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Well-defined steady-state response does not imply CICS

Eugene P. Ryan[a], Eduardo D. Sontag[b,*]

[a]Department of Mathematical Sciences, University of Bath, Bath BA2 7AY, UK
[b]Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA

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Abstract

Systems for which each constant input gives rise to a unique globally attracting equilibrium are considered. A counterexample is provided to show that inputs which are only asymptotically constant may not result in states converging to equilibria (failure of the converging-input converging state, or "CICS" property).

Keywords: Step response; Global attractivity; Homoclinic orbits; Asymptotically autonomous systems

1. Introduction

Consider a controlled finite dimensional system

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

under suitable regularity assumptions, and assume that the following property holds: for each constant input $u \equiv a$, there is a unique steady-state x_a (that is, $f(x, a) = 0$ has the unique solution $x = x_a$), and every solution of the system $\dot{x} = f(x, a)$ converges to this state x_a. In the terminology of [1–3], we say that the system has a "characteristic" or a *monostable steady-state step response*.

Given that (1) admits a characteristic, it is natural to ask if the following *converging-input converging-state* (*CICS*) property must then also hold: for every convergent input $u(\cdot)$ (that is to say, $u(t) \to a$ as $t \to \infty$, for some value a), every bounded solution of $\dot{x} = f(x, u)$ converges to x_a.

Such a property is especially interesting when studying cascades of systems, in which the input u to the system being studied is itself the output of another system. That is, there is another system $\dot{z} = g(z, v)$, $u = h(z)$, and v is an external input to the cascade. In that context, one would like to know whether each of the f and g systems having a characteristic implies that the cascade also does. Suppose that $v \equiv a$. If the g system has a monostable response, its state converges to some value: $z(t) \to z_a$, so that also, assuming continuity of the read-out map h, $u(t) \to b := h(z_a)$. If the CICS property holds for the f subsystem (and assuming that its trajectories are bounded), then $x(t) \to x_b$, and therefore the complete state $(z(t), x(t))$ converges to (z_a, x_b), establishing that the cascade also admits a characteristic.

These questions have a long history in control as well as in dynamical systems theory, see for example the early work of Markus [6], and are closely related to the topic of "asymptotically autonomous" systems, see for example [4] Appendix A (by Z. Artstein). The latter are time-varying systems $\dot{x} = F(x, t)$ for which $F(x, t) \to F_0(x)$ as $t \to \infty$, for some time-invariant vector field F_0, where the convergence is assumed to hold in an appropriate technical sense. Clearly, one may view $f(x, u(t))$, for any fixed given input $u(\cdot)$, as a time-varying vector field $F(x, t)$, and, if $u(t) \to a$ as $t \to \infty$, one may define $F_0(x) := f(x, a)$; in this manner, "$u(t) \to a$" translates into "$F(x, t) \to F_0(x)$," and the questions addressed here amount to relating the behaviour of solutions of $\dot{x} = F(x, t)$ to that of solutions of the limit system $\dot{x} = F_0(x)$. For other related work, see for example [7–9,11,12,5].

There are several known sufficient conditions that guarantee the CICS property for systems which admit characteristics. One such condition is *stability* of the equilibria x_a. That is, not only

* Corresponding author. Tel.: +1 732 445 3072; fax: +1 915 975 8674.
E-mail addresses: epr@maths.bath.ac.uk (E.P. Ryan), sontag@math.rutgers.edu (E.D. Sontag).

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do trajectories of $\dot{x} = f(x, a)$ approach x_a as $t \to \infty$, but the "small excursion" Lyapunov stability condition holds as well: for each neighbourhood U of x_a, there is another neighbourhood V such that solutions starting in V do not exit U (later in this paper we discuss a weaker condition, which is implied by but does not imply the stability condition). The conjunction of stability and global attractivity of x_a is, of course, equivalent to global asymptotic stability of x_a under which condition the CICS property is a particular consequence of Theorem 2 in [6]. A different condition ensuring the CICS property is that of *monotonicity*: the conclusions hold provided that the system is monotone as an input/output system in the sense of [2]; the paper [2] made stability into part of the definition of characteristic, but [3] showed one need not assume stability in order to conclude the CICS property.

In view of these different sufficient conditions, it is natural to ask if it is always true that the CICS property holds for systems with characteristics. The main goal of this note is to provide a negative answer to that question by means of a counterexample. The counterexample is two-dimensional (one-dimensional systems are always monotone, so no one-dimensional counterexamples could exist) and quite explicit. The construction is provided in the next section. For completeness, in the last section we review a simple criterion which guarantees the CICS property for systems with characteristics.

