Iterative Quantization

Using Codes On Graphs

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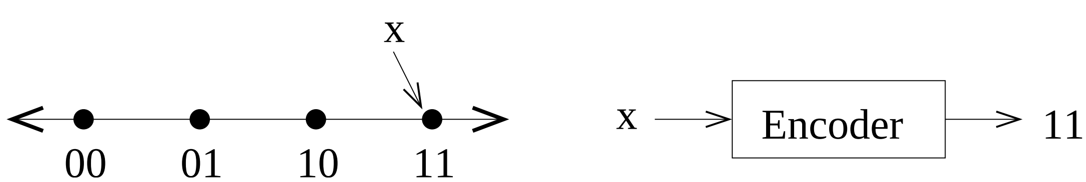
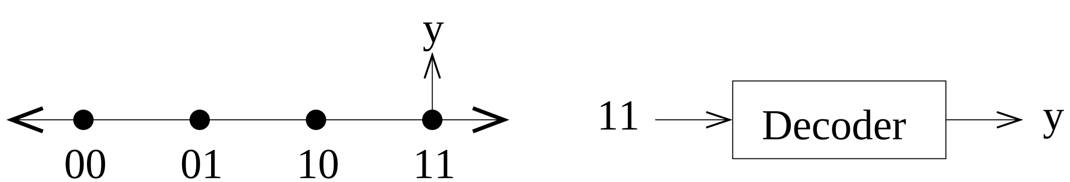
Lossy Data Compression:

- Encoding: Map source $\vec{x} = [x_1\ x_2\ ...\ x_n]$ to bits.



- Decoding: Map bits to $\vec{y} = [y_1\ y_2\ ...\ y_n]$.



- Goal: Construct mapping so that $\vec{y}$ "near" $\vec{x}$.

Lossy Compression Performance:

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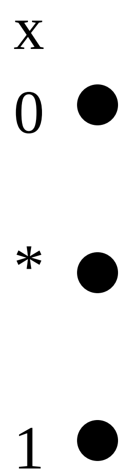
How do we approach $R(D)$ with low complexity?

Compressing a source with "erasures"



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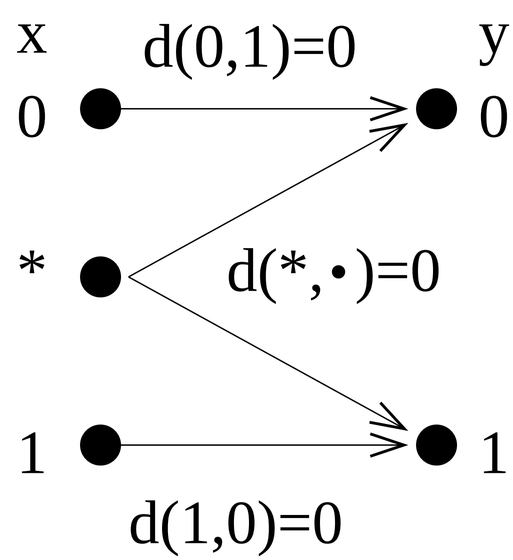

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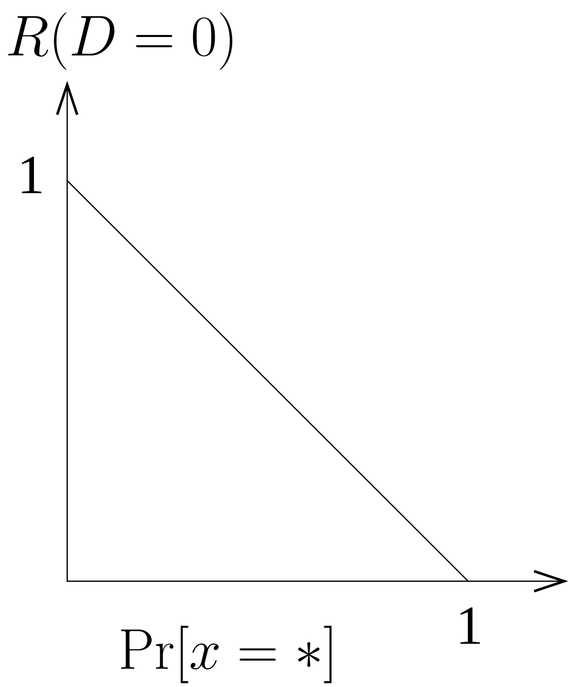
Quantizing $0 \to 1$ or $1 \to 0$ causes non-zero distortion

Performance for erasures models



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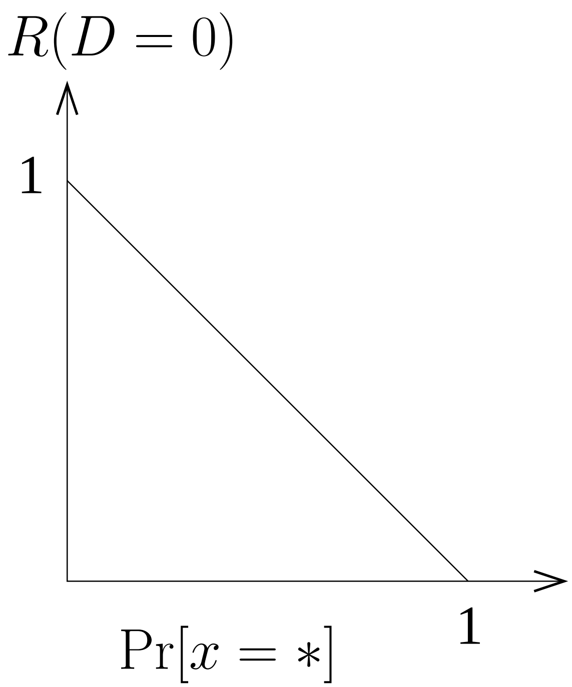
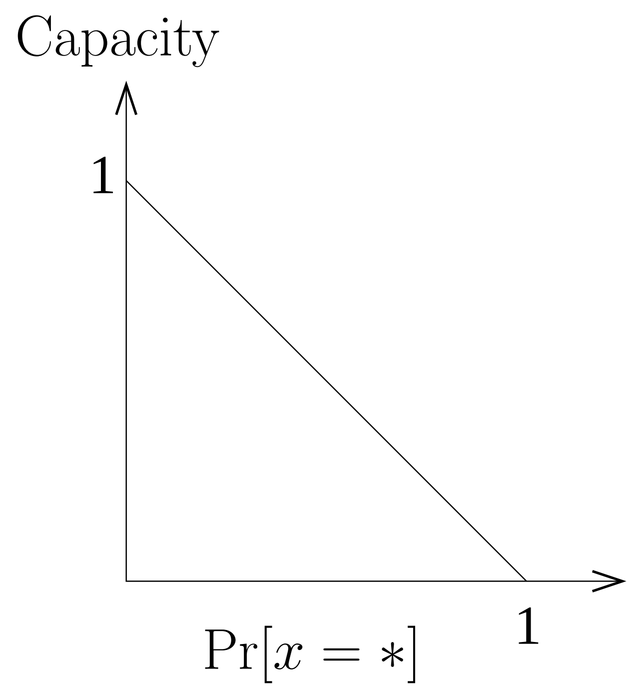
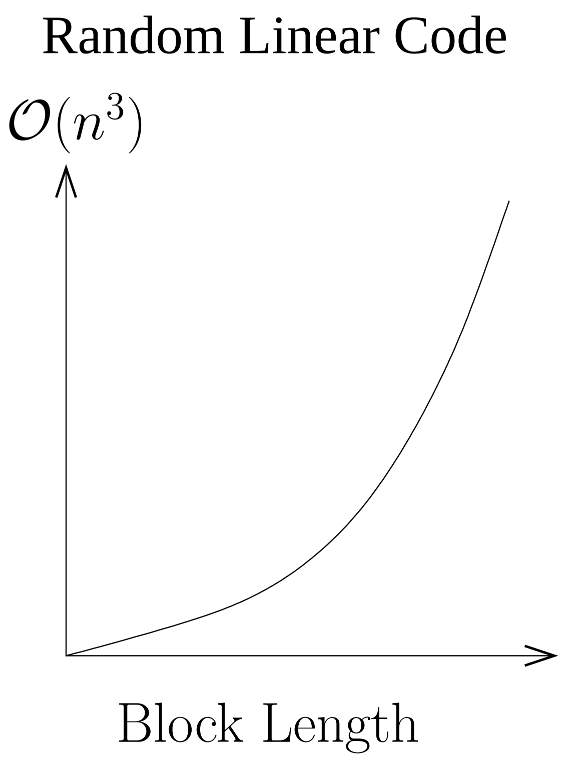




