (pressure and temperature), true airspeed, ground speed and flight Mach number, trajectory information on pressure altitude, latitude, longitude, gross aircraft mass, fuel flow rates, engine spool speeds, combustor pressure, exhaust gas temperature, and engine pressure ratio. All values are also associated with a time stamp.

C. Flight Phase Identification

The trajectory of each flight is divided into different phases using the FDR reported values of aircraft trajectory parameters such as, latitude, longitude, pressure altitude and ground speed, and their derivatives. The phases identified are as follows:

- *Departure taxi*: This is the phase from the first motion of the aircraft up to the start of the takeoff roll.
- *Takeoff roll and wheels-off*: The start of the takeoff roll is identified by a sudden increase in aircraft acceleration. The wheels-off condition is identified by the pressure altitude relative to the departure airport's Above the Mean Sea Level (AMSL) elevation becoming zero and then subsequently increasing continuously and rapidly as the aircraft climbs.
- *Ascent/climb*: Climb follows wheels-off, until the aircraft reaches its cruising altitude.
- *Cruise*: Start of cruise is detected when the pressure altitude levels out.

- *Descent*: Top-of-descent is identified by the onset of a rapid and continuous decrease in aircraft altitude near the destination airport.
- *Wheels-on*: Touchdown (wheels-on) is identified by the pressure altitude relative to the arrival airport AMSL elevation becoming zero.
- Arrival landing roll and taxi: This is the entire phase post wheels-on and till the aircraft comes to a stop.

Fig. 1 depicts a typical trajectory, with the different phases identified.



Figure 1. A typical profile of the pressure altitude versus time and the different flight phases.

D. Landing and Take Off Cycle

The LTO cycle comprises all those phases of the flight trajectory which are below 3000 feet Above Ground Level (AGL). These include the takeoff roll, the climbout to 3000 feet AGL, the approach from 3000 feet AGL, and the ground taxi/idle phase. The ground taxi/idle phase includes both the departure and the arrival taxi phases.

The ICAO databank uses standard values of thrust settings and times in mode to certify engine fuel burn and emissions. The ICAO databank assumes that, irrespective of the aircraft/engine type and the airport of operation, the takeoff roll occurs at a constant 100% thrust setting for 42 s, the climbout at a constant 85% thrust setting for 132 s, the approach at a constant 30% thrust setting for 136 s. Certification idle at a constant 7% thrust setting for 1560 s. Certification is done on an uninstalled engine at standard sea level static ISA (SLS-ISA) conditions. The certification gives the values of fuel flow rates, fuel burn, and emission indices for the 4 different LTO cycle phases.

III. LANDING AND TAKE OFF CYCLE TIMES IN MODE AND FUEL BURN

In this section, the operational values of times in mode, fuel flow rates and total fuel consumed in the LTO cycle are calculated for the different aircraft/engine types using the FDR data. These values are then statistically compared with the corresponding ones reported in the ICAO databank.

A. Methodology

The analysis consists of the following steps:

- Since the ICAO databank reports values at SLS-ISA conditions for an uninstalled engine, the FDR reported values for the fuel flow rates (which reflect the at-altitude conditions for an installed engine) are first converted to equivalent values referenced to SLS-ISA conditions for an uninstalled engine. This procedure enables the appropriate comparison of the ICAO databank and the FDR derived values. The Boeing Fuel Flow Method (BFFM2) [8] (explained in appendix A) is used for this translation to SLS-ISA uninstalled conditions.
- For each aircraft/engine type, the values of the operational times in mode, mean fuel flow rates and the fuel burnt in the different phases of the LTO cycle are calculated using the FDR reported parameters and referenced to SLS-ISA uninstalled conditions, for different flights. These values are then averaged over the different flights to give the mean values of the times in mode, fuel flow rates and fuel consumed for a particular aircraft/engine type in the different phases of the LTO cycle. 95% confidence intervals are also assigned.
- Lastly, these FDR derived mean values are statistically compared with the corresponding values in the ICAO databank. The two-sided Wilcoxon signed rank test for paired samples is used to decide if the FDR derived values are different from the ICAO values [14]. A significance level (α) of 5% is assigned to the test.

B. Results

The results of the analysis are presented in Fig. 2 and Fig. 3. They show the FDR derived operational mean values (as bar plots), the 95% confidence intervals (as error bars), and the ICAO databank values for different aircraft types in different phases of the LTO cycle. Hatched bars indicate FDR derived values that are *not* statistically significantly different from the ICAO databank values at the 5% significance level. The unhatched bars imply a statistically significant difference.