2. The counterexample

It is well known that large-time behaviour of solutions of an asymptotically autonomous system can differ markedly from that of solutions of its autonomous limit system: examples can be found in [11,12]. In the context of the present note, the property that, for every constant input, there should exist a unique attracting state is a distinguishing feature that adds subtlety to the planar counterexample, the construction of which can be summarised as follows.

First, we determine a locally Lipschitz function $f : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ such that, for every constant $a \in \mathbb{R}$, $(0, 1)$ is a globally attractive equilibrium of the autonomous system $\dot{x} = f(x, a)$. For each $r^0 > 1$, we then proceed to construct an input u, with $u(t) \to 0$ as $t \to \infty$, such that, for all initial data $x^0 \in \mathbb{R}^2$ with Euclidean norm $|x^0| = r^0$, the solution of the initial-value problem $\dot{x} = f(x, u)$, $x(0) = x^0$, is bounded and has the unit circle S^1 as its ω-limit set.

In order to define our system, we start by introducing an auxiliary function. Let $h : \mathbb{R} \to [0, \infty)$ be any smooth function such that $h(y) = 1$ for all $y \in [1, 2]$ and $h(y) = 0$ for all $y \notin [\frac{1}{2}, 4]$, and let $g : [0, \infty) \times \mathbb{R} \to [0, \infty)$ be given by

$$g(r, u) := \begin{cases} u^p h((r-1)/u), & u > 0, \\ 0, & u \leqslant 0, \end{cases} \tag{2}$$

with constant $p > 2$. We remark that g is a locally Lipschitz function. To see this, note that g is smooth on each of $[0, \infty) \times (0, \infty)$ and $[0, \infty) \times (-\infty, 0)$, and is continuous on all of $[0, \infty) \times \mathbb{R}$. Let k be an upper bound on h and $|h'|$ (such a bound exists since $h(y) = 0 = h'(y)$ for all $y \notin [\frac{1}{2}, 4]$). Then, for all (r, u) with $0 < |u| \leqslant 1$, we have $|\partial g/\partial r(r, u)| \leqslant k$, $|\partial g/\partial u(r, u)| \leqslant (p + 4)k$, and $|\nabla g(r, u)| \leqslant k\sqrt{1 + (p + 4)^2}$, which implies that g is uniformly Lipschitz around $u = 0$.

Consider the system on $\mathbb{R}^2$ (with Euclidean norm $|\cdot|$)

$$\dot{x} = f(x, u),$$

with $f = (f_1, f_2) : \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}^2$ given by

$$f_1(x, u) := \begin{cases} -x_1(|x| - 1)^{p+1}/|x| \\ \quad -2x_2(1 - (x_1/|x|)) \\ \quad -x_2 g(|x|, u), & |x| \geqslant 1, \\ 2(x_1 - 1)x_2, & |x| < 1, \end{cases}$$

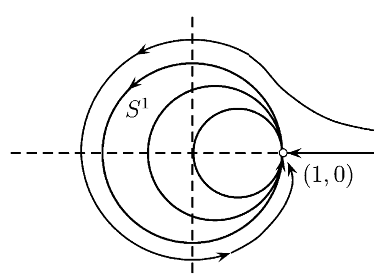
$$f_2(x, u) := \begin{cases} -x_2(|x| - 1)^{p+1}/|x| \\ \quad +2x_1(1 - (x_1/|x|)) \\ \quad +x_1 g(|x|, u), & |x| \geqslant 1, \\ -(x_1 - 1)^2 + x_2^2, & |x| < 1 \end{cases}$$

(where $x = (x_1, x_2)$). We claim that f is locally Lipschitz. Since g is locally Lipschitz, to prove the claim it suffices to show that the u-independent vector field F given by $F(x) := f(x, u) - (-x_2, x_1)g(|x|, u)$ is locally Lipschitz. Let S^1 denote the unit circle centred at 0 in $\mathbb{R}^2$. Observe that F is continuous at all points $(x, u) \in S^1 \times \mathbb{R}$ and continuously differentiable on $(\mathbb{R}^2 \backslash S^1) \times \mathbb{R}$ with bounded derivative on $(K_1 \backslash S^1) \times K_2$ for every compact neighbourhood K_1 of S^1 and every compact $K_2 \subset \mathbb{R}$. It follows that F is locally Lipschitz.