Coding Complexity



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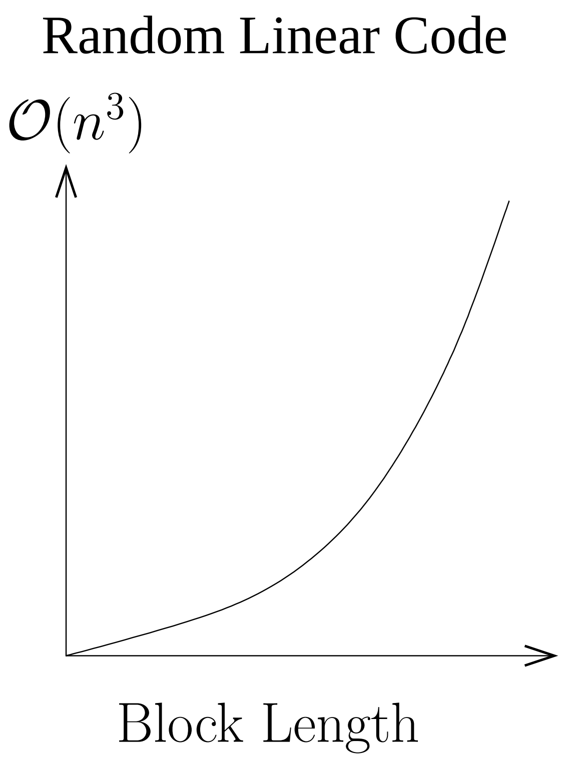
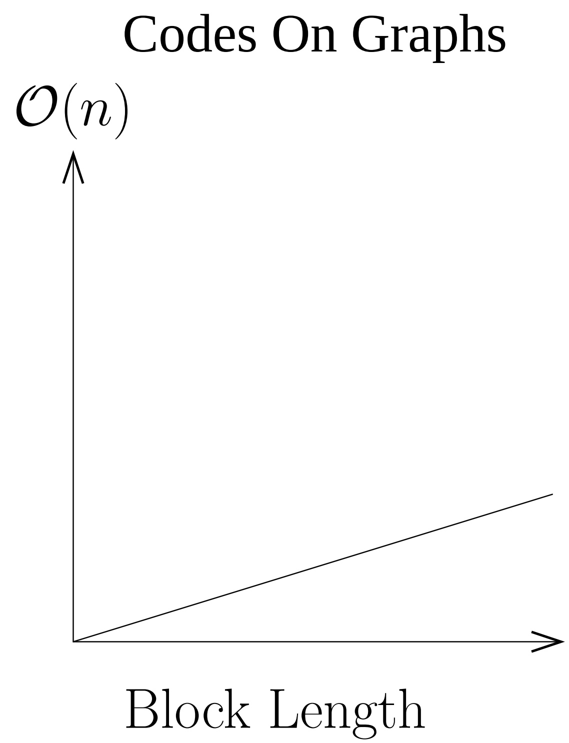
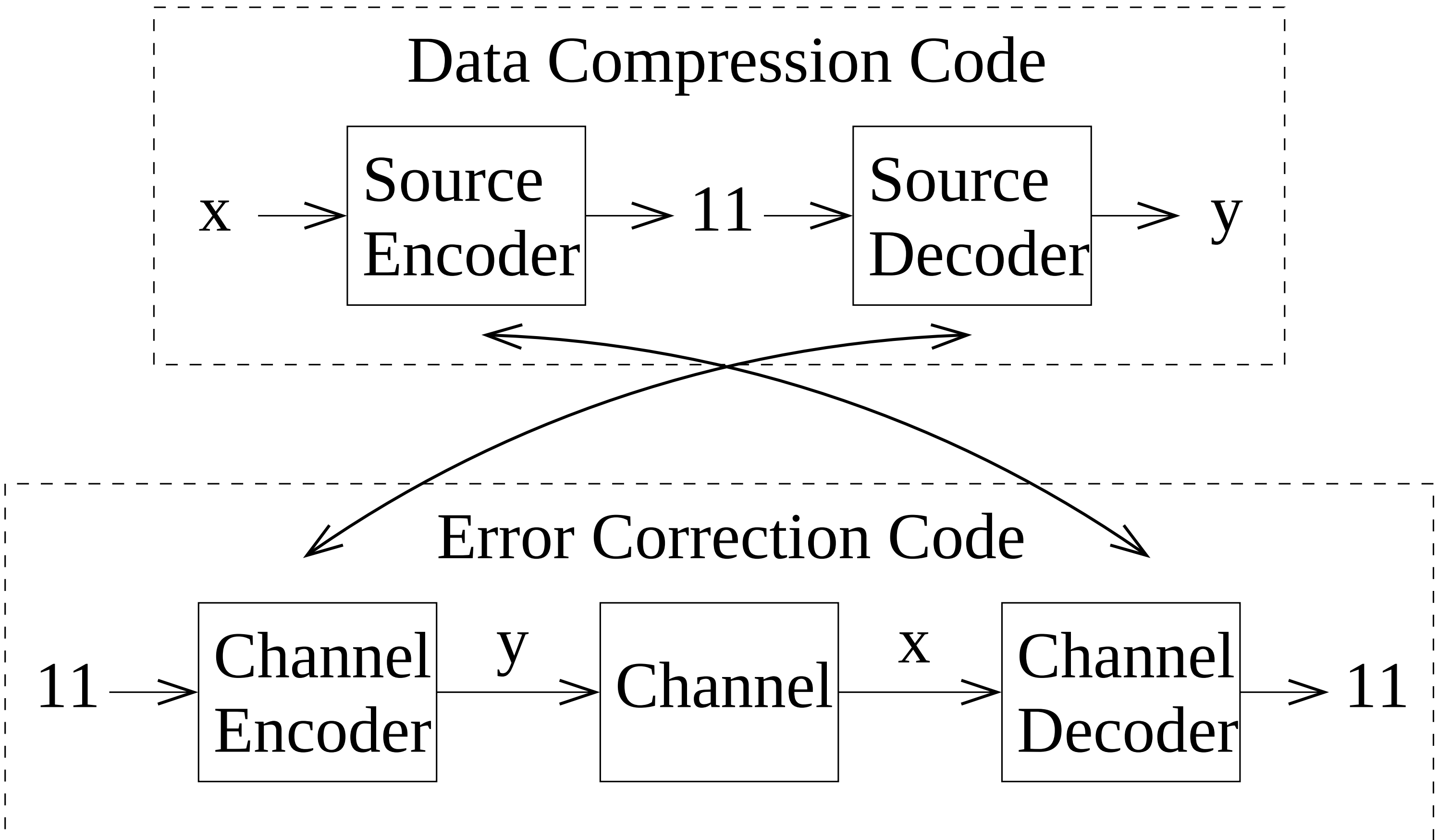
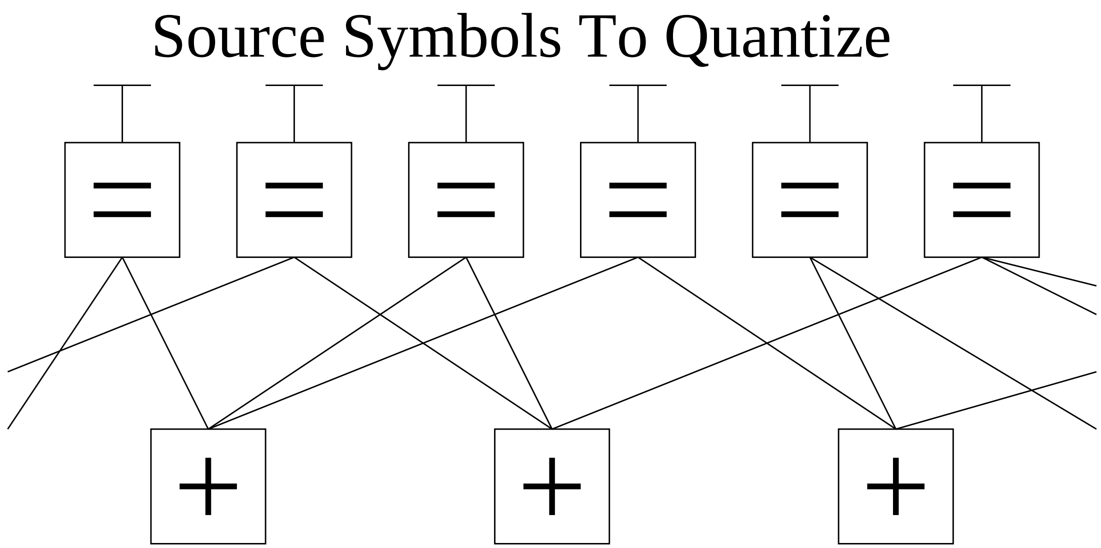




Data Compression ↔ Error Correction Duality



An LDPC Quantizer



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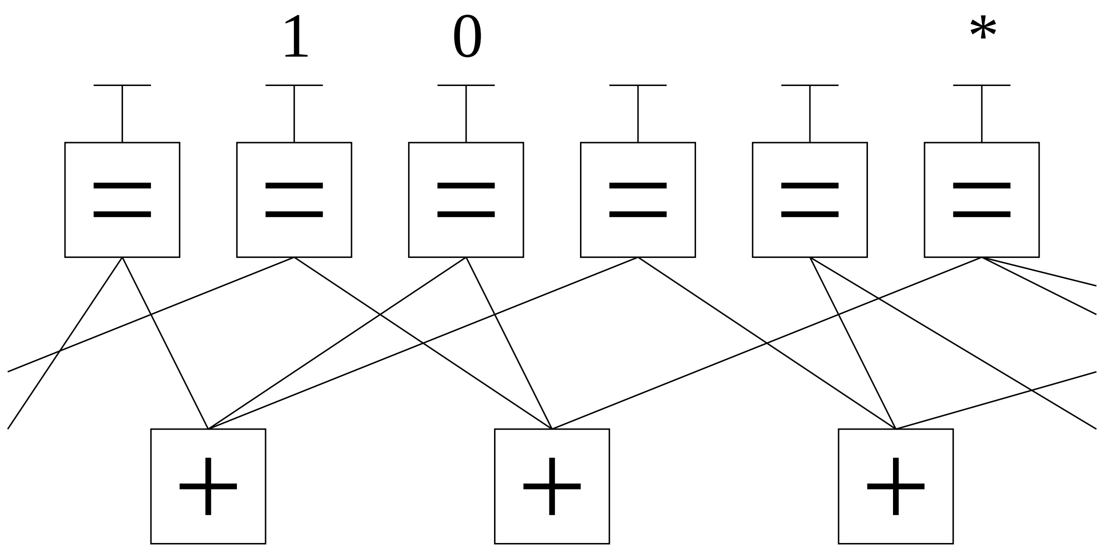

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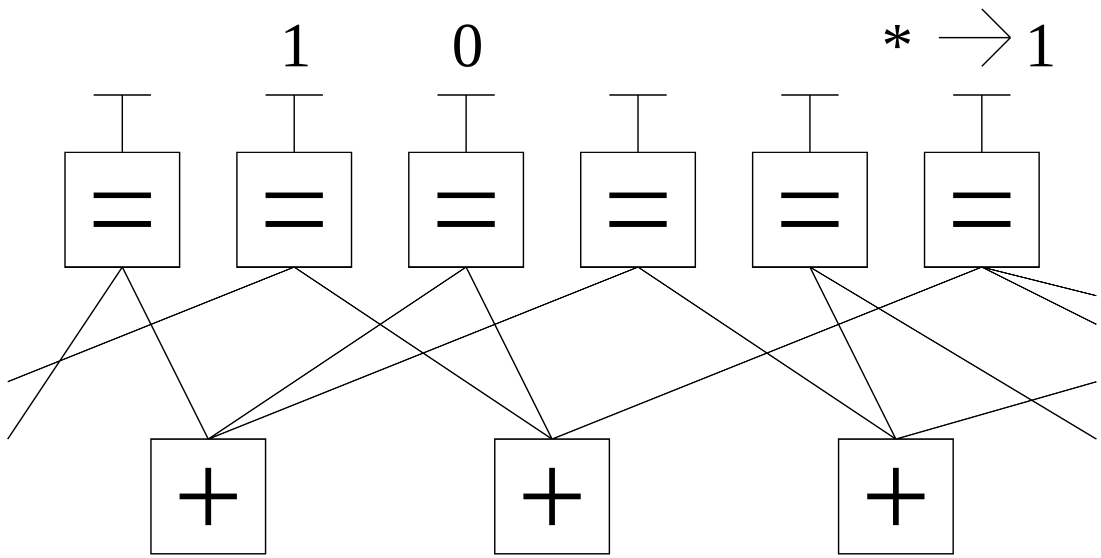

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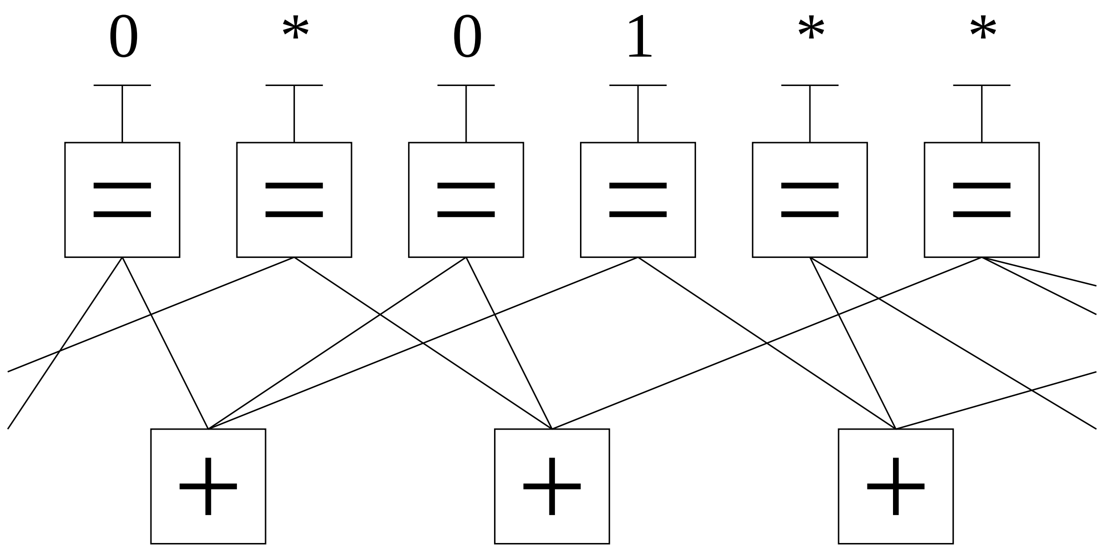

