Indoor navigation using WiFi round-trip time Berthold K.P. Horn

- Fine Time Measurement (FTM) Round Trip Time (RTT)
- IEEE 802.11mc
- Background: Timing Advance (TA) in GSM and LTE
- Recovering initiator position from distances to responders
- Dilution of Precision (DOP) a.k.a. Noise Gain
- Non-Gaussian, non-stationary measurement error
- Best spatial arrangement for responders
- Some issues, problems, and currrent limitations
- When will it happen?



What's is indoor location good for?

- Navigation: Turn-by-turn wayfinding. Show me the way to the meeting.
- Asset Management: Where is the fork-lift? wheel-chair? ultra-sound machine?
- Geo-fencing: Animal tagging. Child safety. Disable company laptops.
- Location based marketing: Alert consumer to nearby item or trigger coupon.
- Emergency Services: Locate situation within building. Civic address and floor.
- Internet of Things: Dim the light in the room I am in. Lock the correct door.
- Warehousing and robotics: Guide autonomous forklifts.
- Network management: Pinpoint equipment failures and activate replacements.
- Smart cities: Public parking structures, street lighting, security cameras.

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Background: Timing Advance in cellular networks

- Timing Advance in GSM
- Timing Advance in LTE

Both use time-division duplex (TDD)

Unlike CDMA which uses code-division duplex (CDD)

GSM 310:260 36454:53371 (0xD07B) SW (39) | GSM dBm -103 BER 0 -GSM ARFCN 605 BSIC 55 TA * EDGE 00 00 00 00 GPS 44.09028 -70.77584 alt 186 | acc 28 vel 0.0 azi -1 Sat 15/18 \ BASE N/A |

GSM 44.06219 -70.72611 s 3133 R I LTE (no registered cells)

- IP 192.168.1.13 45.46.100.69 \ BAT 80% 3.999 V 27 C (SIM: T-Mobile) -2G 310:410 / 2G EDGE /
- 3 AP max -47 a8:9a:93:91:04:3e

GSM: 310:260 36454:53371 TA 10: 5540 m 2018-11-22 17:58:15.673

Google

302

14:56 ^{79°} M 🔗 🗹 •

• O LTE 68%

H1

VoLTE 310:260 12870:21878274 (0x14DD602) NW VoLTE voice connection (GSM) LTE EARFCN 2250 TA * LTE 00 00 00 12 \ GPS 21.33276 -157.92477 alt 9 / acc 17 vel 0.0 azi -1 Sat 0/22 | BASE N/A \ GSM 21.34093 -157.92948 s 1523 R L 0:0 /

LTE 21.34093 -157.92948 s 1523 R I IP 192.0.0.4 136.22.79.149 -BAT 68% 3.93 V 37 C (SIM: T-Mobile) -LTE 310:260 12870:21878274 (0x14DD602) 90 \ LTE SS 19 RP -103 RQ -11 SN 30.0 CQ * (S) -Device DOES support WIFI RTT

Salt Lake Blvd

nt Base Pearl rbor-Hickam

Google

Kuntz Ave

Elliott St

Daniel K. Inouye

LTE: 310:260 12870:21878274 TA 11: 1648 m 2019-02-21 14:56:47.448

Image: A state of the sta

CDMA 445:16:19681 (0x4CE1) SW L (1198) \ CDMA dBm -91 EcIo -7 | LTE EARFCN 5090 LTE 00 00 00 11 / FUSED 44.06815 -71.07408 h 471 / acc 7 vel 0.00 azi 0 Sat 12/18 -BASE 44.05944 -71.09111 d 1672 SW \ MMAP 44.10829 -71.16970 s 3614 | IP 10.200.221.181 166.182.80.117 \ BAT 57% 3.856 V 32 C Li-ion |