With zero input $u = 0$ ($a = 0$ in the notation in the Introduction), the system has a globally attractive (but not stable) equilibrium at $x_a = (1, 0)$, as will follow from a more general result shown below for arbitrary constant inputs. In particular, $S^1 \backslash \{(1, 0)\}$ is a homoclinic connection of $(1, 0)$ to itself. More generally, the collection of punctured circles

$$\{x | \; |x - (b, 0)| = 1 - b\} \backslash \{(1, 0)\}, \quad 0 \leqslant b < 1,$$

constitutes a family of such homoclinic connections filling the unit disc. For all inputs u, this family of homoclinic connections persists (as the vector field on the closed unit disc coincides with the zero input case).



Exterior to the open unit disc, the system representation, in polar coordinates, is given by

$$\dot{r} = -(r - 1)^{p+1}, \quad \dot{\theta} = 4\sin^2(\theta/2) + g(r, u). \tag{3}$$

Therefore, in view of (2), for every *constant* input $u = a \neq 0$ the vector field differs from the zero-input case only when $a > 0$ and then only on the annulus $\{(r, \theta)|a/2 < r - 1 < 4a, 0 \leqslant \theta < 2\pi\}$. Since $r(\cdot)$ is strictly decreasing (with limit 1) along all solutions exterior to the closed unit disc, for $u = a > 0$ we have $r(t) - 1 \leqslant a/2$ for all t sufficiently large and so the qualitative behaviour of the above figure is ultimately exhibited. We may now conclude that, for every constant input, $(1, 0)$ is a globally attractive equilibrium.

Fix $r^0 > 1$ arbitrarily and write $c := 2(r^0 - 1)$. The following construction yields a bounded input u, with $u(t) \to 0$ as $t \to \infty$ (and $u \in L^q([0, \infty))$ for all $q > p$), such that, with initial data $(r(0), \theta(0)) = (r^0, \theta^0)$, $0 \leqslant \theta^0 < 2\pi$, the corresponding solution of (3) has S^1 as its ω-limit set.

The initial-value problem $\dot{r} = -(r - 1)^{p+1}$, $r(0) = r^0$, has unique solution

$$t \mapsto r(t) = 1 + \frac{r^0 - 1}{(1 + p(r^0 - 1)^p t)^{1/p}} \tag{4}$$

(strictly decreasing and converging to 1 as $t \to \infty$). Observe that (by (4)), if $r(t) = 1 + (c/2^n)$, then $r(t + \tau_n) = 1 + (c/2^{n+1})$, where

$$\tau_n = \frac{2^{np}(2^p - 1)}{p\, c^p} > \frac{2^{np}}{c^p} \quad \forall n \in \mathbb{N}.$$

Define the sequence (t_n) by

$$r(t_n) = 1 + (c/2^n) \quad \forall n \in \mathbb{N}.$$

Then, $t_1 = 0$ and, by the previous observation,

$$t_{n+1} - t_n = \tau_n \quad \forall n \in \mathbb{N}.$$

Define the function $u : [0, \infty) \to (0, \infty)$ by the property

$$n \in \mathbb{N}, \quad t \in [t_n, t_{n+1}) \;\Rightarrow\; u(t) = \frac{c}{2^{n+1}}.$$

Let (r, θ) be the solution (on $[0, \infty)$) of (3) with input u and initial data $(r(0), \theta(0)) = (r^0, \theta^0)$, $0 \leqslant \theta^0 < 2\pi$. For all $n \in \mathbb{N}$, $(r(t) - 1)/u(t) \in (1, 2]$ for all $t \in [t_n, t_{n+1})$ and so

$$g(r(t), u(t)) = u^p(t) = \frac{c^p}{2^{(n+1)p}} \quad \forall t \in [t_n, t_{n+1}).$$

Since

$$\dot{\theta}(t) = 4\sin^2(\theta(t)/2) + g(r(t), u(t)) \geqslant g(r(t), u(t)) \quad \forall t \geqslant 0,$$

it follows that

$$\theta(t_{n+1}) - \theta(t_n) \geqslant \frac{\tau_n c^p}{2^{(n+1)p}} > \frac{2^{np}}{2^{(n+1)p}} = \frac{1}{2^p} \quad \forall n \in \mathbb{N}.$$

Therefore, the increasing function θ is unbounded. Since $r(t) \to 1$ as $t \to \infty$, we may infer that S^1 is the ω-limit set of the solution (r, θ).

3. A sufficient condition

In this section, we provide a sufficient condition—which, obviously, is violated in our counterexample—which guarantees the CICS property when characteristics exist. Our condition is not original; indeed, it may be viewed as a consequence of a more general result given by Thieme, cf. [11]. Nonetheless, it seems appropriate to provide it here, since it is much more elementary, and easier to state and check, than the general result in [11], and we can provide a simple self-contained proof.