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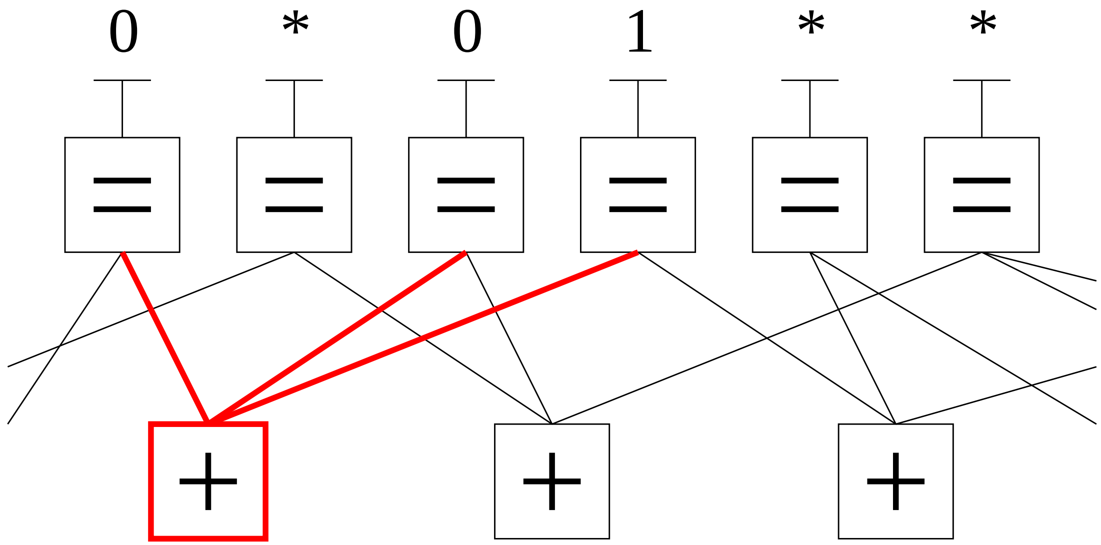

Problems with LDPC Quantizers:

Theorem: LDPC quantizers fail if parity check density $< o(\log n)$.
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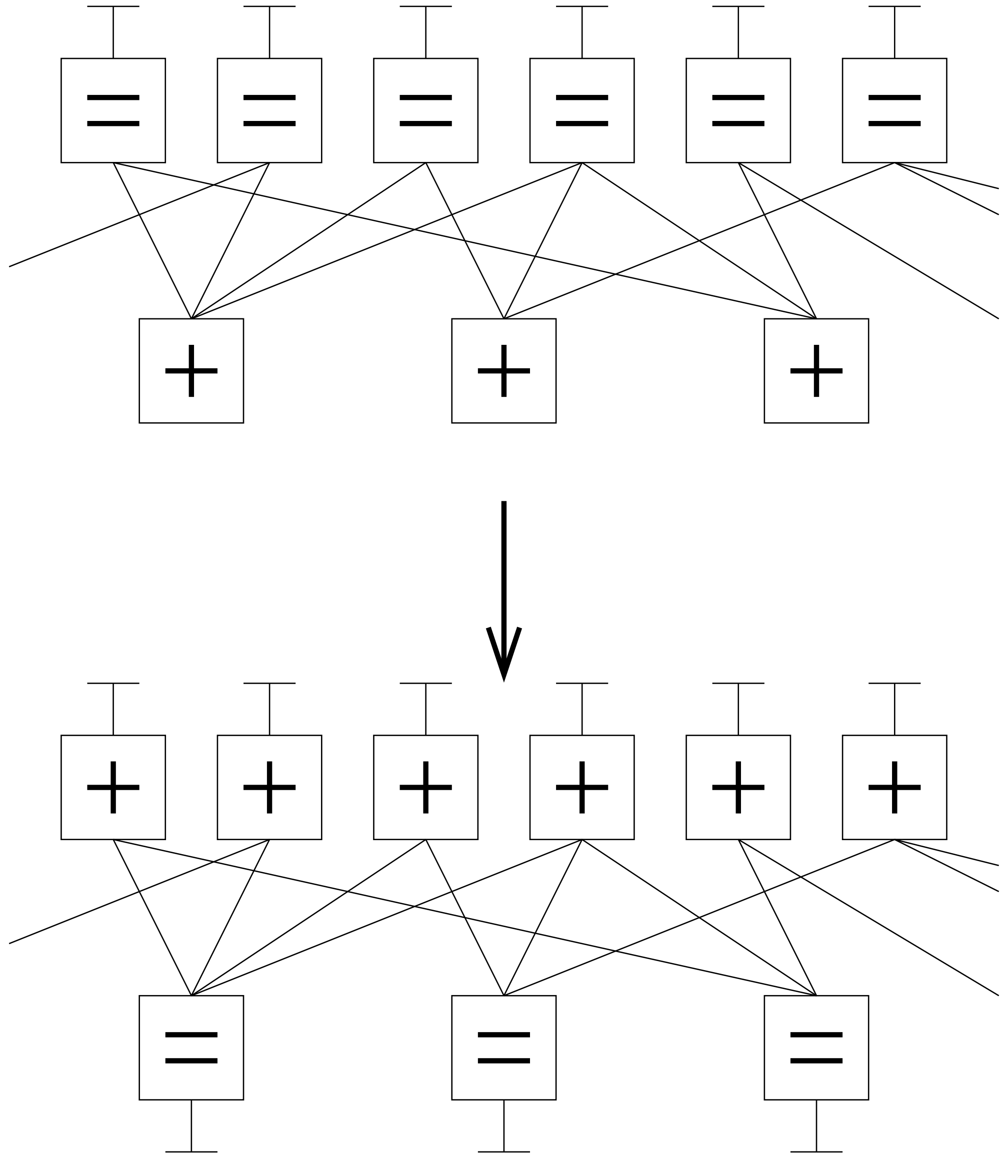
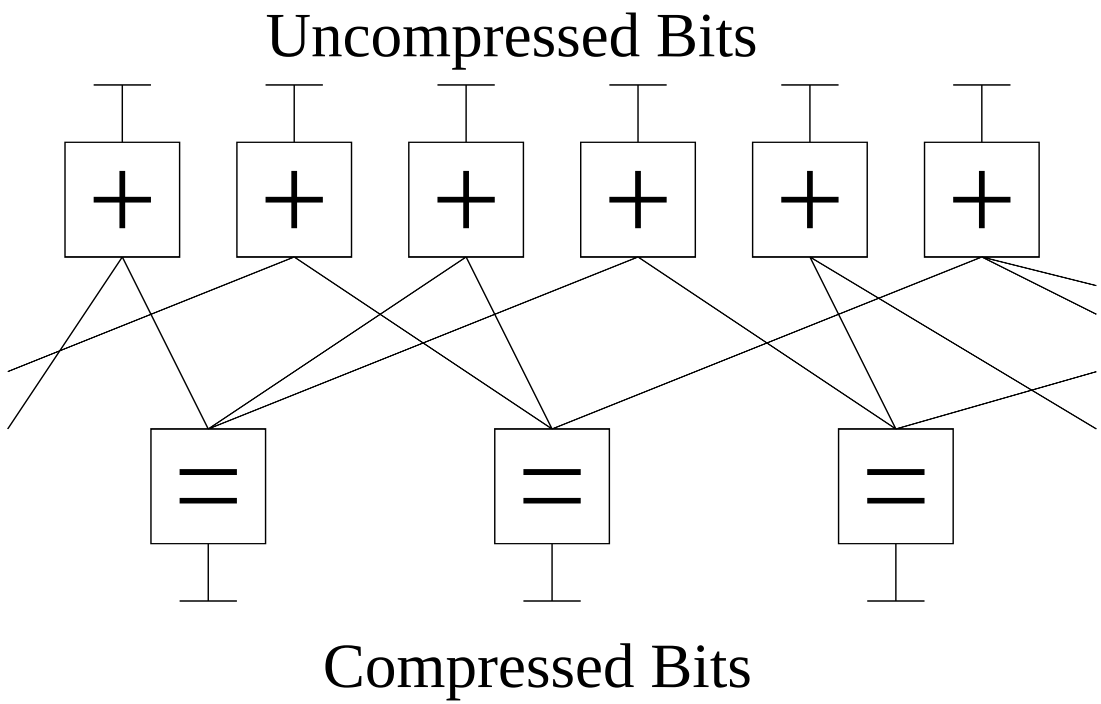
- Successful quantization requires $d \propto \log n$



Dual LDPC Quantizer



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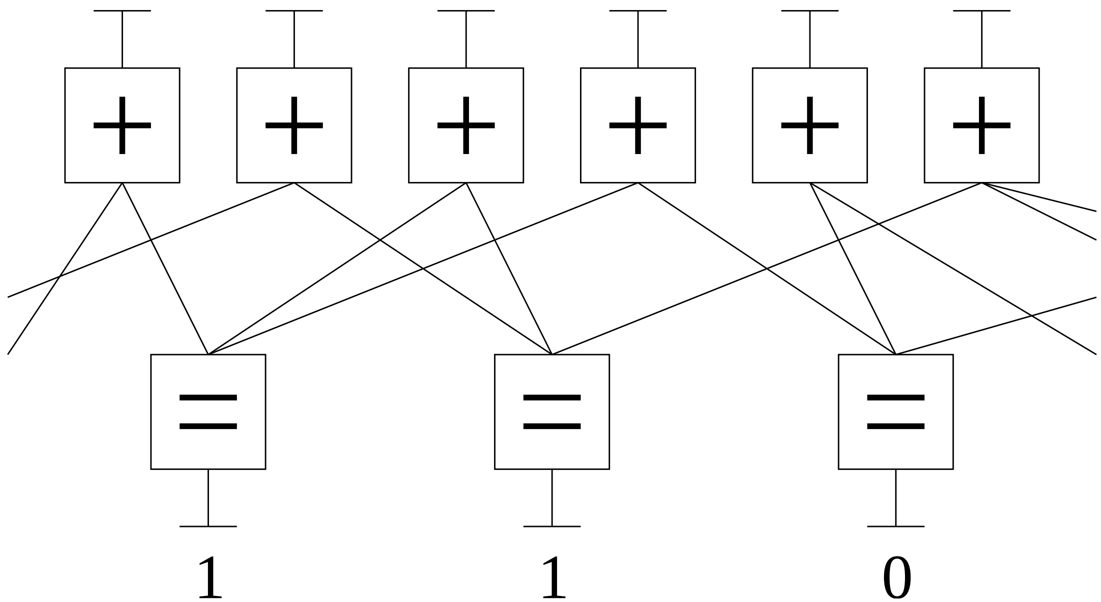