Fig. 2 (top) shows the times in mode calculated using the FDR time-stamps corresponding to the LTO phases. As expected, the time in mode is found to be the greatest during ground taxi, followed by approach, climbout and takeoff roll. A similar trend is observed in the ICAO databank values of the times in mode. In most cases, there is a statistically significant difference between the average operational values of times in mode and those reported in the ICAO databank (except the approach phase for the A319 and B777, and climbout for the two A340 variants). The ICAO databank is found to typically overestimate the mean times in mode. The extent of overestimation is found to be large for the taxi and the climbout phases (as large as 52% for climbout of the A319). However, for the approach phase, many aircraft types (the A319, A330-202, A330-243, A340-541, the GE-powered B767, B777) show an operational time in mode that is larger than the ICAO value.

The error bars indicate the extent of variability in times in mode within the same aircraft type. This variability is partly due to operational factors, and cannot be captured by models of aircraft performance alone. For example, the times in mode in the taxi phase are heavily influenced by factors like airport layout, congestion, weather conditions, and position in the departure queue. The use of operational data from the FDR helps us better estimate the extent of this variability.

Fig. 2 (middle and bottom) shows similar graphs for the fuel flow rates and the mass of the fuel consumed (or fuel burn), respectively during different phases of the LTO cycle. The fuel flow rates are on an average per engine basis, whereas the fuel mass consumed is for all the engines combined. It is important to remember that the fuel flow rates and fuel mass consumed have been corrected to SLS-ISA conditions for an uninstalled engine for comparison with the ICAO databank values.

Since the FDR reports values for the fuel flow rates, the fuel mass is calculated by multiplying the fuel flow rates and the time interval between two consecutive FDR recordings at each instant of time and integrating over the whole phase. In most of the cases, the ICAO databank values differ statistically significantly from the operational values of the fuel flow rates (except for the climbout of the A321 and the Avro RJ) and the fuel mass consumed (except for the takeoff of the A330-202 and A340-313, and approach for the A340-541 and B767 (GE engines)).

In most cases (particularly in taxi and approach), the ICAO databank overestimates the mean fuel flow rates. However, for many of the aircraft types, in the climbout and takeoff phases, the ICAO databank values are less than the operational values. The general trend of the fuel flow rates suggests that the fuel flow rate is the least during taxi, followed by the approach phase. Takeoff roll and climbout have the highest levels of fuel flow rates. This follows from the fact that takeoff and climbout occur at high thrust settings whereas taxi and approach involve low thrust settings.

The trend in the fuel flow rates is opposite to that observed in the times in mode. On the other hand, the general trend for the fuel mass consumed shows that it is the greatest during taxi (except for the A330-202 and A340-313, which have the maximum fuel burn in climbout) and the least during takeoff. This shows that in the taxi and takeoff phases (which are the ground phases in the LTO cycle), the times in mode dominate over the fuel flow rates. This result supports a finding in [12] that the time in mode is the most important contributor to surface fuel burn. In the airborne phases of the LTO cycle, the climbout fuel burn is greater than that of approach, suggesting that the fuel flow rates dominate over the times in mode in the climbout and approach phases. Again, in most of the cases, the operational fuel burn is less than that reported in the ICAO databank (except for the climbout of the A340-313 and the takeoff of the A330-223).

Fig. 3 (top) shows the total fuel burn (all the engines combined) in the LTO cycle, summed over all the phases, for the different aircraft types in the FDR dataset. It is found that in all the cases, the ICAO databank value of the total LTO fuel



Figure 2. LTO cycle: operational and ICAO databank values for (top) times in mode, (middle) fuel flow rate (per engine), and (bottom) fuel mass consumed (all engines).

burn is statistically significantly different from the operational value derived from the FDR. For all the aircraft types, the ICAO databank overestimates the total fuel burn. The extent of overestimation can be as large as 42%. A comparison of the LTO cycle total fuel burn with the Maximum Take Off Weight (MTOW) of the aircraft type shows that the total LTO

fuel burn is 0.6-0.8% of the MTOW. In other words, the LTO cycle total fuel burn scales almost linearly with the MTOW. It is worth noting that the results presented in this paper for the LTO cycle times in mode, fuel flow rates and fuel mass consumed (fuel burn) are qualitatively similar to those reported in an independent study by Patterson et al. [11].



Figure 3. LTO cycle: operational and ICAO databank values for (top) total fuel mass consumed (all engines) in the complete LTO cycle, (middle) NO_x emission indices, and (bottom) NO_x mass produced (all engines).