We assume given a system (1), whose states $x(t)$ evolve on Euclidean space $\mathbb{R}^n$, for some $n \in \mathbb{N}$, and whose inputs take values $u(t)$ on some (locally compact) metric space U. We assume that $f : \mathbb{R}^n \times U \to \mathbb{R}^n$ is continuous and is locally Lipschitz on x uniformly on compact subsets of U. By an *input* we mean a locally essentially bounded Lebesgue-measurable function $u : [0, \infty) \to U$. Under these conditions, one knows (see for instance [10, Chapter 2]) that, for any initial state $x^0 \in \mathbb{R}^n$ and input u, there exists a unique solution of (1) with $x(0) = x^0$, maximally defined on some interval $[0, \sigma_{x^0,u})$, which we denote by $x(\cdot) = \varphi(\cdot, x^0, u)$; we write

$$S(x^0) := \{\varphi(t, x^0, u)|t \in [0, \sigma_{x^0,u})\}$$

for the forward orbit of x^0 under the input u.

Fix a $u : [0, \infty) \to U$ which converges: $u(t) \to a$ as $t \to \infty$, where $a \in U$ is so that $f(x_a, a) = 0$ for some $x_a \in \mathbb{R}^n$. For notational simplicity and without loss of generality, we may assume $x_a = 0$. We will use the notation $|\mu|$ to indicate the distance $\mathrm{dist}(\mu, a)$ from any $\mu \in U$ to a, and also for norm in $\mathbb{R}^n$, and write u_a for the constant input $u_a(t) \equiv a$. We write $g(x) = f(x, a)$ and let φ_a denote the flow generated by $\dot{x} = g(x)$ (and so $\varphi_a(\cdot, x^0)$ is the maximal solution, forwards and backwards in time, of $\dot{x} = g(x) = f(x, a)$ with initial state x^0, and, for $t \geqslant 0$, $\varphi_a(t, x^0) = \varphi(t, x^0, u_a)$).

For any given $x^0 \in \mathbb{R}^n$ for which $S(x^0)$ is bounded (in which case, $\sigma_{x^0,u} = +\infty$), we consider the ω-limit set of the trajectory of $\dot{x} = f(x, u)$ starting from x^0:

$$\Omega(x^0) = \{\xi|\varphi(t_j, x^0, u) \to \xi \text{ for some sequence } t_j \to \infty\}.$$

This is a nonempty compact set, and $\varphi(t, x^0, u) \to \Omega(x^0)$ as $t \to \infty$, because of the boundedness assumption. The following useful fact is as in Markus [6].

Lemma 3.1. *If $S(x^0)$ is bounded, then the set $\Omega(x^0)$ is invariant (both forward and backward in time) under the flow φ_a of $\dot{x} = g(x)$.*

Proof. Pick any point $\xi \in \Omega(x^0)$, and a sequence $t_j \to \infty$ so that $\varphi(t_j, x^0, u) \to \xi$. Pick any $T \in \mathbb{R}$ for which $\zeta := \varphi_a(T, \xi)$ is defined. We will prove that $\zeta \in \Omega(x^0)$ (which will imply that $\varphi_a(T, \xi)$ is defined *for all* $T \in \mathbb{R}$, because the maximal solution is defined globally once that it is known that it remains inside a compact set) and the invariance result follows. So, we need to show that there is a sequence $T_j \to \infty$ such that $\varphi(T_j, x^0, u) \to \zeta$, that is, for any $\varepsilon > 0$, we need to find a J such that $|\varphi(T_j, x^0, u) - \zeta| < \varepsilon$ for all $j > J$. Let $\varepsilon > 0$ be arbitrary. By continuity of $\varphi(T, \cdot, \cdot)$ with respect to states and uniform norms on inputs (see e.g. [10, Theorem 1]), and since $\zeta := \varphi(T, \xi, u_a)$, we know that there is some $\delta > 0$ so that $|\varphi(T, x, v) - \zeta| < \varepsilon$ whenever $|x - \xi| < \delta$ and $t \mapsto v(t) \in U$ is so that $|v(t)| < \delta$ for all t. Pick $J \in \mathbb{N}$ so that, for all $j > J$, $T_j := t_j + T \geqslant 0$, $|\varphi(t_j, x^0, u) - \xi| < \delta$ and $|u(t + t_j)| < \delta$ for all

$t \geqslant 0$. Noting that $\varphi(T_j, x^0, u) = \varphi(T, x, v)$ with $x = \varphi(t_j, x^0, u)$ and $v(\cdot) = u(\cdot + t_j)$, it follows that $|\varphi(T_j, x^0, u) - \zeta| < \varepsilon$ for all $j > J$. □

Now, convergence $\varphi(t, x^0, u) \to 0$ (as $t \to \infty$) is the same as asking that $\Omega(x^0) = \{0\}$ which, in conjunction with Lemma 3.1, yields the following condition that guarantees the CICS property.