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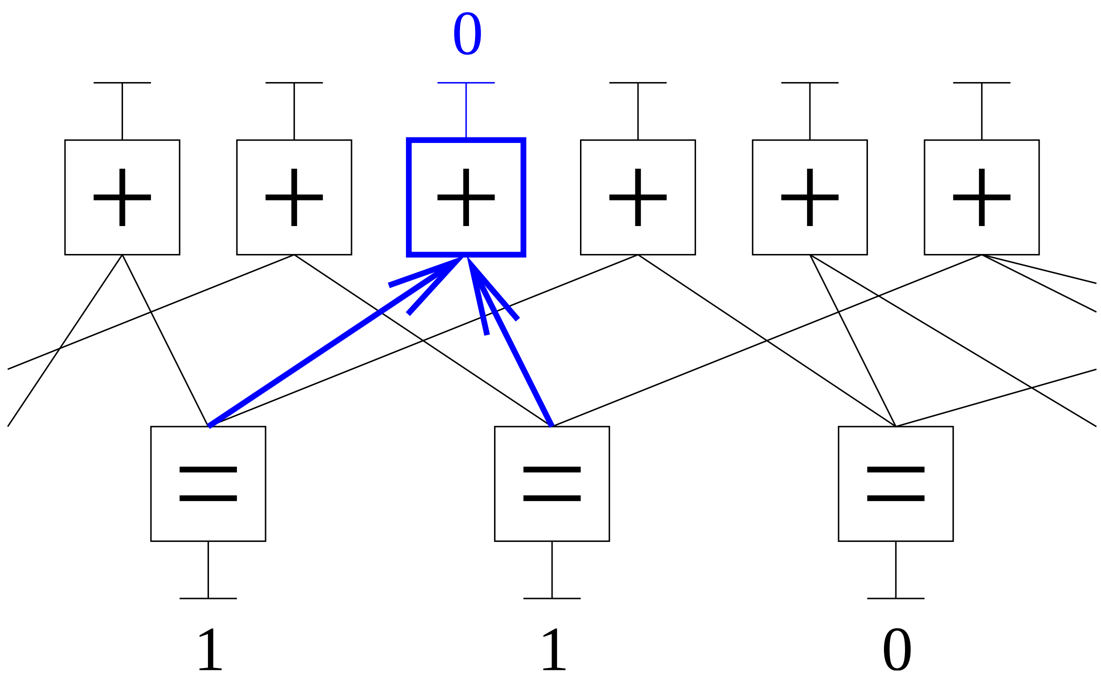

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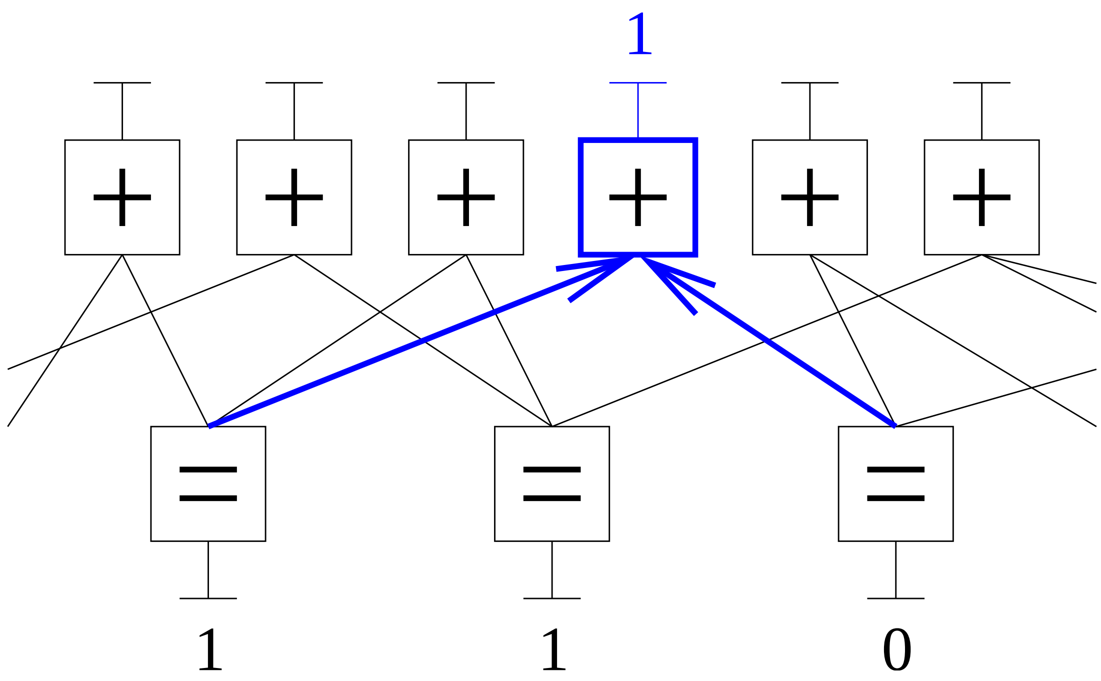

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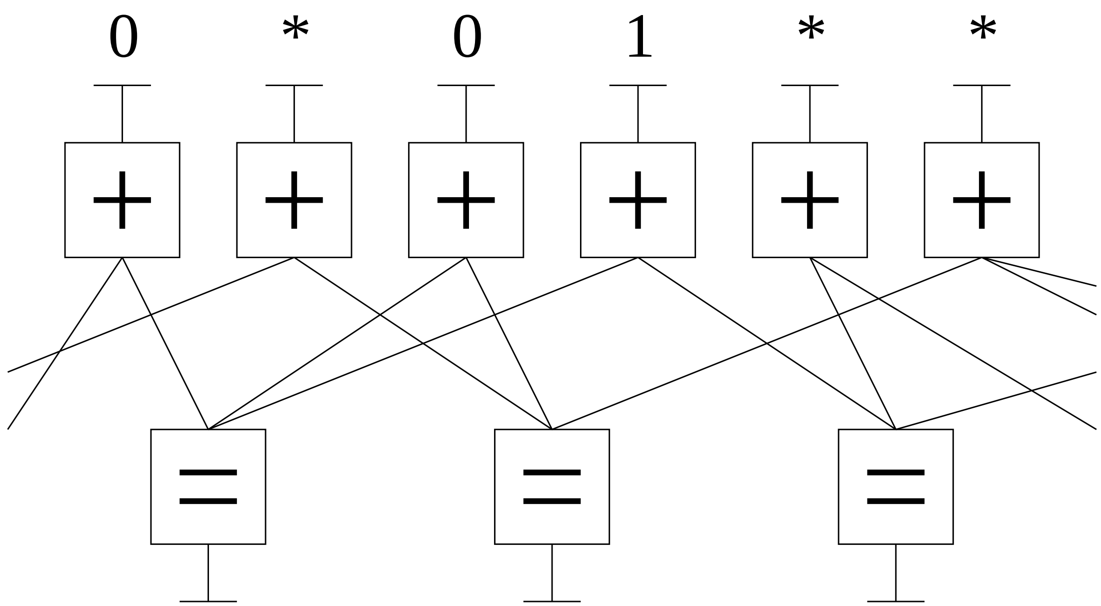

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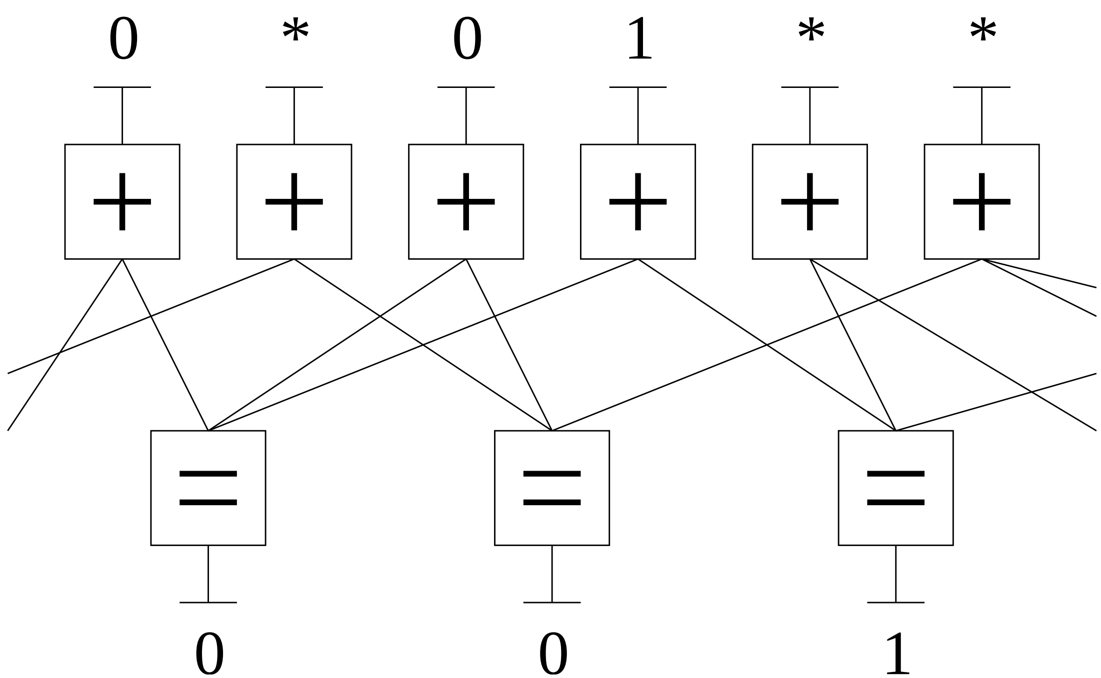
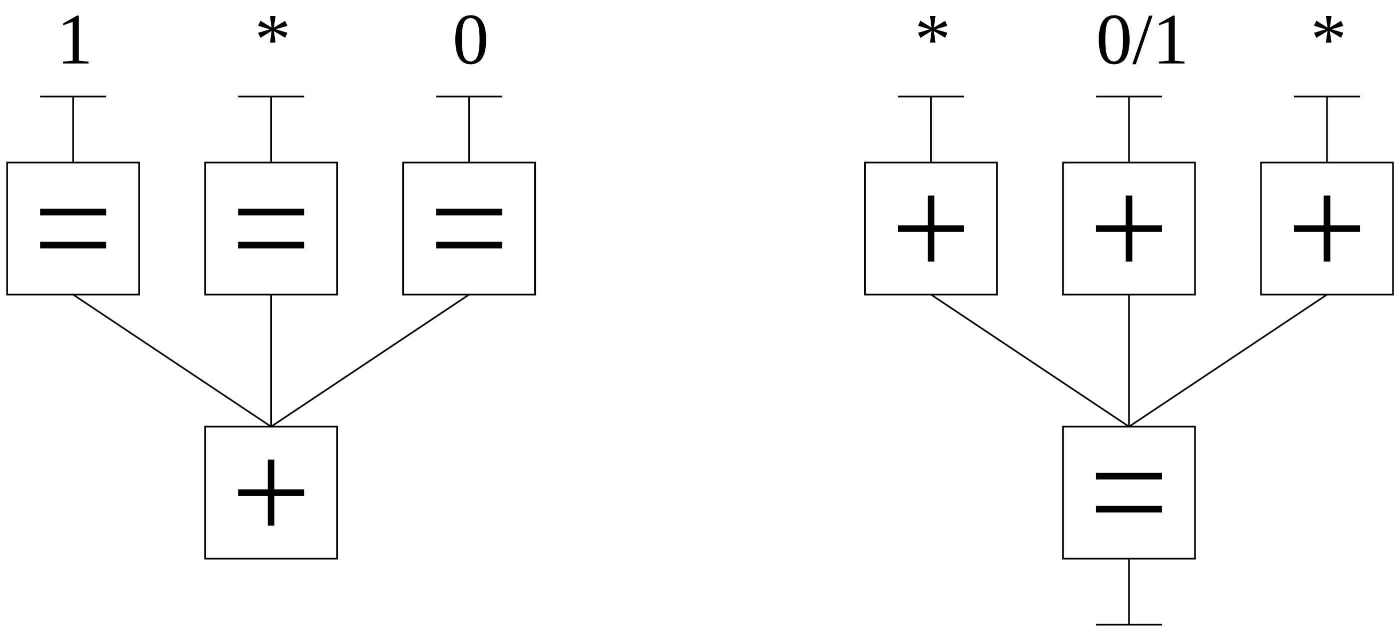


Optimal Decoding/Quantization Duality:

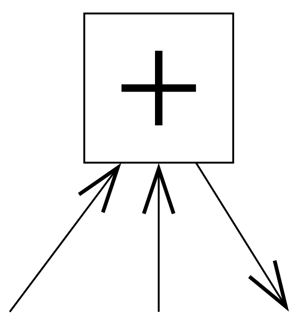


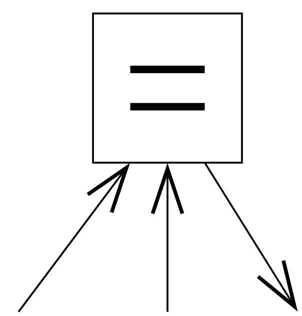
Any data encoded with $\mathcal{C}$ and received with erasures $\vec{e}$ can be decoded $\Leftrightarrow$ any source with erasures $\vec{e}^{\perp}$ can be quantized by $\mathcal{C}^{\perp}$.