IV. LANDING AND TAKE OFF CYCLE EMISSIONS

In this section, the operational values of the LTO cycle emissions of carbon dioxide, water vapor and nitrogen oxides (NO_x) are calculated using FDR parameters and compared with those reported in the ICAO databank. Carbon dioxide (CO_2) and water vapor (H_2O) are the major products of jet

fuel combustion. The operational values of carbon monoxide (CO), unburnt hydrocarbons (HC), particulate matter and other emissions are not studied in this paper. However, the techniques presented in this paper can be easily extended to account for these emissions as well.

A. Methodology

The methodology adopted is as follows:

- The first step in estimating emissions from fuel burn is the calculation of Emission Indices (EI) (mass of emission produced per unit mass of fuel burnt). As explained in [8], assuming complete combustion of jet fuel, the EI for CO₂ is taken to be a constant 3,155 g/kg, and that for H₂O is assumed to be a constant 1,237 g/kg.
- The calculations for estimating the NO_x emissions are more involved, as the EI of NO_x varies with the fuel flow rate. To calculate the EI for NO_x from the FDR reported values of fuel flow rates, the Boeing Fuel Flow Method (BFFM2) [8] is used. The BFFM2 uses the linear relationship between the logarithm of ICAO databank reported NO_x EI and the logarithm of ICAO databank reported fuel flow rates at four thrust settings to interpolate for the values of NO_x EI at fuel flow rates not reported in the ICAO databank. Traditionally, the BFFM2 uses fuel flow rates derived from the Base of Aircraft Data (BADA) airplane performance models to estimate the EIs. This study uses the FDR derived operational values of fuel flow rates in the place of the BADA-obtained fuel flow rates, to estimate the EIs. More details on the BFFM2 are presented in the appendix A.
- Multiplying the EIs with the mass of fuel burnt gives the mass of the different emissions produced.
- As in the case of fuel burn, the values of EI and the mass of the emissions produced are referenced to SLS-ISA conditions for an uninstalled engine so that comparison with the corresponding values in the ICAO databank is possible.
- The values of the EIs and the mass of emissions are then averaged over the different flights to give their mean values for a particular aircraft/engine type in the different phases of the LTO cycle. 95% confidence intervals are also assigned.
- Lastly, these FDR derived mean values are compared with the corresponding values in the ICAO databank. Again, the two sided Wilcoxon signed rank test for paired samples is used to statistically compare if the FDR derived values are different from the ICAO values [14]. A significance level (α) of 5% is assigned to the test.

B. Results

The results of the emissions estimation are shown in Fig. 3. Since the carbon dioxide and water vapor emissions are directly proportional to the mass of fuel consumed, the conclusions drawn for the fuel burn (in Section III-B) hold for the carbon dioxide and water vapor emissions as well. Consequently, the ICAO databank values differ from the operational values in a statistically significant manner in most cases, with the ICAO databank overestimating the mean emissions of carbon dioxide and water vapor.

The general trend for the NO_x EIs (Fig. 3 (middle)) shows that the EIs are the lowest for the taxi phase followed by

the approach phase. The highest EIs are for climbout and takeoff. The trends in NO_x EIs closely follow those in the fuel flow rates, as would be expected from the linear relation between the logarithm of NO_x EI and the logarithm of fuel flow rate. This phenomenon is also expected from the physics of engine operation. Phases which operate at higher thrust settings result in higher fuel flow rates and higher combustor temperatures, which in turn result in higher NO_x EIs. Again, the qualitative trends are similar to those observed in the ICAO databank. In all cases, the operational EIs differ statistically significantly from those reported in the ICAO databank. In the taxi, approach and takeoff phases, the ICAO databank NO_x EIs are greater than the FDR derived values (except for the A330-223 and B777). The extent of overestimation can be as large as 64% (for the Pratt and Whitney engine-powered B767 in the takeoff phase). An interesting observation is that the operational (FDR derived) EIs in climbout are greater than those reported in the ICAO databank (except for the A340-541).

With respect to the mass of NO_x emissions produced (Fig. 3 (bottom)), the maximum NO_x emissions are produced during climbout, a consequence of the high values of both fuel burn and NO_x EIs during this phase. Again, the operational values statistically significantly differ from the ICAO databank values (except in the cases of taxi for the A330-223, approach for A340-541, climbout for the GE engine powered B767, and takeoff for the A340-313). The ICAO databank overestimates the mean levels of NO_x mass produced (except in the cases of taxi for the A330-223, A340-313 and takeoff for the A330-223). The extent of overestimation is as large as 83% (for the Pratt and Whitney engine-powered B767 in the takeoff phase).