LTE 311:580 2822:84515841 (0x5099C01) 81 / LTE SS 8 RP -128 RQ -15 SN -3.4 CQ * (S) /

Merriman State Forest

> Conway Ommon Lands State Forest

- GSM 310:260 19298:16247450:3 (0xF7EA9A) GSM N/A \
- LTE EARFCN 5035 LTE 00 00 00 01 /
- FUSED 21.07841 -156.99653 h 23 -
- acc 3 vel 0.00 azi 0 -
- MCC:MNC 310:260 Fi Network
- MMAP 20.78655 -156.55677 s 3565
- IP 192.0.0.4 75.85.137.247
- BAT 40% 3.693 V 36.7 C Li-ion -

LTE 310:260 12830:22731268 (0x15ADA04) 428 LTE SS 12 RP -116 RQ -12 SN 30.0 CQ * (S)

Wailea-Makena

Haleakala

Maul Channe

LTE: 310:260 12830:22731268 TA 511: 79716 m

Background: using Received Signal Strength:

• Estimate distance based on RSSI (dBm)

Does signal strength obey the inverse square law?

• "Fingerprinting" a venue

Tedious. Repeat when access point configuration changes

•

802.11mc

- Fine Time Measurement (FTM)
- Round Trip Time (RTT)

WiFi 802.11mc

FTM: Fine Timing Measurement

total round trip

Naround time

<u>Clients range relative to APs/beacons</u> But how does that relate to 'location'?

- Requires knowledge of AP locations
- Requires a map with frame of reference

WiFi Indoor Location Device

Android RTT FTM (API Level 28)

get permissions: ACCESS_FINE_LOCATION, CHANGE_WIFI_STATE register BroadcastReceiver for: SCAN_RESULTS_AVAILABLE Call WifiManager.ScanStart(...) [†]

receive ScanResults in BroadCastReceiver
create RangingRequest from ScanResults

Call WifiRttManager.startRanging(...) [‡] RangingResultsCallBack onRangingResults

[†] Throttled to 4 calls per 2 minutes (and "grey listed")

[‡] Disabled when app is in background

8:44 🍡 😳 🗗 🛟 🔸

FTMRTT

0	b8:08:cf:a0:8b:f6	-55	d	÷
1	b8:08:cf:a0:88:04	-88	d	
2	b8:08:cf:a0:7a:ad	-80	d	
3	b8:08:cf:a0:7a:Od	-82	d	
4	b8:08:cf:a0:7a:a3	-106	d	
5	38:8b:59:c4:f0:a0	-58	d	0
6	38:8b:59:c4:f3:55	-76	d	
7	38:8b:59:c4:f7:ee	-87	d	1
8	4c:ed:fb:b7:5a:3c	-76	d	
9	Oc:9d:92:b5:c4:1c	-81	d	2

♥⊿ ₿ 99%

3.33 s 0.40 36811 1.94 s 0.41 36812 4.58 s 0.35 🛃 36813 3.90 s 0.59 🦄 36814 5.31 s 2.07 😂 36815 4.02 s 0.23 google@... 8.80 s 0.06 google@... 1.22 s 0.16 google@... 7.95 s 0.13 ASUS_38... 7.95 s 0.09 ASUS_18...

1:19

FTMRTT • 10 RTT responses from 16 APs ^

0 dBm: -56 dis: 3.03 s: 0.24 "ASUS_38_5G" 1 dBm: -85 dis: 5.34 s: 0.20 "ASUS_18_5G" 2 dBm: -77 dis: 5.87 s: 0.42 "google@176" 3 dBm: -73 dis: 2.12 s: 0.28 " 4 dBm: -88 dis: -0.53 s: 0.57 "🦄 36814" 5 dBm: -91 dis: 8.17 s: 0.95 "😂 36815" 6 dBm: -89 dis: 10.95 s: 1.10 "google@176" 7 dBm: -91 dis: 12.97 s: 0.28 "google@176" 8 dBm: -92 dis: 6.22 s: 1.60 " 36811" 9 dBm: -102 dis: 2.95 s: 4.71 "36812"

Mobile Network: US Cellular LTE - Band 5 (85. Connected to WIFI (google@176)

Solution Methods

 $\|\mathbf{s} - \mathbf{r}_i\| = d_i$ for i = 1, 2, ... n

s is the unknown position of the initiator, and d_i is the measured distance to the responder at \mathbf{r}_i .