Message-Passing Rules for Erasure Decoding:

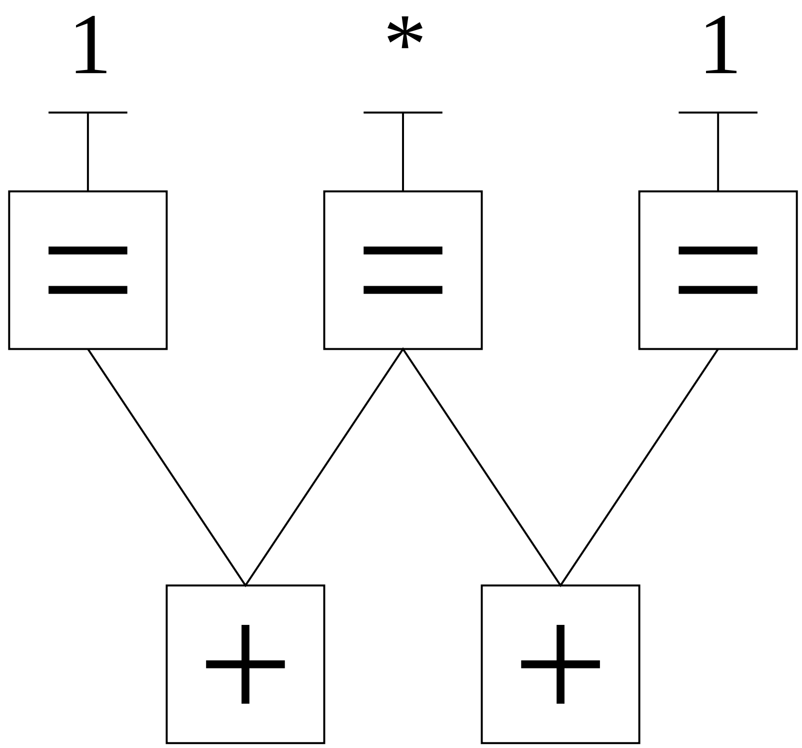
+	0	1	*
0	0	1	*
1	1	0	*
*	*	*	*

=	0	1	*
0	0	#	0
1	#	1	1
*	0	1	*





Erasure Decoding For LDPC Code:



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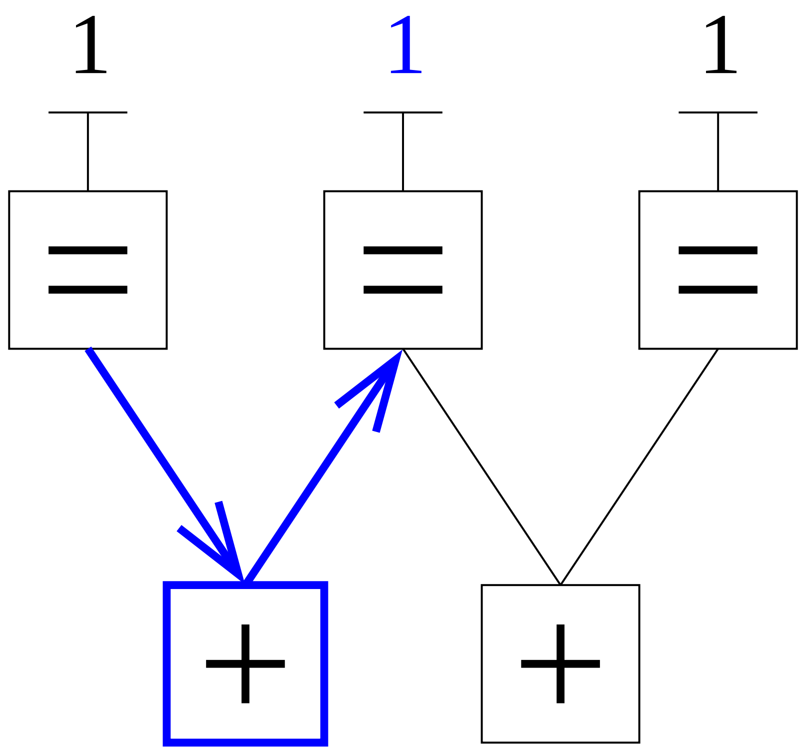
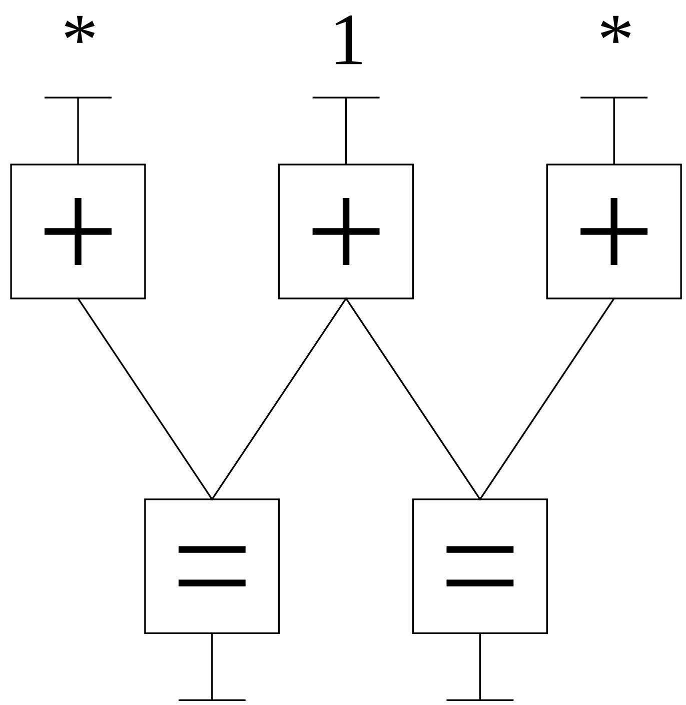


Erasure Quantization For Dual LDPC Code:



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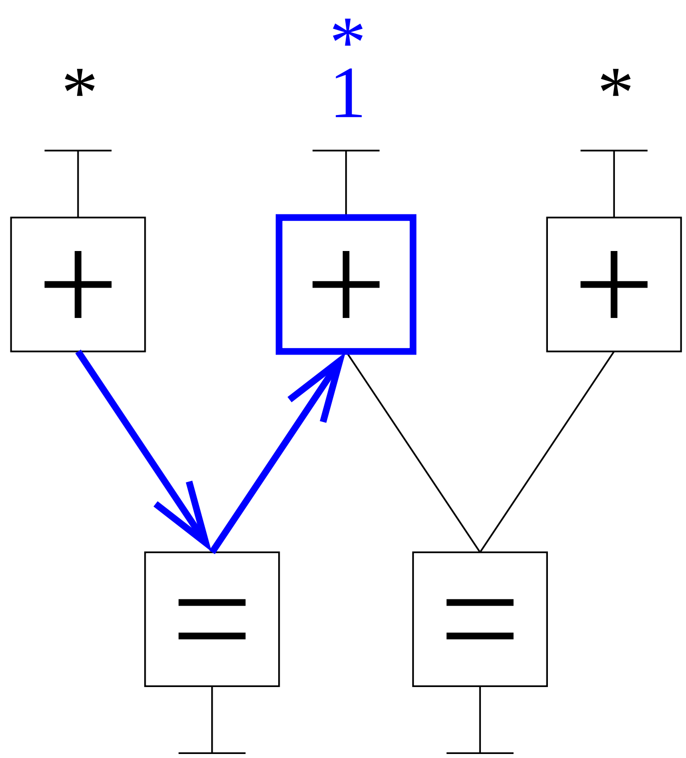

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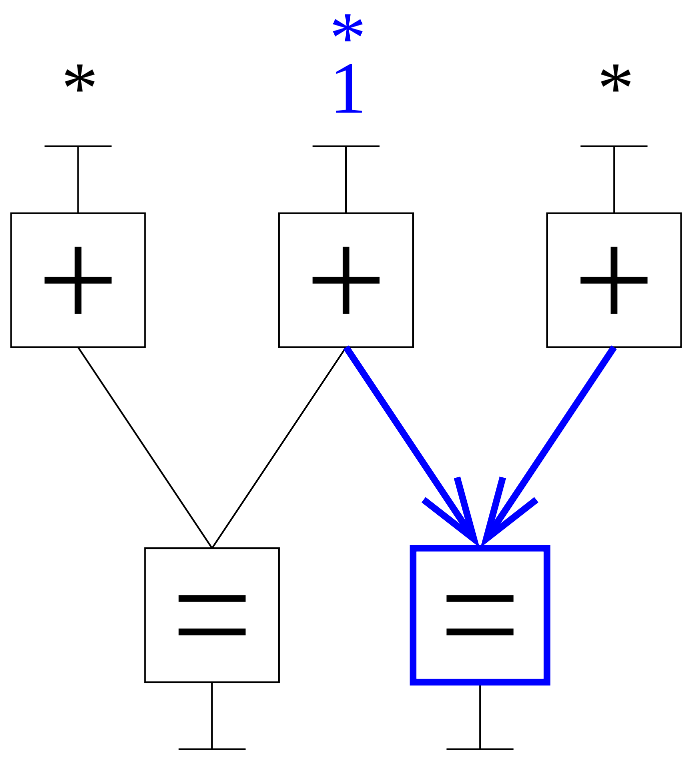