V. CONCLUSIONS AND FUTURE WORK

In this paper, operational data from the FDR were used to compute times in mode, fuel flow rates, fuel burn, NO_x EIs and emissions of carbon dioxide, water vapor and NO_x in the four different phases of the LTO cycle. The mean operational values were also statistically compared with those reported in the ICAO Engine Emissions Databank.

The mean operational values in the different phases of the LTO cycle were found to behave qualitatively similarly to those in the ICAO databank. However, in almost all of the cases, the actual mean operational values differed in a statistically significant manner from those reported in the ICAO databank, and the latter were found to typically be greater than the former. Our analysis showed that these differences cannot be attributed to ambient atmospheric conditions (pressure, temperature, airspeed) or engine installation effects, as the operational values were converted from their at-altitude conditions to SLS-ISA conditions before the statistical comparison with the ICAO databank values. The differences found between the ICAO databank and operational values can lead to an inaccurate estimation (more specifically, an overestimation) of fuel burn and global aircraft emission inventories, which currently rely on the ICAO databank to estimate emissions.

Another advantage of using operational FDR data is the ability to characterize the variability of values among the same aircraft/engine type by assigning confidence intervals around the mean operational values. This is in contrast to the ICAO databank, which assumes constant parameter values during a complete phase in the LTO cycle. This variability accounts for effects like differences in engine performance due to maintenance and aging, weather conditions on a particular day, airport traffic and congestion levels, airline operating procedures, differences in pilot behavior, etc. Although it is hard to quantify the effect of each of these factors independently, the confidence intervals serve to provide an overall estimate of these confounding factors on the estimates of times in mode, fuel burn and emissions.

Future work will focus on using the FDR dataset to develop regression models to estimate engine performance (especially fuel flow rates) from trajectory data of a flight. It is expected that use of operational FDR data will lead to better estimates of engine performance (than current methods which rely on databases) which will further lead to more accurate emission inventories.

APPENDIX

BOEING FUEL FLOW METHOD 2

This appendix describes, in brief, the equations of the Boeing Fuel Flow Method 2 (BFFM2) used to convert the FDR reported fuel flow rates from at-altitude conditions for an installed engine to SLS-ISA conditions for an uninstalled engine. It also describes the method to estimate NO_x EIs from fuel flow rates. The equations are given in more detail in [8].

Conversion to SLS-ISA Conditions

The following equations are used to convert the FDR reported fuel flow rates in the LTO cycle from at-altitude conditions for an installed engine to SLS-ISA conditions for an installed engine.

$$W_{ff} = \frac{W_f}{\delta_{amb}} \left(\theta_{amb}^{3.8} e^{0.2M^2} \right)$$
(1)

$$\delta_{amb} = \frac{P_{amb}}{P_{SL,ISA}} \tag{2}$$

$$\theta_{amb} = \frac{T_{amb}}{T_{SL,ISA}} \tag{3}$$

where,

W_{ff}	FDR reported fuel flow rate converted to
	SLS-ISA conditions (installed engine) (in kg/s)
W_f	FDR reported fuel flow rate at at-altitude
-	conditions (installed engine) (in kg/s)
Pamb	ambient pressure at altitude (in Pa)
$P_{SL, ISA}$	ambient pressure at sea level according to
	standard ISA conditions (= 101325 Pa)
T_{amb}	ambient temperature at altitude (in K)
$T_{SL, ISA}$	ambient temperature at sea level according to
	standard ISA conditions (= 288.15 K)
М	Flight Mach number

Conversion to Uninstalled Conditions

The following equations convert the SLS-ISA converted fuel flow rates for an installed engine to those for an uninstalled engine. The resulting fuel flow rates are referenced to SLS-ISA conditions for an uninstalled engine, the same conditions under which ICAO carries out engine fuel burn and emissions certification.

$$W_{ffu} = \frac{W_{ff}}{r}$$
(4)

$$r = \begin{cases} 1.100 \text{ taxi/ground idle} \\ 1.020 \text{ approach} \\ 1.013 \text{ climbout} \\ 1.010 \text{ takeoff} \end{cases}$$
(5)

where,

Relation between NO_x Emission Indices and Fuel Flow Rates

The BFFM2 assumes a linear relation between the logarithm of the ICAO databank reported NO_x EI and the logarithm of the ICAO databank reported fuel flow rate (Fig. 4). This linear relation is used to calculate NO_x EI values for the values of fuel flow rates not reported in the ICAO databank.



Figure 4. A typical variation of NO_x EI with fuel flow rate, as suggested by the BFFM2.

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