- Intersecting spheres (or circles)
- Reduction to linear equations & pseudo inverse
- Least squares & gradient descent
- Brute force grid search
- Extended Kalman filter linearization & assumption
- Bayesian grid transition & observation models

Reduction to Linear Equations

$$\|\mathbf{s} - \mathbf{r}_i\| = d_i$$
$$(\mathbf{s} - \mathbf{r}_i) \cdot (\mathbf{s} - \mathbf{r}_i) = d_i^2$$
$$\|\mathbf{s}\|^2 - 2 \mathbf{r}_i \cdot \mathbf{s} + \|\mathbf{r}_i\|^2 = d_i^2$$

So we have *n* second order equations in **s**. Subtract pairwise to obtain m = n(n - 1)/2 linear equations:

$$(\mathbf{r}_j - \mathbf{r}_i) \cdot \mathbf{s} = ((d_i^2 - d_j^2) - (R_i^2 - R_j^2))/2$$

where $R_i = \|\mathbf{r}_i\|$. We can rewrite this in matrix-vector form:

 $A\mathbf{s} = \mathbf{t}$

For n = 3 we get three *linearly dependent* equations. For n > 3 can obtain least squares solution:

$$\mathbf{S} = (A^T A)^{-1} A^T \mathbf{t}$$

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BUT: Huge noise gain!

Least Squares and Gradient Descent

Find **s** that minimizes sum of squares of errors

$$e = \sum_{i=1}^{n} (\|\mathbf{s} - \mathbf{r}_i\| - d_i)^2$$

Determine the gradient

$$\nabla(e) = 2\sum_{i=1}^{n} (\|\mathbf{s} - \mathbf{r}_i\| - d_i) \frac{\mathbf{s} - \mathbf{r}_i}{\|\mathbf{s} - \mathbf{r}_i\|}$$

Go downhill

$$\mathbf{r}^{(n+1)} = \mathbf{r}^{(n)} - \gamma \nabla(e)$$

Perhaps better yet, use Newton-Raphson

$$\mathbf{r}^{(n+1)} = \mathbf{r}^{(n)} - H(e)^{-1}\nabla(e)$$

where H(e) is the Hessian matrix of second partial derivatives.

Nature of the measurement "noise"

- Non-Gaussian
- Not "stationary"
- Unknown effects of the environment

Google Wifi 5 GHz band, 80 MHz BW, distances 1, 2, and 3 meters

blue line: slope 1.6

Relative Permittivity of Building Materials

Plasterboard	2.02	
Gypsum	2.41 - 2.60	
Plywood	3.67 - 3.81	
Brick	3.88 - 4.62	
Wood	3.92 - 5.89	
Glass	6.06	
Concrete	7.63 - 9.54	
Tissue	28 - 50	

(*) "Permittivity and Conductivity Measurements of Building Materials at 5.8 GHz and 41.5 GHz," Iñigo Cuiñas and Manuel García Sánchez, *Wireless Personal Communications* **20**: 93-100, 2002.

(*) "Time-Domain Free-Field Measurements of the Relative Permittivity of Building Materials," C.A. Grosvenor, *et al*, *IEEE Transactions On Instrumentation And Measurement*, **58**: 2275, 2009

Horizontal: actual distance (meter). Vertical: measured distance (meter) Red line: slope 1, Green line: slope 1.2 (best fit), Blue line: slope 1.6 Dot color code based on actual RSSI minus predicted RSSI blue = -10dB, green = 0 dB, red = +10 dB Where RSSI prediction is based on actual distance


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Horizontal axis: ratio of measured to true distance



Horizontal axis: ratio of true distance to measured distance Green triangle: observation model for Bayesian update



d 9.616 m s 0.419 m p -90 dBm



1					ľ
					ß

d 7.254 m s 0.204 m p -85 dBm











Bayesian Grid Update

At each time step, measure distances and:

- Apply transition model (random walk), and
- Apply observation model (use Baye's rule)

Bayesian Grid Update

At each time step, measure distances and:

- Apply transition model (random walk), and
- Apply observation model (use Baye's rule)

For single position output, compute:

- Maximum likelihood (peak), or
- Expected value (centroid)



Dilution of Precision (noise gain): $1/\sin(\theta)$



Dilution of Precision (noise gain): $1/\sin(\theta)$



Close up (non linear) case

























Is the location accuracy high enough?