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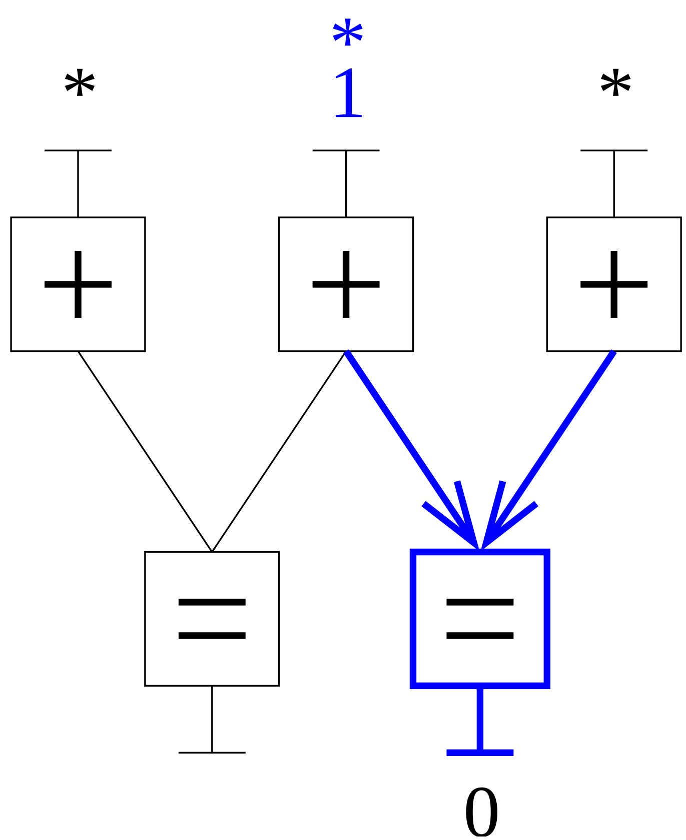

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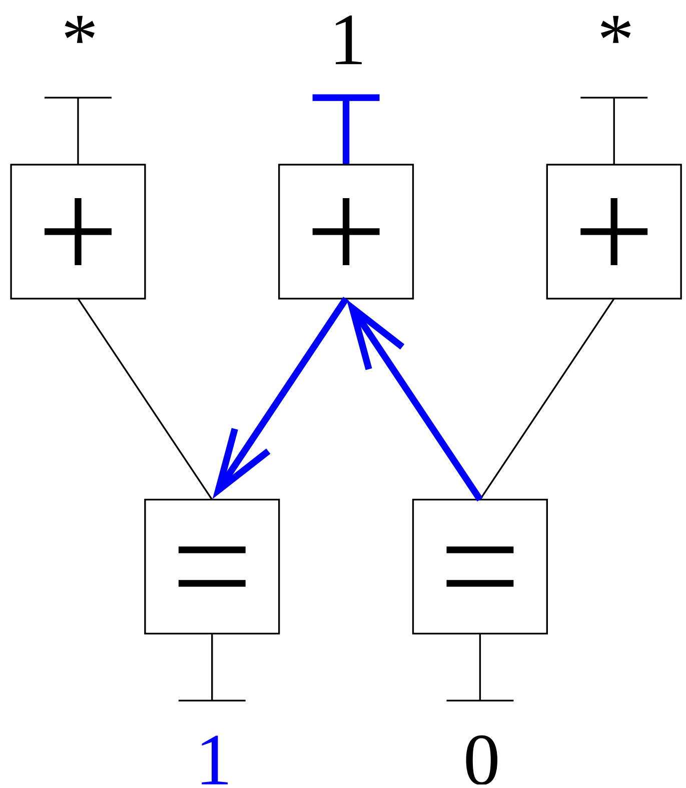
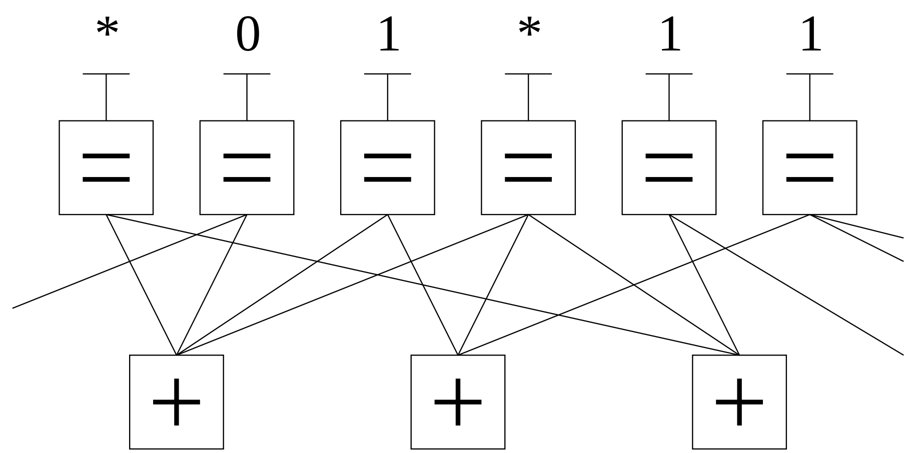


Erasure Decoding vs. Erasure Quantization:

+	0	1	*
0	0	1	*
1	1	0	*
*	*	*	*

=	0	1	*
0	0	#	0
1	#	1	1
*	0	1	*



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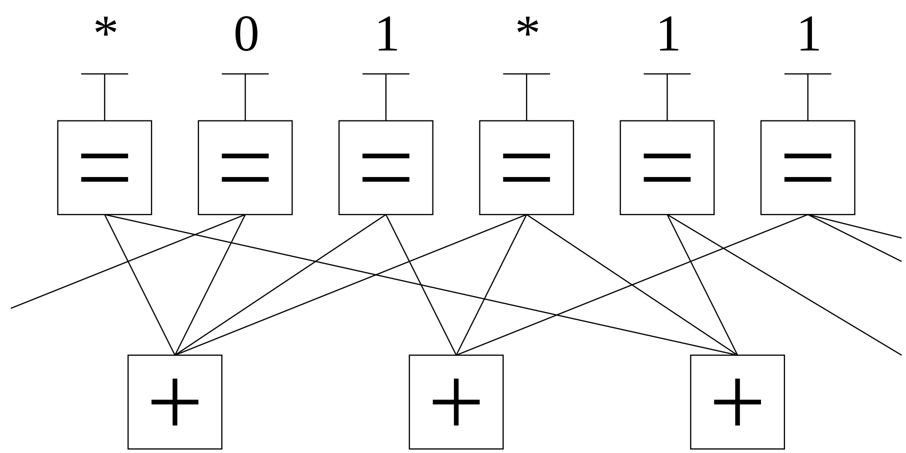

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0	0	#	0
1	#	1	1
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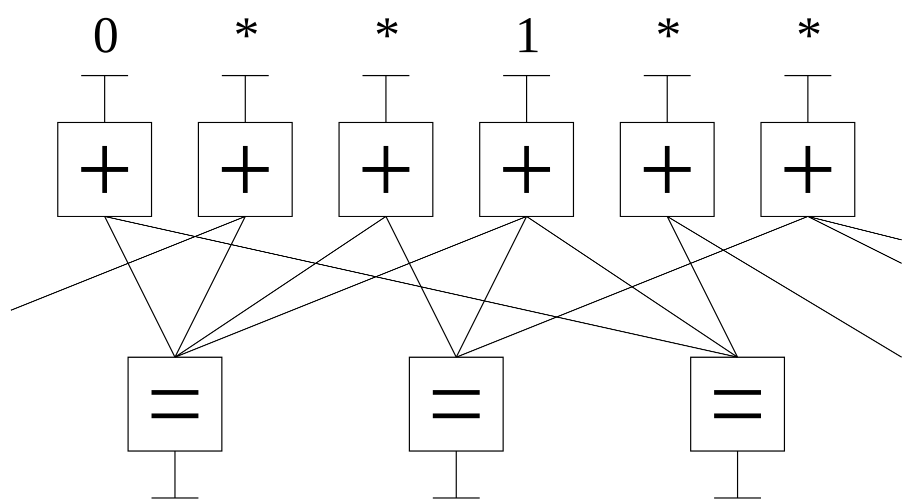

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*	*	*	*	*
∅	∅	∅	*	∅

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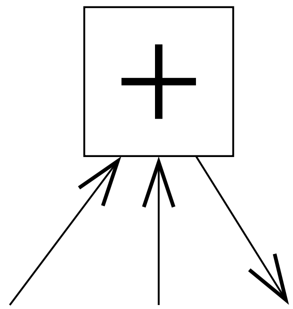

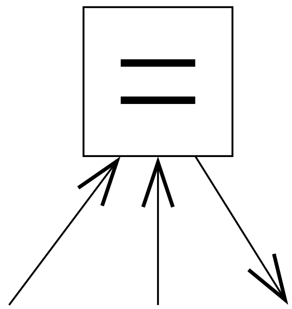

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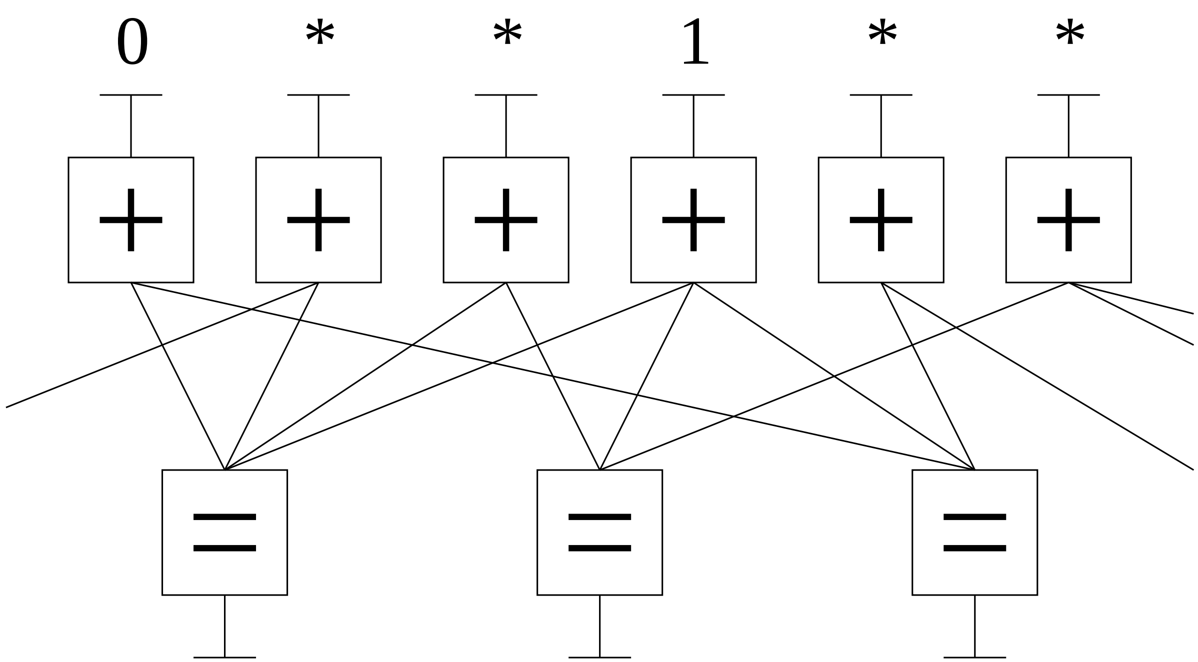