Corollary 3.2. *Assume that $\dot{x} = g(x)$ admits no compact invariant set different from $\{0\}$. Then, for each $x^0 \in \mathbb{R}^n$ for which $S(x^0)$ is bounded, $\varphi(t, x^0, u) \to 0$ as $t \to \infty$.*

There is an equivalent way to state this sufficient condition, which can be interpreted as ruling out homoclinic orbits in a generalised sense. Recall that the α-limit set $A(x^0)$ of the solution $\varphi_a(\cdot, x^0)$ of $\dot{x} = g(x)$, $x(0) = x^0$, consists of all points ζ such that $\varphi_a(t_j, x^0) \to \zeta$ for some sequence $t_j \to -\infty$, and, provided that the solution is bounded, this is a compact invariant set to which $\varphi_a(t, x^0)$ converges as $t \to -\infty$.

Proposition 3.3. *Assume that the system $\dot{x} = g(x)$ has the property that, for each $x^0 \in \mathbb{R}^n$, the solution $t \mapsto \varphi_a(t, x^0)$ is globally defined and converges to 0 as $t \to \infty$. Then, the following two properties are equivalent:*

- *There is a no nontrivial (different from $\{0\}$) compact invariant set.*
- *There is no nontrivial bounded orbit for which 0 is an α-limit point.*

Proof. Assume that there is no nontrivial compact invariant set, and suppose that there exists an $x^0 \neq 0$ with bounded orbit for which $0 \in A(x^0)$. As $A(x^0)$ is a compact invariant set, $A(x^0) = \{0\}$. Therefore, $\varphi_a(t, x^0) \to 0$ as $t \to -\infty$, and we also have that $\varphi_a(t, x^0) \to 0$ as $t \to +\infty$. Therefore, the set C which is made up of the orbit of x^0 together with the origin is a compact invariant set, and is nontrivial since $x^0 \in C$. This gives a contradiction.

Conversely, assume that there is no nontrivial bounded orbit for which 0 is an α-limit point, and suppose that there is a nontrivial compact invariant set C. Pick any nonzero $x^0 \in C$. The orbit of x^0 is a subset of C, so it is bounded; thus $A(x^0)$ is compact and invariant. Pick any $\zeta \in A(x^0)$. Since $\varphi_a(t, \zeta) \to 0$ as $t \to \infty$ and $A(x^0)$ is a closed invariant set, it follows that $0 \in A(x^0)$. So the orbit of x^0 is nontrivial and has 0 as an α-limit point. This gives a contradiction. □

Next, we remark on how these conditions relate to the far deeper results in Thieme's work [11] (see also [12,7]). Restricted to our context (Thieme deals with more complex attractors as well), Corollary 4.3 of [11] says that the statement "$\varphi(t, x^0, u) \to 0$ as $t \to \infty$ for each $x^0 \in \mathbb{R}^n$ for which $S(x^0)$ is bounded" may be concluded from the conjunction of the following two properties, for $\dot{x} = g(x)$: (a) there are no nontrivial homoclinic orbits and (b) the set $\{0\}$ is an isolated compact invariant set. Since the closure of a homoclinic orbit is compact and invariant, (a) would follow from the stronger assumption that there are no nontrivial compact invariant subsets and (b) also follows from this same stronger assumption.

Finally, we re-interpret the above results in the specific context of systems with characteristics. Assume that f is so that (1) admits a characteristic. Then, for each constant input $u \equiv a \in U$, there is a unique steady-state x_a and every solution of $\dot{x} = f(x, a)$ converges to x_a as $t \to \infty$. The final proposition is now an easy consequence of Corollary 3.2 and Proposition 3.3.

Proposition 3.4. *The following properties are equivalent:*

- *for all $a \in U$, $\{x_a\}$ is the only non empty compact set that is invariant under $\dot{x} = f(x, a)$,*
- *for all $a \in U$, x_a is not an α-limit point of any bounded orbit of $\dot{x} = f(x, a)$ other than the equilibrium orbit $\{x_a\}$,*

and each implies that (1) has the CICS property.

Observe that, if x_a is a Lyapunov stable equilibrium of $\dot{x} = f(x, a)$ for all $a \in U$, then the second (and hence also the first) of the above properties holds. Therefore, the fact that stability of all equilibria x_a is sufficient to conclude the CICS property for systems with characteristics is a particular consequence of Proposition 3.4.

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