• 10 m - 20 m X • 1 m - 2 m ? • 0.1 m - 0.2 m

Will it "scale"?

- Hundreds at Honolulu Symphony trying to find their seat
- Knock out Google Wifi mesh with FTM RTT "DOS attack"
- Need "passive" "GPS-like" system
- Can be done if APs communicate and broadcast results.
- Unclear if clock synchronization can be accurate enough.

Privacy

- User needs to give app permission
- User needs to enable location services on device
- STA (phone) need not be connected to AP (WiFi)
- AP need not be connected to WLAN (internet)
- MAC address is randomized
- AP cannot determine RTT
- FTM RTT disabled when app is in background
- But: Neighbour Awareness Networking (NAN)?

What is the competition?

- BLE beacons + phased array direction measurement.
- Low cost, low power, small size.
- Not tied into Wifi AP infrastructure
- Will it work?
- Will it work accurately enough?



18,008 WiFi APs (unique global MAC addresses) in Back Bay

SSID The Promised LAN SSID WayFi SSID NoWires SSID Aloooooha 5g SSID Issa not your wifi SSID Stay-Off-Our-Network SSID Mo money No wifi SSID Don't even try! SSID No Free Parking SSID IWillHackYouAsWell SSID BuzzOff SSID Looking for networks... SSID Loading, Please Wait SSID Access Denied SSID OutOfService SSID Biohazard SSID trojan.exe SSID It hurts when IP SSID Mahalo SSID Unconfigured SSID 4 SSID Abandwidthwaitingfor2 SSID Make Wifi Great Again SSID Pretty Fly for a Wi-Fi SSID FBI Surveillance Team Bravo

SSID NSA Guest SSID Abraham Linksys SSID The password is password SSID ClickHereForWifi SSID ClickHereForAGoodTime SSID FlywireGuest SSID 5GSpot SSID HIDE YOUR KIDS HIDE YOUR WIFI SSID OfCourseIBelieveThat SSID Lobstah SSID Paak the Caa SSID Bizarro iFiW SSID We Can Hear You Having Sex! SSID WittyWifiName SSID Les amis des tutus SSID "Yeah, baby" SSID Engineering, Biotech, and Beer SSID DropitLikeaHotspot SSID DissentIsPatriotic SSID husbandishome306 SSID AgNO3 SSID "来啊" SSID "за гости" SSID "💽"



No RTT responses from 99 APs

U	34.31.71.40.JU.UZ	-00	2432	40	Myspectiummerun4-20
1	82:14:29:ed:96:e0	-60	2412	20	Douglas Farrell's Gu
2	50:09:59:73:f6:28	-61	2462	20	Pretty Fly
3	80:29:94:53:e7:0e	-62	2462	20	TC8717T08
4	08:95:2a:15:f5:24	-64	2412	20	UNSULLIED WIFI
5	b0:39:56:b3:6b:39	-64	2447	20	ks wifi pool_22
6	a8:9a:93:9b:ca:b6	-65	2412	20	MySpectrumWiFib0-2G
7	60:d0:2c:22:f5:a9	-65	2437	20	RitzCarlton_RESIDENC
8	20:e5:2a:06:37:43	-66	2457	20	NETGEAR33
9	50:09:59:c2:1a:23	-66	2412	20	black

10 d8:a7:56:67:f8:be 11 60:d0:2c:23:57:69 12 a0:39:ee:60:18:36 13 10:0d:7f:e4:e1:ab 14 20:4e:7f:56:85:14 15 34:6b:46:4e:4c:1a 17 00:19:be:29:07:ff 18 60:d0:2c:63:57:68 19 4c:60:de:2d:82:3e 20 20:e5:2a:17:75:42 21 12:19:be:29:07:ff 22 94:91:7f:48:3c:b3 23 60:d0:2c:a3:57:6c 24 ec:58:ea:54:bd:ec