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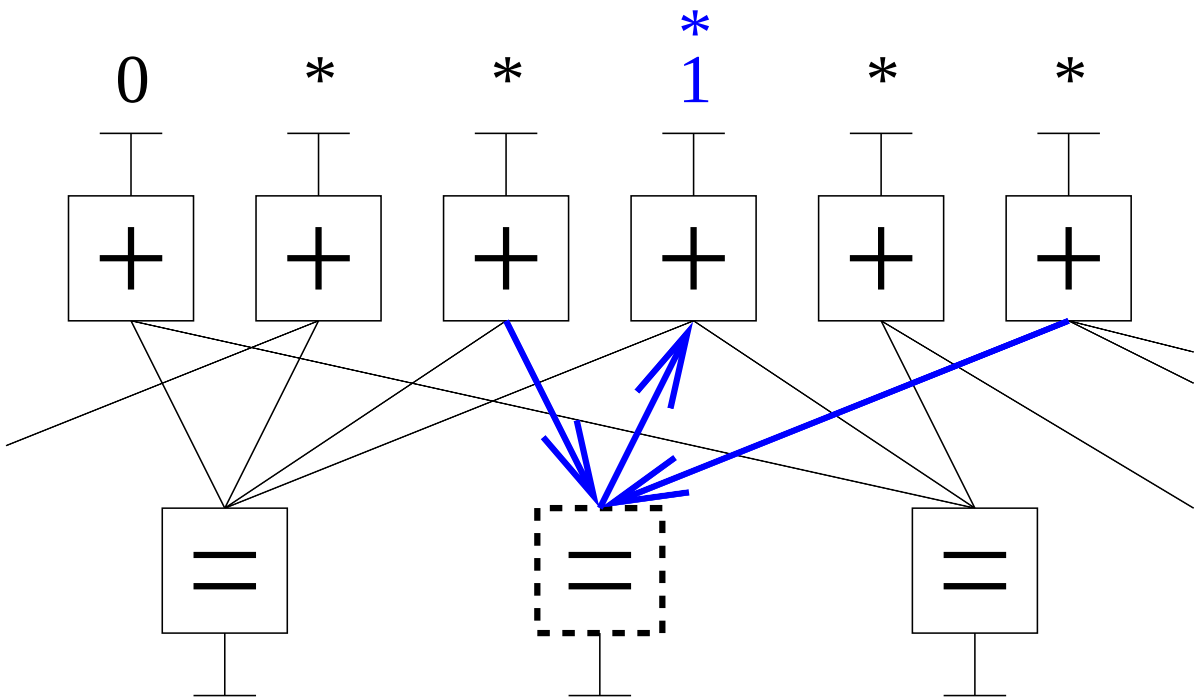

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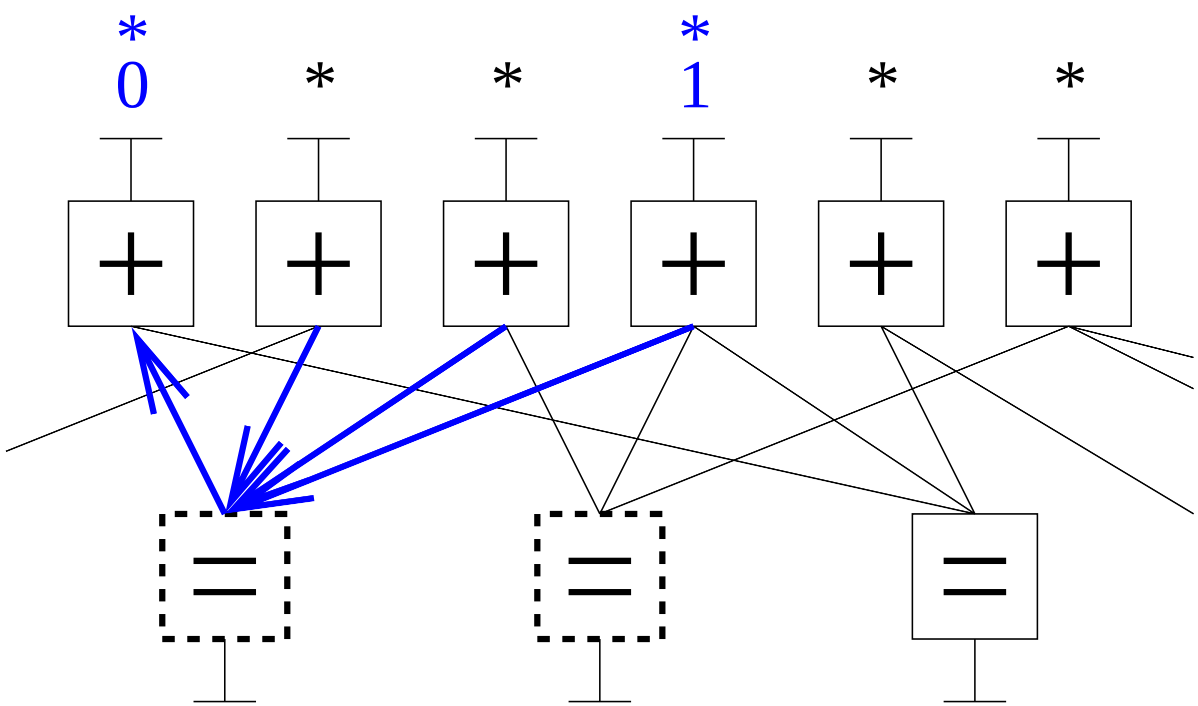

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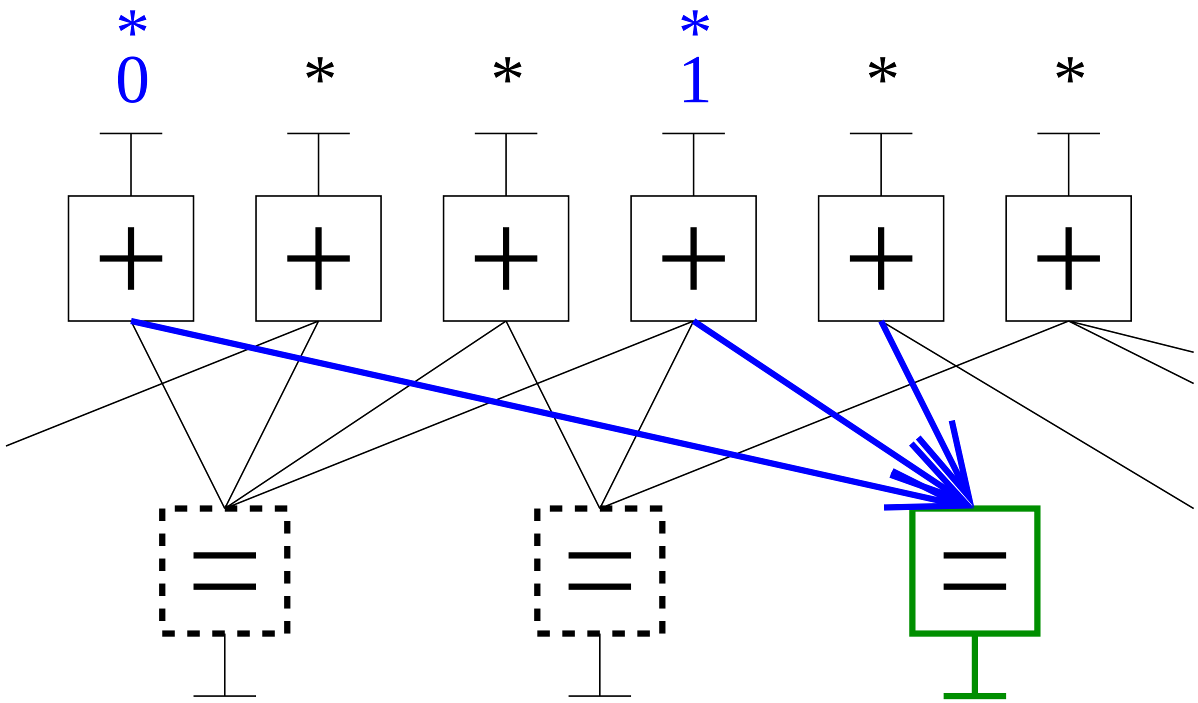

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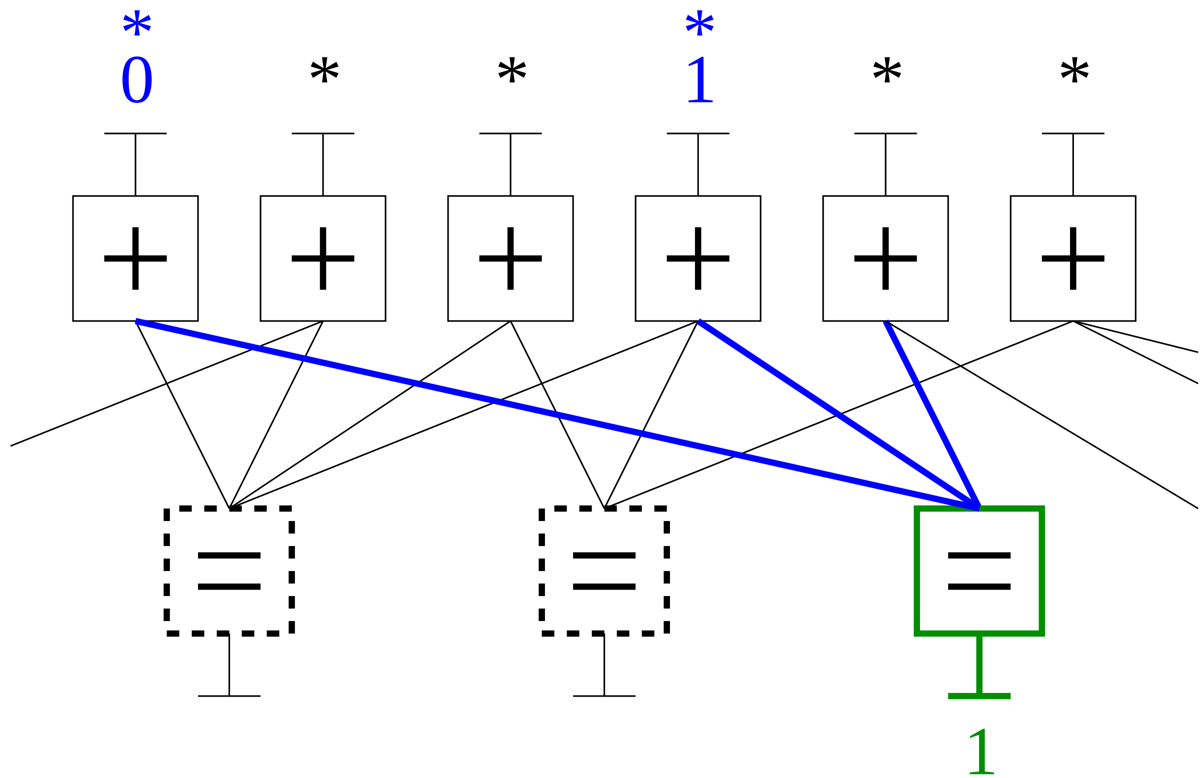