-67 2412 20 MySpectrumWiFib8-2G -68 2437 20 RitzCarlton_RESIDENC... -68 2437 20 MySpectrumWiFi30-2G -68 2462 20 01110111 01101001 01... -69 2417 20 Keei -69 2437 20 MySpectrumWiFi14-2G 16 6c:b0:ce:ee:44:31 -70 2452 20 NETGEAR02 -71 5745 20 BGS -71 2437 20 -72 2442 20 Kaiolu 403 -73 2442 20 NETGEAR30 -74 5745 20 Waikiki.com -75 5200 80 MySpectrumWiFib4-5G -75 5825 20 -76 5825 20

25 ec:58:ea:94:bd:ec -76 5825 20 26 ec:58:ea:d4:bd:ec -76 5825 20 27 ec:58:ea:14:bd:ed -76 5825 20 RitzCarlton RESIDENC... 28 60:d0:2c:63:57:6c -76 5825 20 -76 5825 20 RitzCarlton RESIDENC... 29 60:d0:2c:23:57:6d 30 00:19:be:29:08:07 -76 5745 20 BGS 31 60:d0:2c:e3:57:6c -76 5825 20 32 60:fe:20:dc:91:32 -76 2427 20 HT066 33 12:19:be:29:08:07 -77 5745 20 Waikiki.com 34 20:e5:2a:06:37:42 -77 5765 40 NETGEAR33-5G GPS 21.282213 -157.828692 alt 33 acc 6

No RTT responses from 99 APs



SSID	01110111 01101001 01100110 01101
SSID	0ooSzechuanSauce
SSID	5Ghz Imperial Defence Network
SSID	56k_fax_modem
SSID	=^^=
SSID	AdventurousShark
SSID	aloha.aloha.aloha
SSID	Audi_MMI_4040
SSID	catfood 5GHz
SSID	Central Perk-5G
SSID	Da Hi Life-5G
SSID	Dakine Wifi
SSID	Drama Queen Guest
SSID	Drybar Guests
SSID	Edo
SSID	EMPTEE
SSID	Exhale Hawaii
SSID	FBI Surveillance Van
SSID	Fish Head Jar
SSID	Found Residences Waikiki
SSID	Free Wifi
SSID	GallowsDancer
SSID	GetYourOwn
SSID	goHawaii Free WiFi
SSID	Hang10

SSID Housekeeping SSID Ice Monster Hawaii Guest SSID ISeeYou!!! SSID IU Sucks SSID Jack's Wi-Fi Network SSID Kapu SSID Koi Pond SSID Kokua SSID Mohamed's Wi-Fi Network SSID My WiFi SSID NanaNene SSID RedDirt SSID Roosterhome SSID Sandworms Lair SSID Seaweed SSID Standard Parking SSID Tesla Guest SSID Trina Turk IM SSID Universal SSID Waikiki SSID WhiteDragonHorseInDaHouse SSID 小倉理紗のiPhone SSID 海賊王ですw_5GHz SSID 경찰청 (Korean Police) SSID whawaii.thai_net

- Broadcom 802.11ac Acculocate Access Point
- Intel® Dual Band Wireless-AC 8260
- Marvell AP-8964 802.11ac 4x4 Wave2 Concurrent Dual Band Access Point
- Mediatek MT663X 802.11abgn/ac Ref. STA
- Qualcomm IPQ4018 802.11ac 2-stream Dual-band, Dual-concurrent Router
- Qualcomm IPQ8065 802.11ac 4-stream Dual-band, Dual-concurrent Router
- Qualcomm Snapdragon™ 820 Development Kit
- Realtek RTL8812BU



236 FTM RTT (802.11mc) responders in Back Bay

WiFi Mesh Systems

- eero home WiFi system (Tri-band mesh WiFi)
- Google Wifi AC1200
- Netgear Orbi AC2200 (Tri-band mesh WiFi)
- TP-Link Deco M5 Whole Home Mesh WiFi System ?
- Linksys Velop AC2200 (Tri-band Wifi Mesh System)?
- Ubiquiti AmpliFi HD WiFi System ?
- Luma Whole Home Wifi (2 pack) ?
- ASUS Lyra Trio AC1750 (Mesh WiFi System)?
- HTC Managed Wi-Fi Service with Mesh Wifi ?

• . . .





No RTT responses from 55 APs

Use Wi-Fi

- $\{34.10.09./4.39.01=2402,-$ 66,4s} {34:fc:b9:77:0e: 81=2437,-57,4s};(11)max=-65, {34:fc:b9:73:d6:f1=5300,-

87,10s} {40:e3:d6:19:e0:b1=5660,-89,5s} {34:fc:b9:77:0e:

UHM [(3)]{34:fc:b9:73:d6:e0=2437,-86,9s} {34:fc:b9:74:59:e0=2462,-66,4s} {34:fc:b9:77:0e: 80=2437,-58,4s};(9)max=-68, {40:e3:d6:19:e0:b0=5660,-89,5s} {34:fc:b9:76:aa:



DIRECT-CB-HP OfficeJet Pro 8710

No RTT responses from 55 APs

/5,58};]







suo sponte

suo motu

How far along are we?

- Boston Back Bay (July 2018) $236/18008 \approx 1.3\%$
- Waikiki (January 2019) 37/1275 ≈ 2.9%
- Google WiFi (January 2019) turned on RTT bit in beacon frame

13:58 81° ¹ ¹

<u>98%</u>



& Wifi points

Everything looks good and 3

Wifi points are online.



Your Google Wifi just got better

Google Wifi installed an update at 1/14/2019, 1:12 PM HST. Here's what's new:

- IPv6: Support for devices on the guest

- network.
- IPv6 port opening.
- General stability & performance

improvements.

DISMISS **MORE DETAILS**







Bacon-Concannon Associates

Alloyar Pouner Fed Public Alley 422

Boston Public Garden Two Common Avenue

Four S

Hotel

Nordstrom Rack Newbury

ParkP A Boston Park Plaza St James AV Plaza

aubic NAtilingtone

Grill 23 & Bar

Public Alley A37

Fairmont Copley Plaza, Boston Stuart

Club Café

Isabella St

Cortes St

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- Dilution of Precision (DOP)
- Non-Gaussian, non-stationary error
- Best spatial arrangement for responders
- Some issues, problems, and currrent limitations
- When will it happen?

15:51 🔤 🥶 28° 🎯

FTMRTT

Continue WiFi Scanning 🗌

- 0 b8:08:cf:a0:8b
- 1 b8:08:cf:a0:88
- 2 b8:08:cf:a0:7a
- 3 b8:08:cf:a0:7a
- 4 b8:08:cf:a0:7a
- 5 b8:08:cf:a0:7a
- 6 Oc:9d:92:b9:8f
- 7 38:8b:59:c4:f0
- 8 38:8b:59:c4:f3
- 9 38:8b:59:c4:f7

Range After Scan

Force Ranging

Write Log File







X:

Get GPS Locations

Get Vendors from MAC

Run WiFi Scan Service

Use Fixed List of APs



Correct Range



Show Floor Plans Keeep On Ranging Signal Strength Tweak Verbose Logging



Linear Equations for Four Transponders

With 4 responders we get 6 linear equations. We can chose 3 that are linearly independent:

$$\begin{pmatrix} (\mathbf{r}_0 - \mathbf{r}_1)^T \\ (\mathbf{r}_1 - \mathbf{r}_2)^T \\ (\mathbf{r}_2 - \mathbf{r}_3)^T \end{pmatrix} \mathbf{s} = \mathbf{t}$$

where
$$\mathbf{t} = (t_{0,1}, t_{1,2}, t_{2,3})^T$$
 and $t_{i,j} = ((d_j^2 - d_i^2) - (R_j^2 - R_i^2))/2$.

Can solve 3 linear equations in 3 unknown components of **s** (provided \mathbf{r}_0 , \mathbf{r}_1 , \mathbf{r}_2 , and \mathbf{r}_3 are not coplanar).

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