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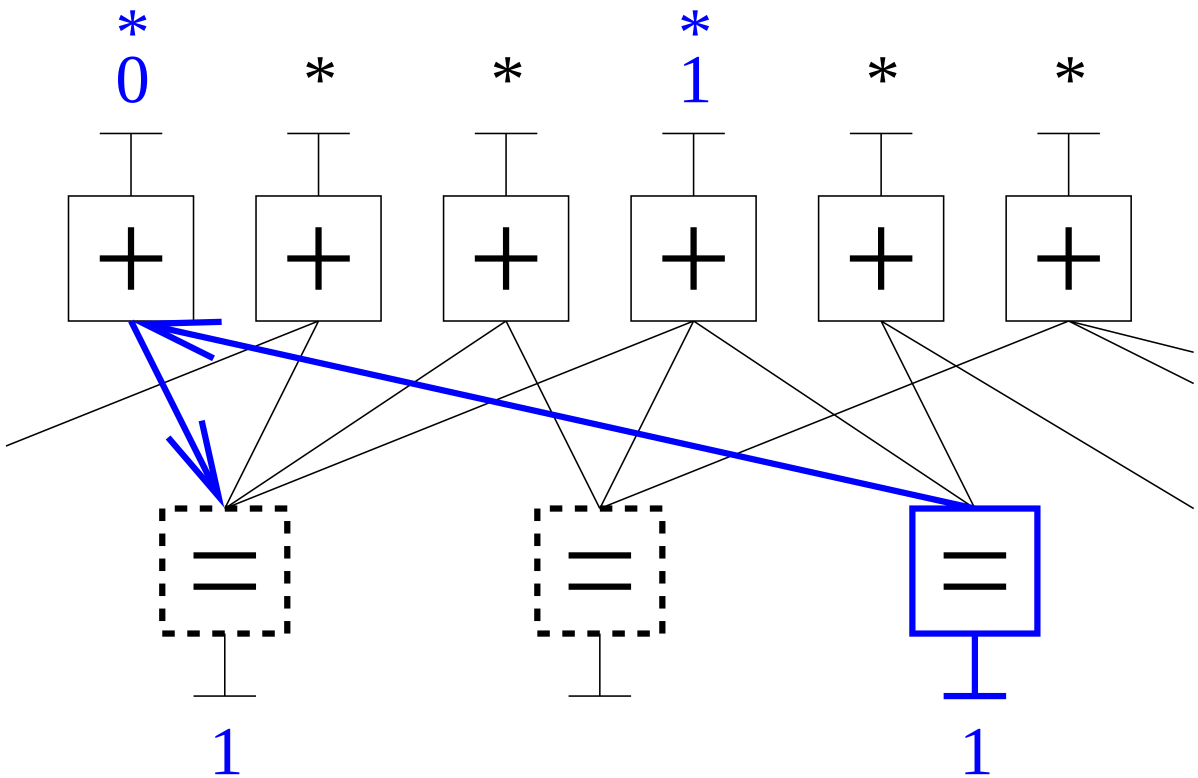

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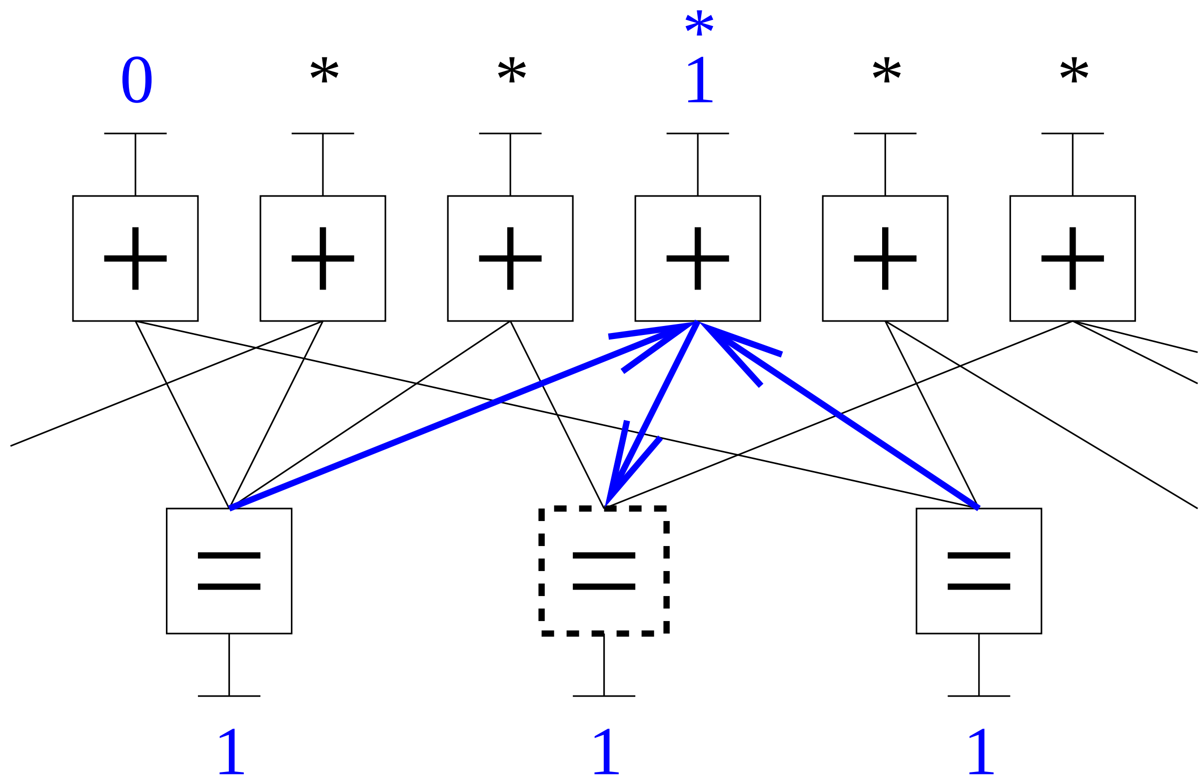

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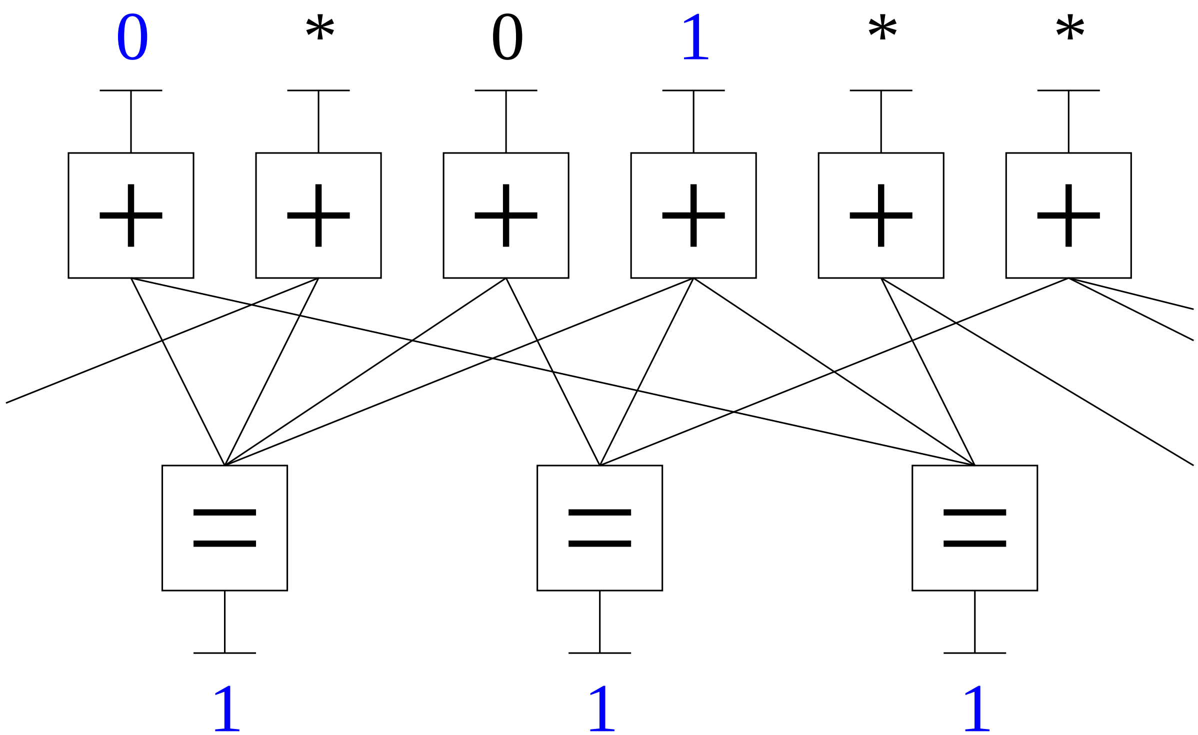

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Corollary: Duals of capacity achieving codes for erasure decoding, achieve the minimum rate for erasure quantization.

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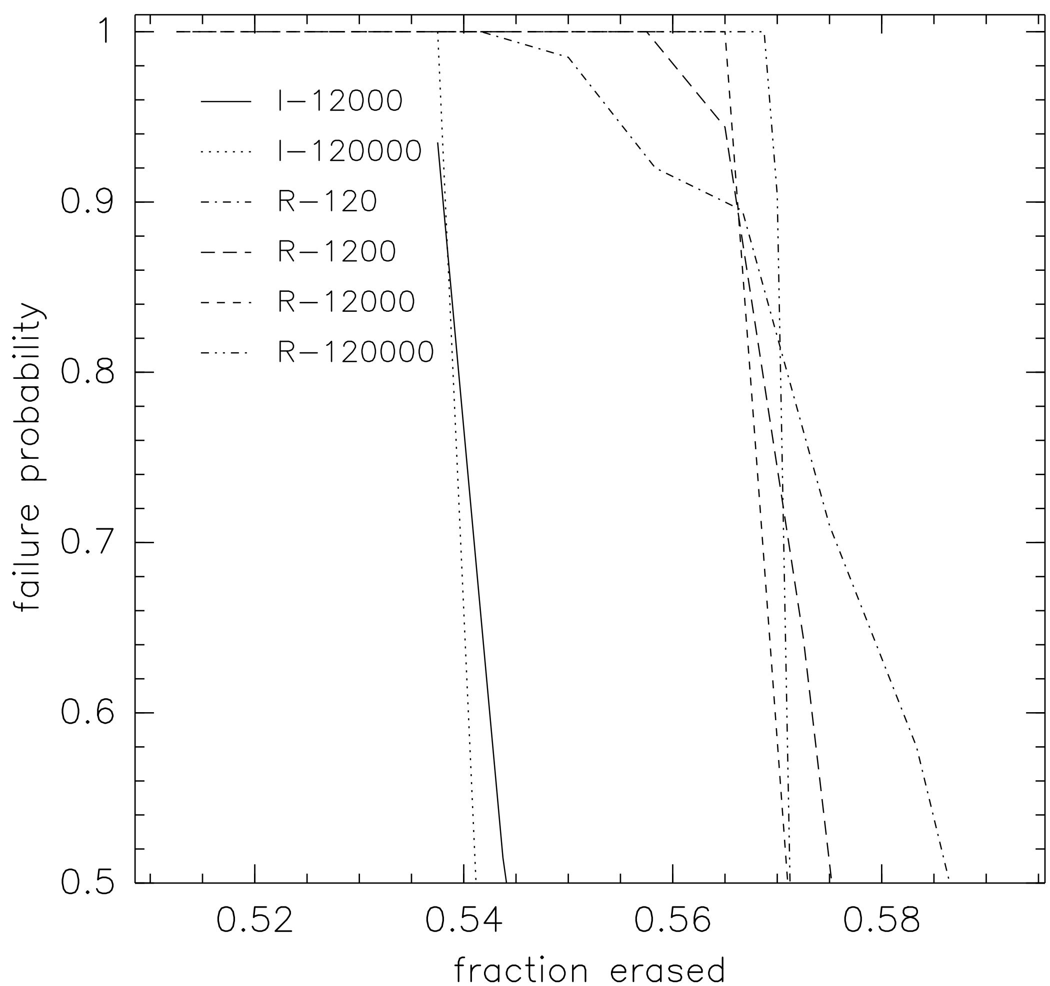

Concluding Remarks:

- LDPC codes = bad quantizers; dual codes = good quantizers.
- Duality between optimal/iterative decoding/quantization.
- Future work: extend analysis/algorithms/duality to all source models and distortion measures.